Evaluating the Decomposition of Tropical Atlantic Drifter Observations

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ABSTRACT

Because the tropical Atlantic is characterized by regions of strong seasonal variability that have been sampled inhomogeneously by surface drifters, Eulerian averages of these Lagrangian observations in spatially fixed bins may be aliased. In the Pacific, this problem has been circumvented by first calculating seasonal or monthly means. In the Atlantic, such an approach is of limited value because of the relatively sparse database of drifter observations. As an alternative, a methodology is developed in which drifter-observed currents and sea surface temperatures are grouped into bins and, within each bin, simultaneously decomposed into a time-mean, annual and semiannual harmonics, and an eddy residual with a nonzero integral time scale.

The methodology is evaluated using a temporally homogeneous SST product and in situ SST observations, and also using simulated drifter observations in an eddy-resolving model of the Atlantic Ocean. These analyses show that, compared to simple bin averaging, the decomposition developed herein yields significantly improved estimates of time-mean values in regions of strong seasonal variability. The methodology can also successfully estimate the distribution of the seasonal harmonics' amplitude and phase throughout much of the tropical Atlantic.

1. Introduction

The tropical Atlantic plays a crucial role in governing and modulating ocean/atmosphere variability across a broad range of time scales. Observations collected by standardized, satellite-tracked surface drifting buoys (Sybrandy and Niiler 1992; Niiler 2001) are an ideal tool to investigate near-surface processes. In the recent few years, a focused effort to seed this basin with drifters has resulted in a dramatic increase in observation density (Lumpkin and Garzoli 2005; Fig. 1). However, the combination of inhomogeneous sampling and strong seasonal variations demands careful consideration when analyzing these data.

At basin scales, drifter observations in the Atlantic are distributed quite homogeneously through the seasons (Fratantoni 2001). However, at the smaller scales needed to resolve major tropical currents, drifter observation density can vary widely from month to month (Lumpkin 2003). A variety of reasons can be blamed: seasonal variations in the strength of surface divergence, deployments from research vessels making repeat cruises in a particular month, or simple random chance in poorly sampled regions. Because tropical currents display very strong seasonal variability, these inhomogeneous observations must be approached cautiously if one intends to calculate time-mean values from them because the potential for bias is obvious. To address this problem in the tropical Pacific Ocean, where drifter observations are far denser, researchers have typically calculated mean properties for particular months or seasons (cf. Bauer et al. 2002). Unfortunately, despite recent efforts, the dataset of tropical Atlantic drifter observations remains far sparser—monthly or seasonal maps of currents would contain large regional gaps. These could be ameliorated by heavy spatial smoothing, but this approach would sacrifice resolving the spatial structure of time-mean cur-

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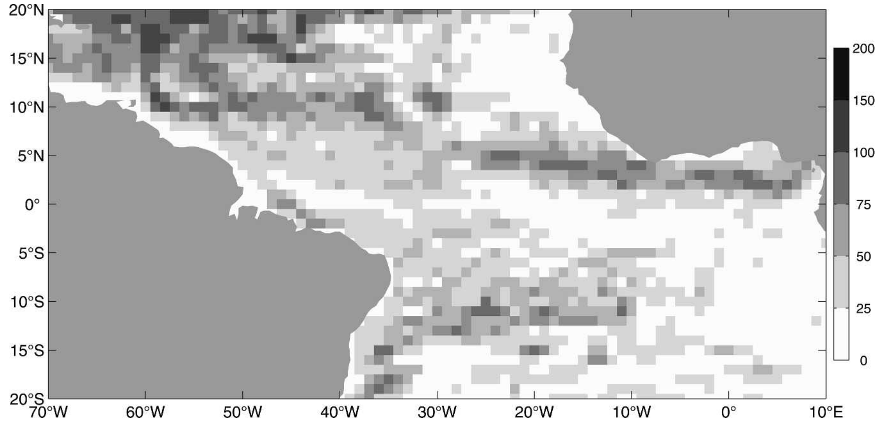

FIG. 1. Density of surface drifter observations of velocity (drifter days per square degree) in the tropical Atlantic Ocean for the period 1 Sep 1997–31 Aug 2003. The density of SST observations is similar. Adapted from Lumpkin and Garzoli (2005).

rents and their variability. Alternatively, one could fill the gaps in monthly or seasonal mean maps by a posteriori temporal interpolation. For example, if the mean currents in a region have been determined for most climatological months, and these values are sufficient to resolve the annual and semiannual variations that dominate tropical Atlantic variability (Philander and Pacanowski 1986; Richardson and Walsh 1986), this information is sufficient to estimate mean currents for unsampled months.

As an alternative to calculating regional means and then interpolating (spatially or temporally) to fill gaps, one could simultaneously decompose regional Lagrangian time series into components including a time-mean value and seasonal variations, using a tapered, weighted least squares approach that provides explicit error bars quantifying the success of the decomposition. This approach was recently introduced for these data by Lumpkin (2003), who showed that application of this methodology to the Atlantic drifter observations produced significantly different annual mean values than simple averaging in regions with strong seasonal variations that had been sampled inhomogeneously through the seasons. In this paper, the methodology is rigorously evaluated for the first time. This evaluation is done for sea surface temperature in the real ocean, where satellite-derived products can be used to quantify which method is superior, and for the velocities of simulated drifters in a numerical model for which the gridded output gives the true time-mean fields. This methodology is applied to the dataset of tropical Atlantic drifters in Lumpkin and Garzoli (2005), which does not include an explicit evaluation of the methodology. In this paper we examine drifter observations in the tropical Atlantic Ocean (20°S–20°N, 70°W–15°E) for the period September 1997 to August 2003. A few drifter observations were made in this region in the period October 1990–September 1997, but more than 95% of the total dataset was collected during the period considered here.

2. The decomposition

To derive pseudo-Eulerian estimates from the Lagrangian observations, drifter observations are grouped in 2° × 1° spatial bins, rotated so that they are aligned with the major axis of velocity variance (see Fig. 5 of Lumpkin and Garzoli 2005). This rotation tends to orient the bins along the major currents, rather than across them, so that more observations can be averaged in each bin without the sacrifice in resolution that 2° square bins would entail. Within each bin, the observations are treated as a time series composed of a time-mean value, cyclical components, and residual $n(t)$ according to

$$y(t) = x_0 + \sum_{i=1}^{2}\left[x_i^s \sin\left(\frac{2\pi t}{P_i}\right) + x_i^c \cos\left(\frac{2\pi t}{P_i}\right)\right] + n(t) \tag{1}$$

(Lumpkin 2003), where P_i is the period of the cyclical components, chosen as one year and one-half year to fit annual and semiannual fluctuations. Although seasonal variability includes other frequencies, such as interannual amplitude and phase modulation, much of the seasonal variance (~70% for velocity; Richardson and Walsh 1986) can be accounted for by these two harmonics. Including these sinusoids explicitly in the decomposition avoids biasing the means where the seasonal cycle is strong and the observations are inhomogeneously distributed through the seasons.

In matrix formulation, (1) can be written

$$\mathbf{Y} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{2}$$

where matrix $\mathbf{A}$ contains the coefficients for the unknowns x_o, x^c, and x^s contained in vector $\mathbf{x}$. This system may be solved by tapered, weighted least squares techniques to yield

$$\tilde{\mathbf{x}} = \mathbf{R}_{xx}\mathbf{A}^{\mathrm{T}}(\mathbf{A}\mathbf{R}_{xx}\mathbf{A}^{\mathrm{T}} + \mathbf{R}_{nn})^{-1}\mathbf{y} \tag{3}$$

(this equation and its derivation can be found in Wunsch 1996), where $\tilde{\mathbf{x}}$ is the Gauss–Markov estimator for $\mathbf{x}$, $\mathbf{R}_{xx}$ is the covariance matrix of the unknowns, and $\mathbf{R}_{nn}$ describes the variance structure of the eddy signal $n(t)$. This methodology, common in the inverse model literature, produces posterior estimates of the covariance structures of $\mathbf{x}$ and $\mathbf{n}$, allowing generation of formal statistical error bars for the amplitudes x and maps of the resolution of $y(t)$ into these amplitudes.

For this study, diagonal elements of $\mathbf{R}_{xx}$ were assumed to be equal to the squared range (the square of the difference between the maximum and minimum observation) of $y(t)$ and off-diagonal elements were set to zero. These choices are consistent with any of the unknown coefficients either accounting for the full range of observed variability (for the seasonal fluctuations), or falling within the mean plus standard deviation of the observations (for the unbiased time-mean value), and the assumption that the coefficients are independent. Elements of $\mathbf{R}_{nn}$ were given by

$$R(\tau) = \mathrm{var}(y)\cos(\pi\tau/2T_d)\exp[-(\pi\tau/2\sqrt{2}T_d)^2] \tag{4}$$

(Garraffo et al. 2001; Lumpkin et al. 2002) where var(y) is the variance of y and $T_d = 10.33$ days, equivalent to an integral eddy time scale of five days (a typical choice for the Lagrangian time scale; cf. Fratantoni 2001). This structure for $\mathbf{R}_{nn}$ is consistent with the assumption that all of the observed variance may be due to noise, and that observations separated by less than five days are not statistically independent.

Including the seasonal harmonics explicitly in the decomposition avoids biasing the means where the seasonal cycle is strong and the observations are inhomogeneously distributed through the seasons. Figure 2 shows two examples where this effect is severe.

The bin centered on 7°N, 47°W is located in the southeastward-flowing North Brazil Current (NBC) retroflection. Within this bin, 128 drifter days were collected by 26 different drifters from late 1997 to late 2002. The binned-mean zonal speed is 17 cm s^{-1}, with a standard deviation of 37 cm s^{-1}. If observations are treated as independent over an integral time scale of five days, the resulting standard error for the mean estimate is $37/(128/5)^{1/2} = 7$ cm s^{-1}. However, observations in this bin are quite inhomogeneous with respect to seasonal variations: over 60% were made between March and June (fractional year 0.15–0.5). Applying the tapered, weighted least squares (LS) decomposition (cf. Wunsch 1996) as described above, one obtains a much larger mean: 44 ± 7 cm s^{-1}. This estimate is larger than the simple binned mean because a disproportionally large fraction of the observations are on or near the annual cycle's minimum, associated with the boreal spring weakening of the NBC retroflection and reversal of the western North Equatorial Countercurrent (NECC; Garzoli and Katz 1983; Richardson and Walsh 1986). The reader may be alarmed to note that this estimate differs from the simple binned mean by four standard errors. We note that the standard error estimate assumes that all observations are equally independent, and that variations about the mean are because of white noise. (By using an integral time scale of five days to reduce the number of degrees of freedom, noise that is correlated over this duration is acknowledged implicitly.) The presence of lower frequency variability, such as seasonality, violates this assumption. Similar situations can be found in some bins for mean SST, such as at 10°N, 27°W (Fig. 2) where winter-biased observations yield a binned mean SST of 24.3° ± 0.4°C, while the simultaneous fit of a mean, annual and semiannual components yields a mean of 25.9° ± 0.6°C. For comparison, the National Oceanic and Atmospheric Administration (NOAA) optimum interpolation (OI) version 2 SST product (Reynolds et al. 2002) at this location has a mean value of 26.6°C, with winter minima less cold than suggested by the drifter observations.

3. Evaluating the decomposition

Does the tapered, weighted least squares (LS) decomposition produce better estimates of mean currents and SST than simple bin averaging? How successfully does the decomposition extract annual and semiannual amplitudes, given the relatively sparse density of drifter observations in much of the tropical Atlantic Ocean?

These questions can be readily addressed for the decomposition of SST observations. A variety of products and in situ observations offer temporally homogeneous SST time series in the tropical Atlantic. These can be separated into mean, annual, and semiannual ampli-

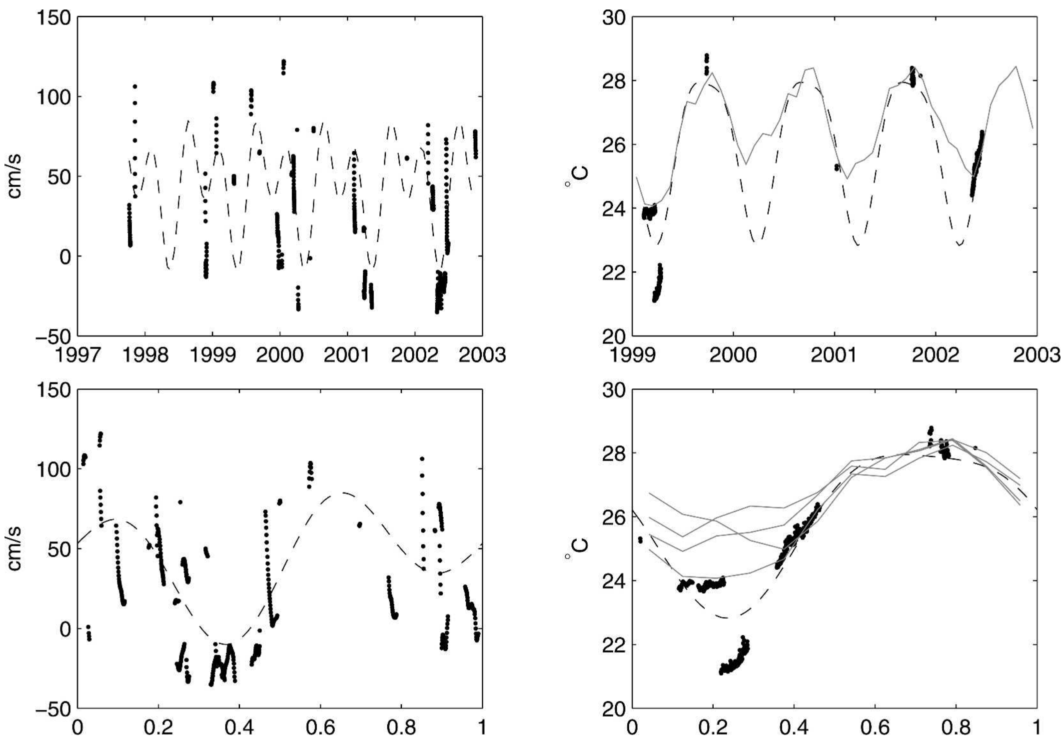

FIG. 2. (left) Zonal drifter speeds in the 2° zonal by 1° meridional bin centered on 7°N, 40°W, as a function (top) of year and (bottom) of fraction of year (0 = 1 Jan, 0.5 = 30 Jun). The least squares fit of a mean, annual and semiannual harmonics, and eddy residual is shown (dashed). (right) SST observed by drifters in the bin centered on 10°N, 27°W (points), a least squares fit to these observations (dashed), and coincident monthly SST from the NOAA OI.v2 product (gray line; R02).

tudes that can be compared to results of the decomposition applied to the drifter SST observations.

Evaluating the decomposition of velocity is more difficult—there exists no comparable high resolution, homogeneous, databased climatology of tropical Atlantic currents. At lower resolution, ship drifts have been used to generate a climatology of monthly surface currents (Richardson and Walsh 1986), but discrepancies between ship drifts and drifter speeds could be attributed to differences in spatiotemporal sampling and to intrinsic differences of the platforms. These discrepancies are described in Lumpkin and Garzoli (2005). As an alternative, the methodology can be tested by applying it to simulated drifter observations in a high resolution model of the Atlantic, and comparing results to the "true" annual mean surface currents and their seasonal variations, which can be calculated directly from the homogeneous Eulerian, gridded model output. This evaluation is conducted here. The goal of this effort is not to evaluate the realism of the model, although a model with reasonable eddy kinetic energy levels is required since mesoscale variability is the dominant noise in Lagrangian velocity time series, but rather to determine whether the LS decomposition yields better estimates of the mean amplitudes, and their associated error bars, than simple binned averages and standard error bars.

a. *Evaluating the decomposition of tropical Atlantic SST*

The NOAA optimum interpolation, Version 2 SST product (Reynolds et al. 2002, hereafter R02), September 1997 to August 2003, is used to evaluate the decomposition of relatively sparse drifter SSTs. R02 SST is derived primarily from satellite observations, adjusted to match in situ observations (including drifters) where they are coincident (Reynolds and Smith 1994). This product is an ideal tool for evaluating the methodology, as it is derived from a far denser set of observations, yet should not contain large-scale biases with respect to the drifter observations. While a truly independent product would be preferable for evaluating the SST observations themselves, such an evaluation is not the goal here.

Large-scale features of the time-mean R02 SST are consistent with time-mean SST from the drifters, either calculated by the LS decomposition or by simple bin averaging (Fig. 3). The mean SST field derived by the

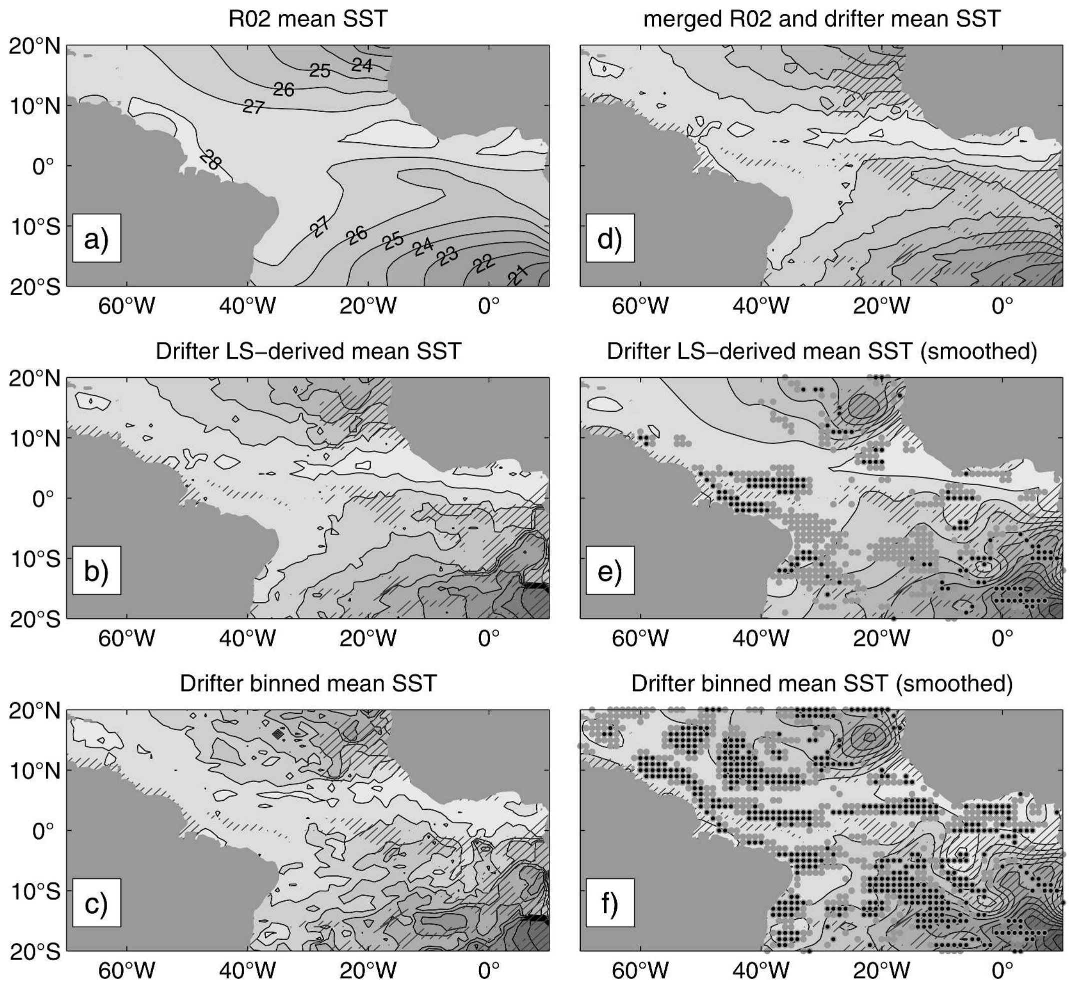

FIG. 3. Time–mean SST (°C), 1 Sep 1997–31 Aug 2003, from (a) the NOAA OI.v2 product (R02), (b) LS decomposition of the drifter observations, (c) simple binned means of the drifter observations, (d) merged R02 and drifters (see text), (e) OI-smoothed amplitudes from the LS decomposition, and (f) OI-smoothed binned means. Hatched regions indicate <25 drifter days of observations per bin. In (e) and (f), gray (black) points indicate bins for which the smoothed SST differs from R02 mean SST by more than 1 (2) standard errors.

decomposition is much smoother than the noisy-looking bin averaged field. This is true even with a single, noniterative application of the decomposition, suggesting that the LS method helps alleviate noise contamination. For bins with more than 25 drifter days of observations, the root-mean-square (rms) difference between R02 SST (Fig. 3a) and the binned mean field (Fig. 3c) is 0.68°C. The rms difference between R02 mean SST and the LS decomposition-derived field (Fig. 3b) is 0.44°C.

Both drifter-derived SST fields contain more high-wavenumber structure (consisting of both true structure and noise) than the R02 fields, which have been smoothed via OI assuming a Gaussian autocorrelation of *e*-folding scales 850 km (zonal) and 615 km (meridional; Reynolds et al. 2002). For comparison, the drifter-derived amplitudes were smoothed using the same scales (Figs. 3e,f). The rms difference between R02 and the OI-smoothed binned mean SST is 0.66°C, slightly smaller than for the unsmoothed drifter-derived field. For the smoothed version of mean SST from the LS decomposition, the rms difference with Reynolds is nearly unchanged (0.45°C). Because the decomposition produces error bars, which are sensitive to the observations being "clumped" in a single season, they are a better estimate of how well time-mean SST was determined by the observations: differences between OI-smoothed binned means and R02 mean SST were smaller than one standard error bar in only 48% of the bins, while the OI-smoothed decomposition-derived drifter means were within one error bar of the R02 mean SST values in 66% of the bins.

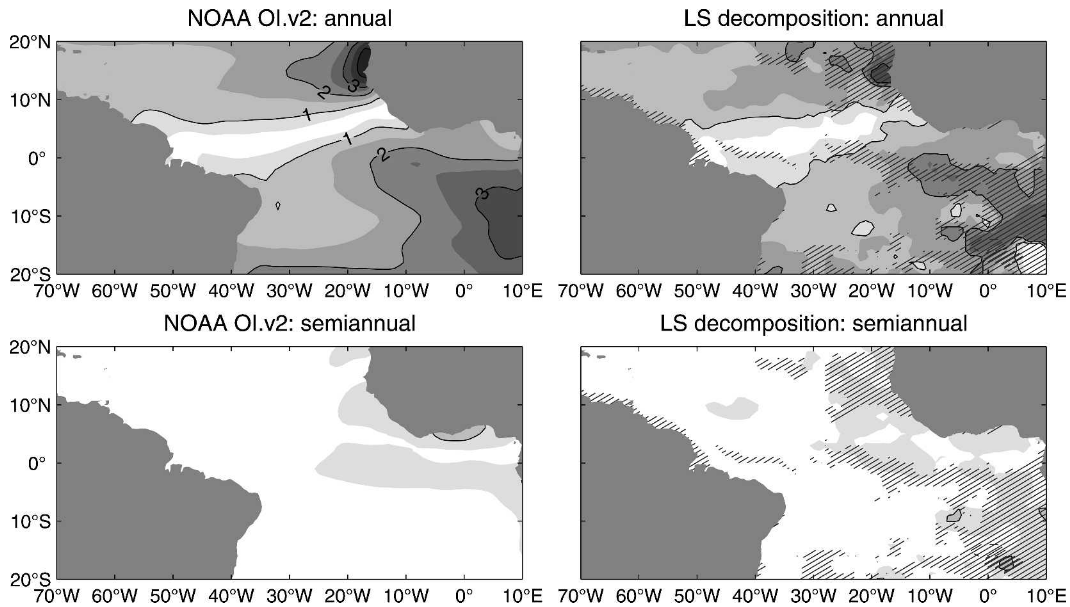

FIG. 4. Amplitude of the (top) annual and (bottom) semiannual components of SST derived (left) from NOAA OI.v2 and (right) from the LS decomposition of drifter observations. Line contours are shown for every 1°C. Hatched regions indicate <25 drifter days of observations per square degree.

In addition to deriving improved estimates of the time-mean SST, the LS decomposition provides explicit amplitudes for the annual and semiannual variations, which can be compared to amplitudes derived from a simple least squares fit of sinusoids to the R02 fields (Fig. 4).

Amplitudes derived by the decomposition are also consistent with other products, such as National Centers for Environmental Prediction (NCEP; Fig. 2 of Lumpkin 2003). Unfortunately, in the study domain the largest annual SST variations are found in the eastern subtropics, where drifter observations are too sparse to resolve them. Elsewhere, amplitudes are extremely small, with maxima of <1°C on and north of the equator. Drifter-derived semiannual SST amplitudes are small everywhere, exceeding 1°C only in a few bins.

By combining the drifter-derived time-mean and seasonal amplitudes, time series of climatological SST can be generated. Even where the observation density is relatively poor, the methodology produces a reasonable time series compared to the R02 SST product and in situ observations. For example (Fig. 5), in the equatorial bin at 23°W, decomposition of the 34 drifter days of observations yields a mean SST of 26.80° ± 0.42°C with annual/semiannual amplitudes of (1.08 ± 0.67)/(0.57 ± 0.65)°C, respectively. At this location, a subsurface mooring, part of the Pilot Research Moored Array in the Tropical Atlantic (PIRATA), measured SST with a mean of 26.61°C and annual/semiannual amplitudes of 1.50°/0.67°C. Collocated R02 SST has a mean SST of 26.81°C and annual/semiannual amplitudes of 1.37°/0.51°C. (Note that PIRATA-derived values are for different time spans than September 1997–August 2003, the span used for drifter and R02 estimates. Coverage for various PIRATA mooring SST time series is shown in Fig. 5.) Although the annual amplitude has been underestimated by the decomposition, a reasonable mean SST is derived despite a boreal spring bias in the drifter observations (the simple binned mean is 27.18°C). In regions where the drifter observations are denser, annual and semiannual SST amplitudes compare favorably to in situ observations. For example, the PIRATA mooring at 8°N, 38°W (Fig. 5) measured SST with annual/semiannual amplitudes of 1.16°/0.26°C; the collocated bin contains 132 drifter days of observations which yield annual/semiannual components with amplitudes of (1.31 ± 0.33)/(0.22 ± 0.34)°C.

These results indicate that the LS decomposition produces better estimates of time-mean SST than simply averaging drifter observations within a bin, and that the methodology can estimate time series of climatological seasonal SST even in relatively sparsely sampled regions.

The methodology can easily be adapted to incorporate information from a product such as R02 SST, although the product would then no longer be usable for evaluating the methodology. The choices made for the diagonal elements of the matrix $\mathbf{R}_{xx}$ reflect one's knowledge of the coefficients x_o, x_i [Eq. (1)]. If the drifter observations are the sole source of information

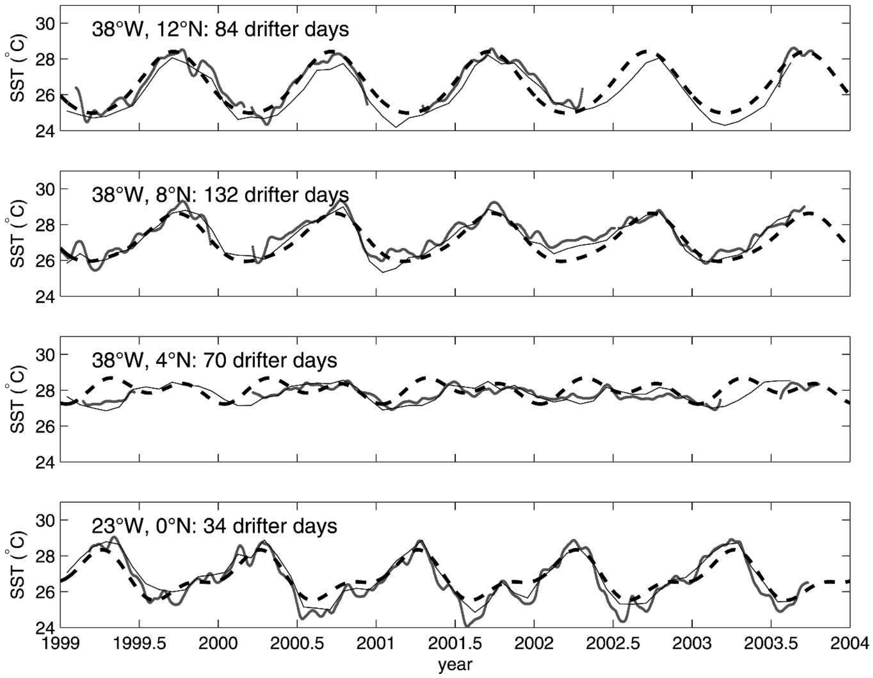

FIG. 5. Time series of SST at four of the PIRATA moorings, low passed at 30 days (gray), with coincident monthly SST from NOAA OI.v2 (solid black) and climatological SST constructed from decomposition of the drifter observations (dashed black). The number of drifter days of observations in the bin is indicated in each panel.

for the time-mean SST and its seasonal variations, then the approach used so far—using the variance of the drifter time series in 2° × 2° bins for the elements of $\mathbf{R}_{xx}$ and $\mathbf{R}_{nn}$—reflects the conservative choice that any of the individual unknown amplitudes (annual, semiannual, or noise) can alone account for the observed variance. One could instead derive prior (predecomposition) estimates of the unbiased time-mean and annual and semiannual amplitudes from the R02 product: because the product is evenly spaced in time, the simple mean of the product's SST estimates within a bin is not biased because of inhomogeneous sampling of the seasonal cycle. After removing this mean, the annual and semiannual amplitudes of the R02 SSTs can be derived by simple least squares fitting. These amplitudes (more specifically, their squared value) can then be used as the diagonal elements of $\mathbf{R}_{xx}$ in the order specified by Eq. (1). If the decomposition is approached this way, it becomes a least squares adjustment from the R02 SST time series. Significant departures from the R02-derived prior amplitudes would indicate problems with the drifter observations (bad data that were not caught during quality control) or with the R02 SSTs (e.g., atmospheric dust contaminating the satellite-derived SST estimates, or optimal interpolation with large correlation scale overly smoothing oceanic fronts). This synthesis of R02 and drifter SST was performed as described above; results are shown in Fig. 3d. In regions where the drifter observations are dense, mean SST is resolved at much higher resolution than in the R02 product; where they are sparse, the field resembles (by construction) the smoothed R02 field. A more interesting version of this approach could combine truly independent SST observations, such as by microwave or from ships, with the drifter observations. Such an application extends beyond the scope of this study, but might be a rewarding avenue for future studies of upper ocean heat advection.

b. *Evaluating the decomposition using simulated drifters*

As a second test of the methodology, decompositions of drifter observations were evaluated using a 1/12° Miami Isopycnal Coordinate Ocean Model (MICOM) simulation of the Atlantic Ocean. The model was identical to that of Garraffo et al. (2001) and Bracco et al. (2003), except forced by monthly climatology from the European Centre for Medium-Range Weather Forecasts (ECMWF), with the domain extended to 70°N and including the Mediterranean Sea, and with Richardson number–dependent diapycnal mixing and entrainment parameterization as in Hallberg (2000). This eddy-resolving model produces surface eddy kinetic energy (EKE) levels consistent with altimetry-derived estimates, and compares favorably to observations regarding North Brazil Current ring shedding and observed Lagrangian time scales in the Tropics (Garraffo et al. 2001). The model domain was 28°S to 70°N, with relaxation to monthly climatological temperature and salinity (Levitus 1982) at these boundaries. Because this relaxation created a buffer zone where EKE is affected, simulated results were not examined south of 10°S. Model results north of 20°N or west of 70°W were not examined. "True" time-mean SSTs and currents were calculated directly from the three-day gridded (Eulerian) model output in 1° by 1° bins.

After five model years of spinup, simulated drifters were released on a 1° by 1° grid on the first day of each month. Each drifter was given a "lifetime" of six months. The drifters were advected by the model's uppermost layer (mixed layer) velocities using a fourth order Runge–Kutta scheme with 16-point space interpolation in the ocean interior and 4-point space near the coasts. Particles were advected with a one-hour time step. The overall set of simulated drifters yielded 6590 drifter years of observations, far more than the actual oceanic dataset (200 drifter years). To evaluate the LS method's performance with a more realistically sparse dataset, a subset of the simulated drifters was selected such that this subset had the same number of drifter days for each month, in 1° × 1° bins, as the actual (nonsimulated) drifter dataset. This was done by randomly choosing one drifter in a bin, extracting the time series of its trajectory in that bin, and repeating with another randomly selected drifter until the required number of drifter days were extracted. This subset contained 200 drifter years of observations.

1) Simulated SST

In the MICOM simulation, the largest basin-averaged monthly mean true SST (28.3°C) was found in January. Monthly mean SST decreased to its minimum (25.7°C) in July, then increased to 26.1°–26.4°C in October–December. The overall mean true SST was 26.5°C.

Because of the sampling scheme, the full set of simulated drifter observations was sparsest in January and densest in July–December, introducing a seasonal bias. The difference between the true time-mean SST (from gridded model output) and the simple binned average SST (from all simulated drifters), averaged over all bins with more than 25 drifter days per square degree, was 0.14°C. For the LS decomposition method this difference decreased to −0.03°C. The overall rms error of time-mean pseudo-Eulerian SST was 0.43°C for simple bin averaging, and 0.08°C for the LS decomposition method.

The subset of simulated drifters with realistic density (200 drifter years) was not significantly biased toward any particular season at basin scale. The overall mean SST from bin averaging was 0.02°C smaller than the true mean SST. The LS decomposition produces a mean SST, which was <0.01°C different from the true mean SST. Also, rms errors for the two methods were 0.58°C (bin averaging) and 0.42°C (LS decomposition).

2) Simulated currents

For the full set of simulated drifters, overall mean zonal and meridional speeds ($\langle u \rangle$, $\langle v \rangle$) differed from the true Eulerian means by (0.46, <0.01) cm s^{-1} for bin averaging and by (<0.01, <0.01) cm s^{-1} for the LS decomposition. These differences are small because the overall mean is an average over currents of opposing sign, and does not reflect large regional differences between Eulerian and pseudo-Eulerian means. The overall rms errors were (3.14, 2.25) cm s^{-1} for bin averaging and (1.17, 0.87) cm s^{-1} for the LS decomposition.

For the subset with same observational density as the real oceanic drifters (200 drifter years overall), time-mean values were calculated from the simulated drifter observations via binned means and the LS decomposition, using two schemes for defining the bins. First, the observations were grouped in 1° × 1° bins. With this approach, simple bin averaging produced time-mean currents with rms errors of (6.1, 4.8) cm s^{-1} (zonal, meridional). The LS decomposition had rms errors of (5.8, 5.3) cm s^{-1}. Then, the observations were grouped in 2° × 1° bins, rotated as described in section 2. The rms errors for the bin-averaged means were (4.4, 2.9) cm s^{-1}. For the LS decomposition the rms errors were (3.4, 2.8) cm s^{-1}. These errors are defined with respect to Eulerian averages in 1° × 1° averages, and indicate that pseudo-Eulerian estimates in the rotated 2° × 1° bins are closer to the true time-mean values at 1° resolution than pseudo-Eulerian estimates in 1° × 1° bins. We stress that this conclusion is implicitly a function of the present density of drifter observations in the tropical Atlantic, and may not be true elsewhere. For the remainder of this section, we focus solely upon results from the subset of simulated drifters with pseudo-Eulerian averages calculated in rotated 2° × 1° bins.

Both the LS decomposition and simple binned averaging produced reasonable distributions of surface speed compared to the true Eulerian model output (Fig. 6), suggesting that either method can be used to derive pseudo-Eulerian mean currents to lowest order.

However, compared to simple binned averaging, the decomposition produced significantly better time-

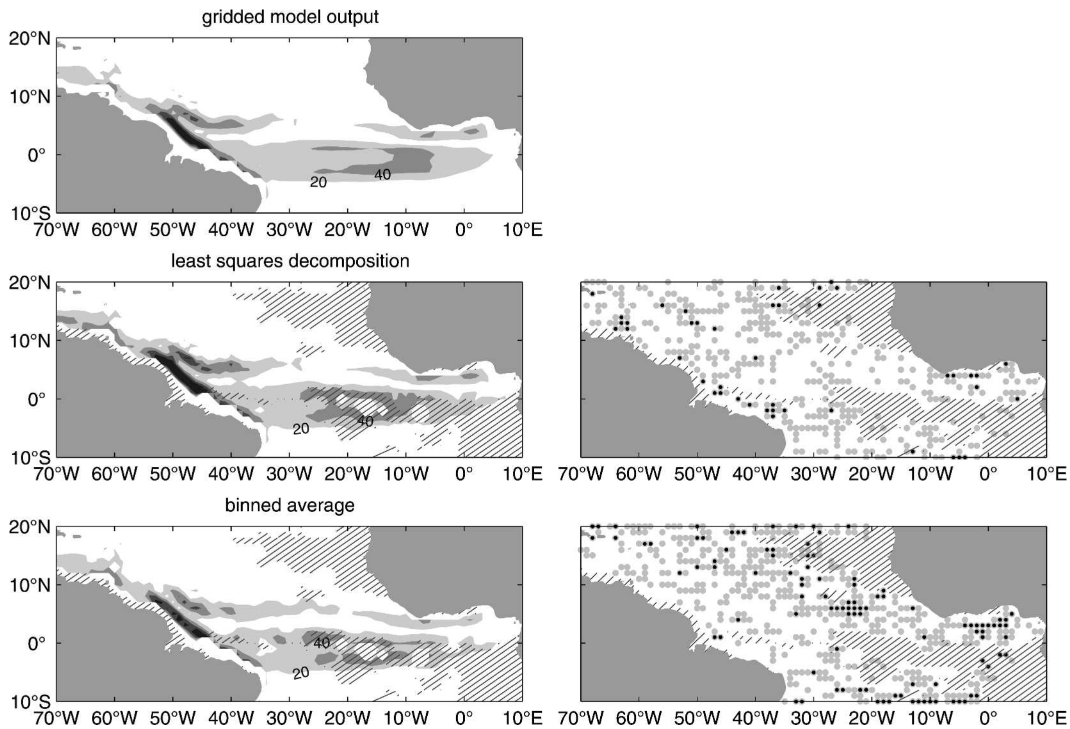

FIG. 6. (left) Annual averaged speed (cm s^{-1}) of surface currents in a numerical model of the tropical Atlantic, calculated directly from (top) the model output in $(1°)^2$ bins, and from simulated drifter observations grouped in $2° \times 1°$ rotated bins using the (middle) LS decomposition method and (bottom) simple binned averaging. Shaded contours are in increments of 20 cm s^{-1}, from 20 (lightest gray) to 80 (black). Hatched regions indicate where drifter observations were <25 drifter days per square degree. (right) Gray (black) points indicate where the drifter-derived means differ from the true annual averaged values by one (two) error bars (top: formal error from LS decomposition; bottom: simple standard error).

mean values in regions where currents have strong seasonal fluctuations. This is shown in Fig. 7 (top). For this figure, errors of the time-mean estimates in individual $2° \times 1°$ bins were sorted by the strength of the annual cycle in that bin (determined from the gridded model output). The rms value of these errors was then calculated within a finite band of amplitude, in order to show rms error as a function of annual amplitude. The error bar on each point is the average of the estimated errors from the two methods (formal error for LS decomposition; standard error for simple binned averaging).

For tropical Atlantic currents, seasonal fluctuations (and errors in the binned averages) were largest along the eastward NECC and in the equatorial branch of the westward South Equatorial Current (SEC). In the NECC at 4°–5°N, 42°–44°W, zonal speed had an annual fluctuation of 69 cm s^{-1}, the binned average had an rms error of 18 cm s^{-1}, and the LS decomposition mean had an rms error of 8 cm s^{-1}. In the SEC at 1°S, 2°–8°W, zonal speed had an annual fluctuation of 26 cm s^{-1}, the binned average had an rms error of 23 cm s^{-1}, and the LS decomposition mean had an rms error of 7 cm s^{-1}.

Within each bin, the LS decomposition produced a formal error estimate (hereafter LSE) for each time-mean value that can be compared to the simple standard error (hereafter STDE), that is, the standard deviation of the observations divided by $(N_{\text{ind}})^{1/2}$, where N_{ind} is the number of independent observations assuming a 5-day integral time scale. Averaged over all tropical Atlantic bins, the LSEs were not significantly different from the STDEs. For example, the overall mean LSE for zonal speed was 5.3 cm s^{-1}; the overall mean STDE was 5.0 cm s^{-1}. LSEs and STDEs were both large where eddy energy is large, because both are a function of the standard deviation. LSEs were generally larger than STDEs in bins that were sampled heavily over a small fraction of the annual cycle, as error estimation in the LS method takes into account the possible presence of significant seasonal variations. As a consequence, LS-derived estimates of the overall speed (e.g., the magnitude of the velocity vector, including both meridional and zonal components) were within one LSE of the true mean in 70% of the bins; simple bin averages were within one STDE of the true mean in only 58% of the bins (Fig. 6).

The LS decomposition also produces annual and

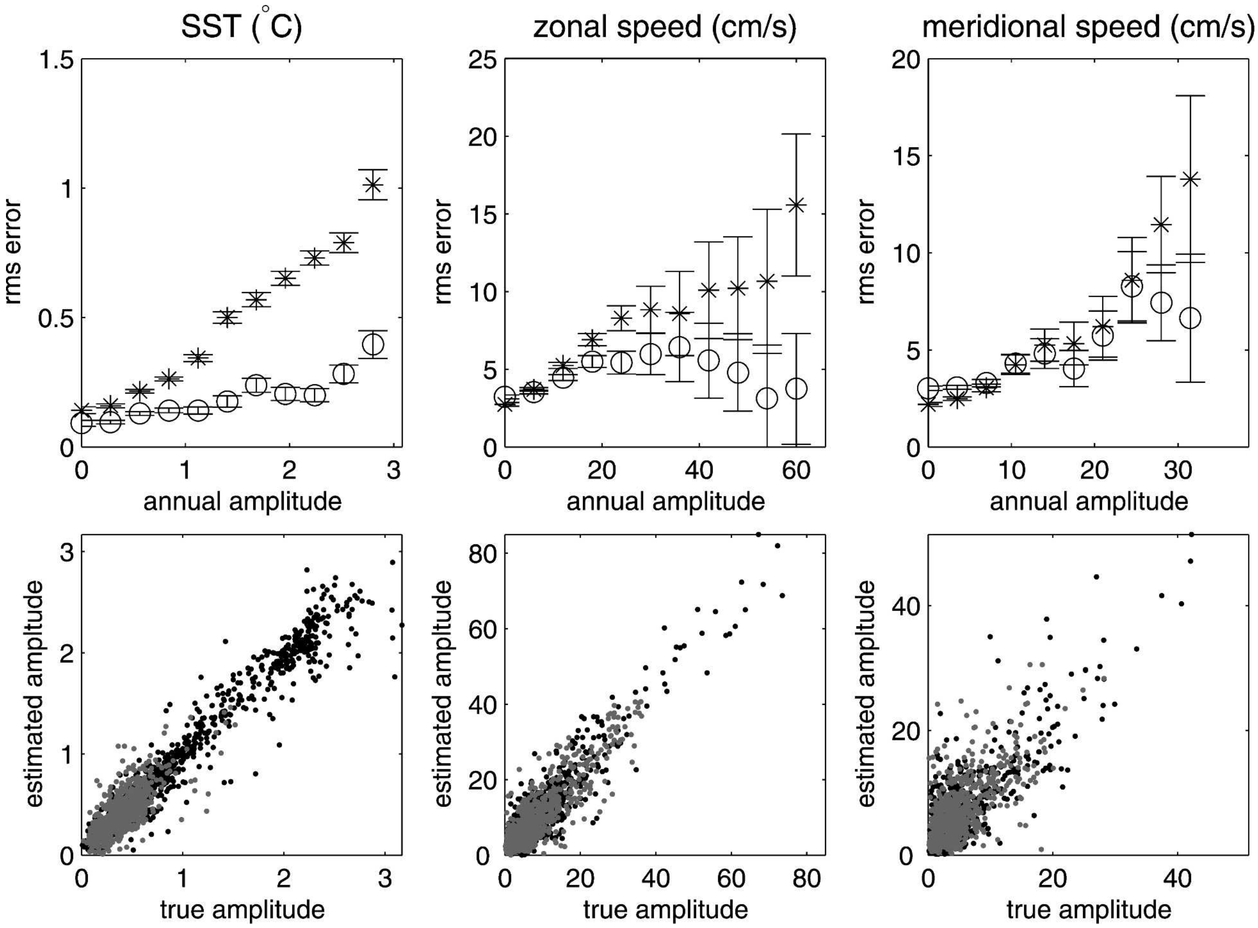

FIG. 7. (top) The rms error (vertical axes) as a function of seasonal amplitude (horizontal axis) for pseudo-Eulerian estimates of time means, estimated by simple binned averaging (asterisks) and LS decomposition (circles). Averaged error bars for the two estimation methods are also shown (see text). (bottom) Estimated amplitude of the annual (black) and semiannual (gray) harmonic from the least squares decomposition in each bin (points), shown vs the true value of these harmonics given by a least squares fit to the Eulerian time series. (left) SST, (middle) zonal speed, and (right) meridional speed.

semiannual amplitudes that are comparable in magnitude (Fig. 7) and distribution to those calculated directly from the gridded model output. For the total speed $(u^2 + v^2)^{1/2}$ the amplitude A of a seasonal harmonic $A = ½(A_x{}^2 + A_y{}^2)^{1/2}$, where x and y denote the zonal and meridional components (Richardson and Walsh 1986). The annual and semiannual amplitudes A calculated from the gridded model output and from the LS decomposition of simulated drifter observations are shown in Fig. 8.

Seasonal amplitudes derived by the LS decomposition tend to be slightly larger than the true values determined from the (Eulerian) model output. The overall mean difference between LS-derived and true annual amplitudes is 1.32 ± 0.09 cm s^{-1}, with a standard deviation of 2.33 cm s^{-1}. For semiannual amplitudes, this difference is 1.39 ± 0.09 cm s^{-1} with a standard deviation of 2.42 cm s^{-1}. This overestimation is not a function of the amplitude itself (e.g., there is no significant difference between averages calculate for bins with amplitudes >10 cm s^{-1} and <5 cm s^{-1}), but it is a strong function of observation density. The overestimation is largest in bins with the fewest drifter days of observations: averaged over bins with 25–50/50–100/100–150/150–200 drifter days, the mean difference between drifter derived and true annual amplitude is 1.68 $\pm$0.15/1.08 $\pm$0.13/0.86 $\pm$0.22/−0.04 $\pm$0.26 cm s^{-1}, respectively. These results indicate that undersampled variability at other frequencies, such as mesoscale variability, is (slightly) contaminating the derived amplitude of seasonal variability—in other words, motion that is not described by a pure annual or semiannual sinusoid is projected onto these harmonics in bins which are not heavily sampled. This is true in most of the study domain (92% of bins contain less than 100 drifter days of observations.) However, these results

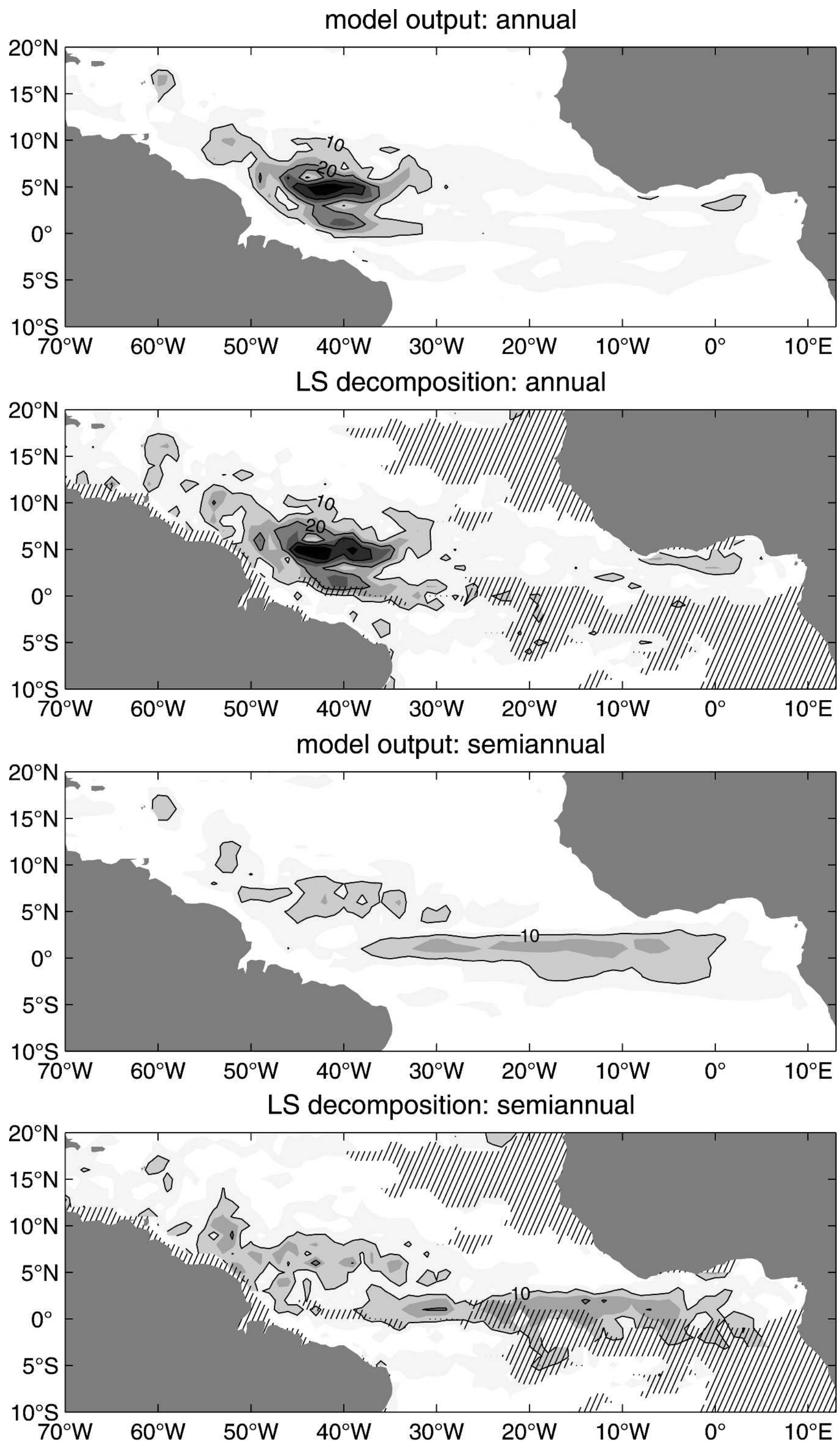

FIG. 8. Amplitudes (cm s^{-1}) of seasonal variations in surface currents calculated directly from gridded model output averaged in $(1°)^2$ bins and by applying the LS decomposition to subsampled simulated drifter observations. Hatches indicate regions with <25 drifter days per square degree: (top) annual model output, (middle) annual LS decomposition, and (bottom) semiannual model output.

also indicate that the decomposition produces better time-mean results than simple bin averaging, and that the methodology at the present density of observations is sufficient to resolve seasonal variations of near-surface currents to within a few centimeters per second throughout most of the tropical Atlantic.

4. Summary

Lagrangian observations collected by drifters can be used to calculate pseudo-Eulerian estimates of time-mean near-surface currents and concurrent sea surface temperature throughout most of a given region. This is typically done by grouping them in spatial bins and averaging the time series within each bin. Error bars on such estimates are typically calculated as the standard deviation of the time series divided by the square root of the number of independent observations, the latter determined from the total number of observations in the bin and a Lagrangian integral time scale set by mesoscale variability. However, in the presence of strong seasonal variability that may have been sampled inhomogeneously by the drifters, simple binned means can be significantly biased. Standard error estimates that assume white noise at frequencies lower than the mesoscale will not reflect this alias.

An alternative methodology for deriving pseudo-Eulerian means in the presence of inhomogeneously sampled strong seasonal variations has been developed and tested in this paper. In this approach, the drifter observations are simultaneously decomposed into time-mean, seasonal and residual (eddy) components using a tapered, weighted least squares (LS) decomposition that provides formal error bars for the time-mean and seasonal amplitudes. The methodology was evaluated by calculating time-mean SST and associated error estimates from drifter observations in the tropical Atlantic Ocean via simple binned averaging and LS decomposition. These were compared to time-mean SST from a temporally homogeneous product (Reynolds et al. 2002) and from in situ observations from fixed moorings. The methodology was also tested by applying it to simulated drifters in a 1/12° simulation of the tropical Atlantic, for which true Eulerian means could be determined from the gridded model output.

This evaluation showed that time-mean SST (Fig. 3) and currents (Fig. 6) are more accurately estimated by the LS decomposition than by binned averaging in regions where seasonal variability is large and observations are scarce and distributed nonuniformly (Fig. 7). The LS-derived error estimates increase with decreasing numbers of independent observations and with increasing signal variance, as with the simple standard error; they also increase if only part of the seasonal cycle has been sampled (e.g., if the seasonal cycle cannot be accurately resolved).

The LS decomposition also produces estimates of the seasonal harmonics' amplitude and phase distribution at much higher spatial resolution than can be achieved by monthly or seasonal pseudo-Eulerian binned means. Semiannual and annual SST amplitudes from the LS decomposition compare favorably to amplitudes calculated directly from the Reynolds et al. (2002) product (Fig. 4) and are able to reproduce the amplitude and phase of SST variations observed at the PIRATA moorings (Fig. 5). The methodology was also applied to a subset of the numerical drifters, subsampled so that the month-by-month observation density was the same as the actual oceanic drifters. Results showed that the methodology can produce reasonable estimates of mixed layer currents' seasonal variations throughout most of the tropical Atlantic domain (Fig. 8).

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