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Weather Bureau

TECHNICAL NOTE 18-NHRL-75

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A Simple Model of the Hurricane Inflow Layer

NATIONAL HURRICANE RESEARCH LABORATORY REPORT NO.75

WASHINGTON, D.C.
November 1965

TECHNICAL NOTES SERIES

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A SIMPLE MODEL OF THE HURRICANE INFLOW LAYER

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ABSTRACT

A simple numerical model of the hurricane inflow layer has been constructed. A pressure profile representative of an actual hurricane was specified. The effects of turbulent mixing (both vertical and horizontal) and of vertical advection on the field of motion were examined. It was found that the use of an eddy viscosity (K_m) proportional to the pressure gradient force and a surface drag coefficient (C_d) as a function of the surface wind speed resulted in the most realistic meridional circulation. The numerical results were compared with observations obtained in hurricane Donna (1960). The similarity between calculations and observations was satisfactory.

1. INTRODUCTION

The mature tropical cyclone may be conveniently divided into three vertical layers (a lower inflow layer, a middle layer where radial motion is small, and an upper outflow layer). The depth of the inflow layer is variable, but aircraft observations show that in the majority of tropical cyclones, most of the inflow takes place below 3,000 meters (Miller, [8], [9]; Colon, [2]; Malkus and Riehl, [7]). The radial motion of the hurricane inflow layer is largely controlled by frictional forces. The evaporation from the ocean, the transfer of sensible heat from the sea surface to the atmosphere, and the low-level convergence of water vapor are necessary elements of hurricane development and maintenance. All are turbulent processes. In addition, the inflow layer produces most of the kinetic energy which is used to maintain the cyclone against losses by frictional dissipation and export by the outflow layer.

In spite of the basic importance of the inflow layer, however, much uncertainty exists as to the nature, magnitude, and variation of its turbulent structure. There are several reasons for this uncertainty, both observational and theoretical. First, few measurements in a hurricane have ever been made by research aircraft below 1,500 ft. because of the safety factor. Second, the measurement of the radial wind component by aircraft is not very accurate because it is a computed quantity, sensitive to small errors in wind direction and in locating the center of the hurricane. Third, no measurement of the Reynolds stresses or of vertical motion have ever been made at any level in a hurricane, although Gray [4] recently made some rough computations of draft scale vertical motions in several hurricanes. Finally, no relation between the diffusion coefficients (completely applicable to hurricane conditions) and the velocity field, or other easily measured quantity, either empirical or theoretical, has ever been formulated. In fact, it is highly probable that the formulation of diffusion processes in terms of a coefficient and the mean velocity field will never be completely adequate to explain these processes within the hurricane.

Numerous investigators have examined the meridional circulation of the hurricane from a theoretical standpoint. Many of these (Krishnamurti, [5, 6]; Estoque, [3]; Rosenthal, [10]; Barrientos, [1]) have specified either the pressure profile or the tangential wind field. The meridional circulation can be determined, once a choice of the surface drag coefficient, C_d, and the vertical (K_m) and lateral mixing coefficients (K_h) has been made. All of these investigators assumed that C_d is a constant and that K_m is at most a function of height; K_h is treated as an absolute constant, although Barrientos experimented with various values of all three and chose the ones which seemed to produce the most realistic meridional circulation. All of the models contain some realistic features, insofar as this can be determined from the observations available. There are certain unresolved questions, however; for example, does the depth of the inflow layer increase as the center of the hurricane is approached? Is the vertical motion concentrated in strong updrafts near the wall cloud or is there a more gradual ascent over a wide area?

Our model differs from those cited in that we have chosen to investigate the influence of a variable drag coefficient (C_d, as a function of the surface wind speed) and a variable eddy viscosity (K_m, as a function of radius and height) upon the meridional circulation of the inflow layer. The pressure gradient force is specified.

2. AN INFLOW LAYER WITH CONSTANT DEPTH

In a recent paper, Rosenthal [10] investigated the field of motion in the hurricane inflow layer from a theoretical standpoint. He assumed steady state, symmetrical flow, neglected lateral mixing, and vertical advective terms, obtaining the system

$$K_m \frac{\partial^2 v_\theta}{\partial z^2} = v_r \zeta_a \tag{1}$$

$$K_m \frac{\partial^2 v_r}{\partial z^2} = \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v_\theta^2}{r} - f v_\theta \tag{2}$$

in which K_m is the kinematic coefficient of eddy viscosity for vertical mixing (assumed to be a constant), v_r and v_θ are the radial and tangential components of the wind, ζ_a is the absolute vorticity, p is pressure, ρ is density, z is the height, r is the radial distance, and f is the Coriolis parameter. These equations were linearized by perturbation techniques, and analytical solutions for v_r and v_θ were obtained. Rosenthal chose K_m equal to 5×10^5 cm.^{2} sec.$^{-1}$, $C_d = 3.0 \times 10^{-3}$, and a pressure gradient force (independent of height) defined by

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{(p_n - p_o)}{\rho} \left(\frac{R}{r^2}\right) e^{-R/r} \tag{3}$$

p_o is the central pressure, p_n is the peripheral pressure, R is the radius of maximum wind (30 km.). $(p_n - p_o)$ was chosen as 60 mb. and the mean density as 1×10^{-3} g. cm.$^{-3}$.

Rosenthal's model simulates many of the characteristics of the typical hurricane. The two main objections, as he has pointed out, are: (1) the depth of the inflow layer decreases radially inward, from a value of about 2.5 km. at r = 250 km. to about 320 m. at r = 10 km. (fig. 1a). This decrease does not seem to be verified by observations, although the radial variation of the depth of the inflow layer cannot be determined with any degree of certainty. (2) The vertical motion near the center (fig. 1b) appears to be too small (Miller, [9]; Gray, [4]). The small vertical motions result from the decrease in the depth of the inflow near the cyclone core, since the radial inflow at the outer radius (250 km.) is quite realistic.

In Rosenthal's model the depth of the inflow layer, h, is given by

$$h = \frac{1}{\lambda} \arctan \left[- \frac{\lambda}{\lambda + \chi} \, v_{eg} \right] \qquad (4)$$

in which χ is defined as C_d/K_m, V_{eg} is the gradient wind, and λ is a function of r

$$\lambda = \left[\frac{\zeta_{ag} \left(f + \frac{2 v_{eg}}{R}\right)}{4K_m^2} \right]^{1/4} \qquad (5)$$

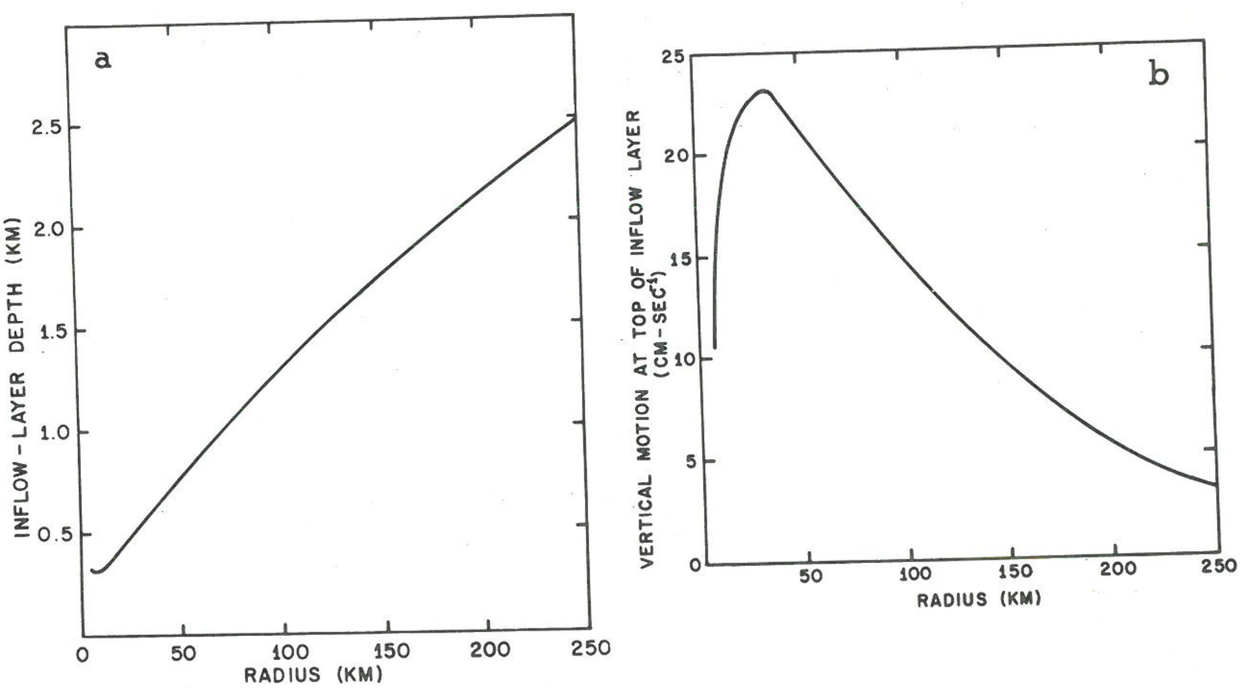

Figure 1. - (a) Depth of inflow layer as a function of radius [10]. (b) Radial profile of the vertical motion at top of inflow layer [10]. K_m = constant.

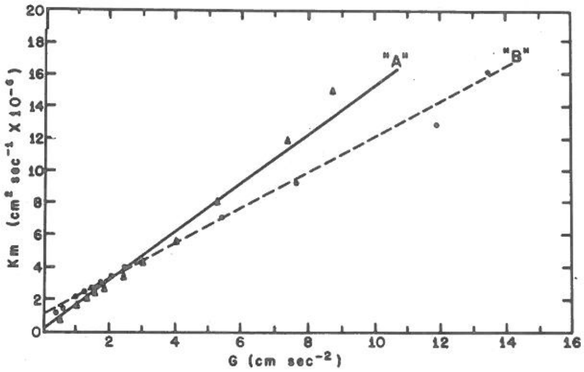

Figure 2. - K_m (for h = constant) as a function of pressure gradient force. (a) Experiment I. (b) Experiment II.

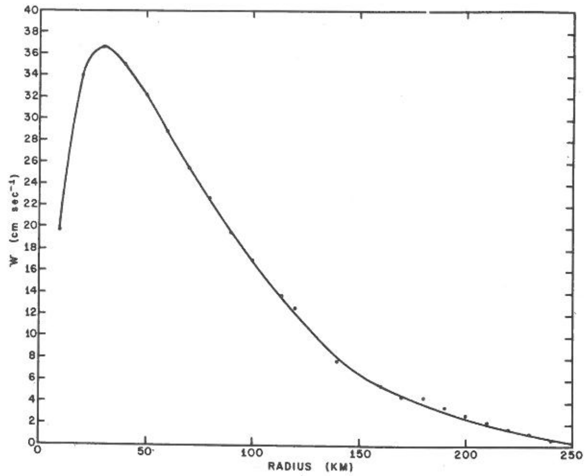

Figure 3. - Radial profile of vertical motion at top of inflow layer (h = constant, K_m = f(r)).

Since we do not really know how h varies with radius, we make the simple assumption that h is a constant, at least inward as far as the region of maximum tangential winds. We may then inquire as to the variation of K_m necessary to keep h constant, using equations (4) and (5). It was found that the eddy viscosity increased from 5×10^5 cm.^{2} sec.$^{-1}$ at a radius of 250 km. to 1.2×10^7 cm.^{2} sec.$^{-1}$ at a radius of 30 km. This increase does not seem unreasonable. While no completely satisfactory formulation of the radial variation of K_m as a function of the wind field has been performed, it is of interest to note that the values of K_m required to keep h constant are proportional to the pressure gradient force (fig. 2). The vertical motion at h (fig. 3) resulting from the use of a variable K_m seems to be more realistic than the values obtained by use of a constant K_m (fig. 1b). The maximum vertical motion at z = h, however, is still too small.

3. A QUASI-TIME DEPENDENT INFLOW MODEL

a. The Basic Equations

It has been shown by Krishnamurti [5] and Estoque [3] that lateral eddy diffusion of the tangential wind has a significant effect on the meridional circulation of the hurricane inflow layer. We now examine the results of including lateral mixing and non-linear terms neglected in the Rosenthal model, while still assuming symmetrical flow and a specified pressure gradient. To do with this we replace equations (1) and (2) with more complete equations (Rosenthal [11]):

$$\frac{\partial M}{\partial t} = - v_r\frac{\partial M}{r\partial r} - w\frac{\partial M}{\partial z} - frv_r + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_m\frac{\partial M}{\partial z}\right) + \frac{K_h}{r}\frac{\partial}{\partial r}\left(r\frac{\partial M}{\partial r} - 2M\right) \quad (6)$$

$$\frac{\partial v_r}{\partial t} = - v_r \frac{\partial v_r}{\partial r} - w \frac{\partial v_r}{\partial z} + \frac{M}{r} \left(f + \frac{M}{r^2}\right) + \frac{1}{\rho} \frac{\partial}{\partial z} \left(\rho K_m \frac{\partial v_r}{\partial z}\right)$$

$$+ \frac{K_h}{r^2} \frac{\partial}{\partial r} \left(r \frac{\partial r v_r}{\partial r} - 2 r v_r\right) - \frac{1}{\rho} \frac{\partial p}{\partial r} \tag{7}$$

where M is the relative angular momentum (rv_e) and K_h is the kinematic eddy viscosity for lateral mixing. Local variations of density are ignored so that the equation of continuity may be written

$$\frac{\partial \rho v_r}{\partial r} + \frac{\rho v_r}{r} + \frac{\partial \rho w}{\partial z} = 0 \tag{8}$$

or

$$v_r = - \frac{1}{\rho r} \frac{\partial \psi}{\partial z} \tag{9a}$$

$$w = \frac{1}{\rho r} \frac{\partial \psi}{\partial r} \tag{9b}$$

where ψ is the Stokes stream function. Equation (7) is multiplied by ρr and then differentiated with respect to z, as suggested by Rosenthal [11] to yield, by use of (9a)

$$\frac{\partial^2}{\partial z^2} \left(\frac{\partial \psi}{\partial t}\right) = - \frac{\partial}{\partial z} (\rho r \Gamma) + \frac{\partial}{\partial z} (\rho r G) \tag{10}$$

where G is the pressure gradient force (invariant with height) and

$$\rho r \Gamma = \frac{\partial}{\partial z} \left(\rho K_m \frac{\partial r v_r}{\partial z}\right) + \rho \frac{K_h}{r} \frac{\partial}{\partial r} \left(r \frac{\partial r v_r}{\partial r} - 2 r v_r\right) + \rho M\left(f + \frac{M}{r^2}\right)$$

$$- \rho r \left(v_r \frac{\partial v_r}{\partial r} + w \frac{\partial v_r}{\partial z}\right) \tag{11}$$

Equations (6), (9a), (9b), and (10), together with a specified pressure field, can be solved numerically for ψ, M, v_r, and w.

b. Initial Conditions

In Experiment I, input data consisted of tangential winds and stream function values computed from Rosenthal's steady state model, with constant C_d and K_m. We then introduced a drag coefficient which depended upon the surface wind speed (fig. 4), and the empirical values of the eddy viscosity for vertical mixing required to make h a constant along the radius. In Experiment II, initial values for v_θ and ψ were computed from our modification of Rosenthal's model, with variable C_d and K_m, and a constant depth of the inflow layer inward to a radius of 30 km., at which point h was allowed to drop rapidly. A value for eddy viscosity for lateral mixing (K_h) suggested by Barrientos [1] was used (3×10^8 cm.^{2} sec.$^{-1}$). By imposing lateral mixing, plus the vertical advective terms, the initial steady state was disturbed, and the stream function and momentum fields became time dependent. To forecast the time changes of these quantities, a grid consisting of 11 points in the vertical and 26 in the radial direction was adopted. The radial grid interval was 10 km., in Experiment I the vertical interval was 250 m., while in Experiment II it was 300 m. In Experiment I a time step of 50 sec. was used, this was reduced to 20 sec. for Experiment II. Forecasts of ψ and M were prepared numerically by use of the finite difference equations described by Rosenthal [11].

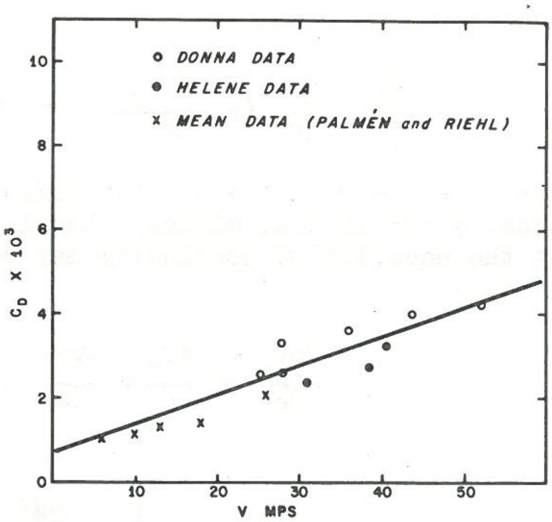

Figure 4. - Surface drag coefficient as a function of wind speed [9].

c. Boundary Conditions

At z = 0 and r = 250 km. we impose the conditions that

$$w = 0 \qquad (12)$$

while at r = 0 and z = h

$$v_r = 0 \qquad (13)$$

At r = 0 and z = 0

$$\psi = 0 \qquad (14)$$

and

$$\frac{\partial \psi}{\partial t} = 0 \qquad (15)$$

In addition the following were also specified:

$$\text{at } z = 0 \qquad \rho K_m \frac{\partial M}{\partial z} = \rho C_d v_e M \qquad (16)$$

$$\rho K_m \frac{\partial r v_r}{\partial z} = \rho C_d v_\theta r v_r \qquad (17)$$

at $z = h$

$$\rho K_m \frac{\partial M}{\partial z} = 0 \qquad (18)$$

$$\rho K_m \frac{\partial r v_r}{\partial z} = 0 \qquad (19)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{v_\theta^2}{r} + f v_\theta \qquad (20)$$

at $r = 250$ km.

$$v_\theta r^{1/2} = \text{constant} \qquad (21)$$

$$\frac{\partial}{\partial r}(r v_r) = 0 \qquad (22)$$

All of the above conditions are reasonably supported by observational data (Miller, [8, 9]; Colón, [2]; Malkus and Riehl, [7]. That implied by equation (22) is perhaps an over-simplification, but the inflow layer of many hurricanes appears to be very nearly non-divergent at this radius.

4. RESULTS

The initial fields of stream function, radial winds, vertical motion, and tangential winds for Experiment I are shown in figures 5a-8a. These values are based on Rosenthal's model with $K_m = 5 \times 10^5$ cm.^{2} sec.$^{-1}$ and $C_d = 3.0 \times 10^{-3}$. Figure 6a shows the decrease of the inflow layer inward. In figure 7a the maximum vertical motion at 2,500 m. is about 28 cm. sec.$^{-1}$ at a radius of 30 km. There is some upward motion at the center.

The values of eddy viscosity for vertical mixing shown in figure 2 are vertical means. It is known that near the ground in a neutral atmosphere, the eddy viscosity is satisfactorily represented by

$$K_m = k \ u_* z \qquad (23)$$

where k is von Karman's constant (about 0.4), u_* is the friction velocity, and z is the height. We therefore used equation (23) to determine K_m near the ground; K_m was allowed to increase linearly with height up to the point where it reached the value indicated by figure 2. Above that level, K_m was taken as a constant along z, with its radial value being determined from figure 2. C_d, which was allowed to vary with the surface wind speed, ranged from 1.5×10^{-3} at $r = 250$ km. to 3.3×10^{-3} at $r = 20$ km.

The results after 11 hr. are shown in figures 5b-8b. At that time, a steady state had been very nearly attained. The depth of the inflow layer was very nearly level (fig. 6b), although the radial motion above 1,500 m.

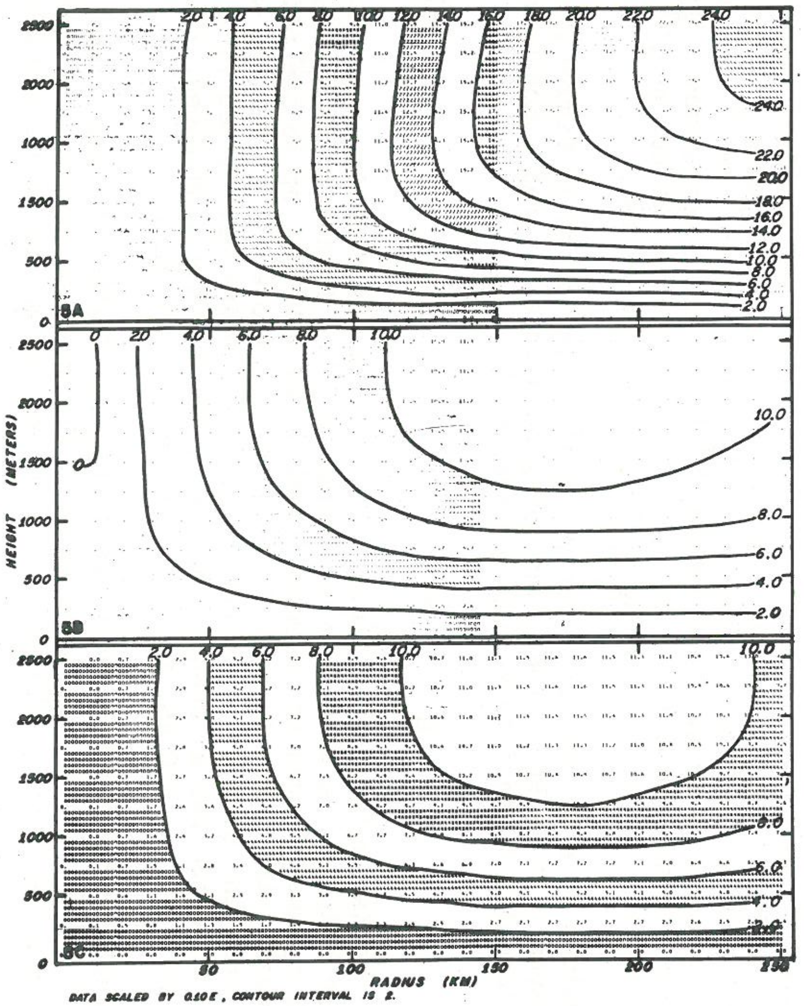

Figure 5. - Experiment I, meridional streamlines, (a) Initial fields, (b) 11 hr., (c) 3 hr. after (b) and with $K_h = 0$.

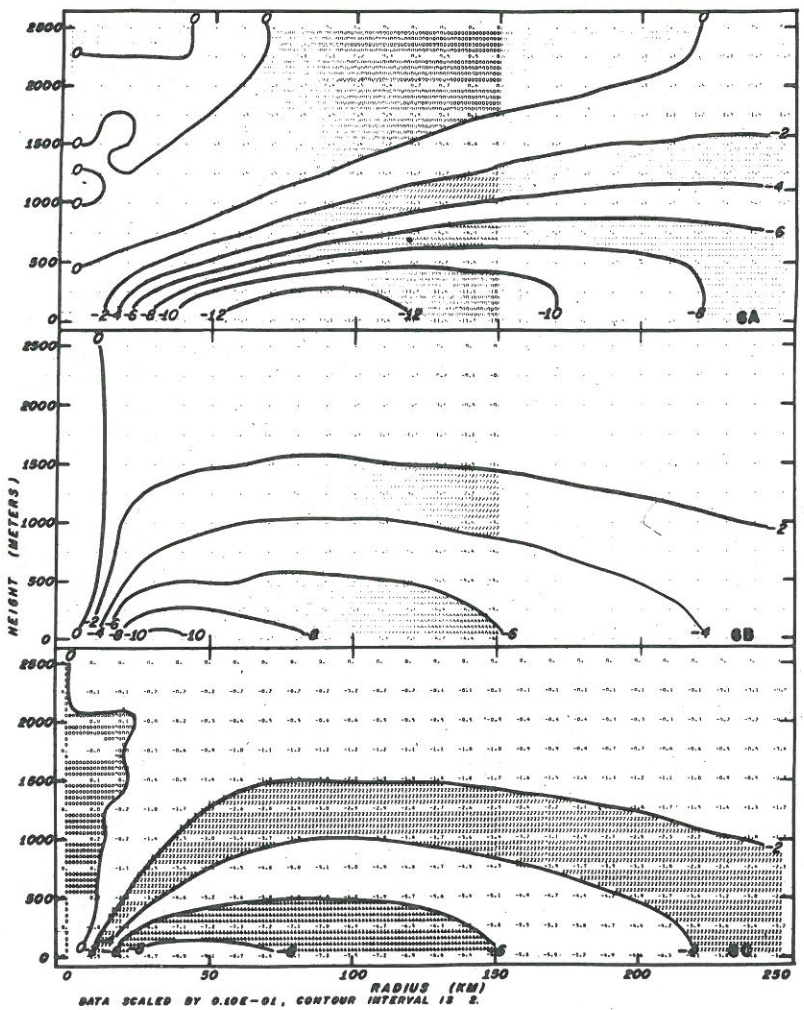

Figure 6. - Experiment I, radial motion (m. sec.$^{-1}$), (a) Initial, (b) After 11 hr. (c) 3 hr. after (b) and $K_h = 0$.

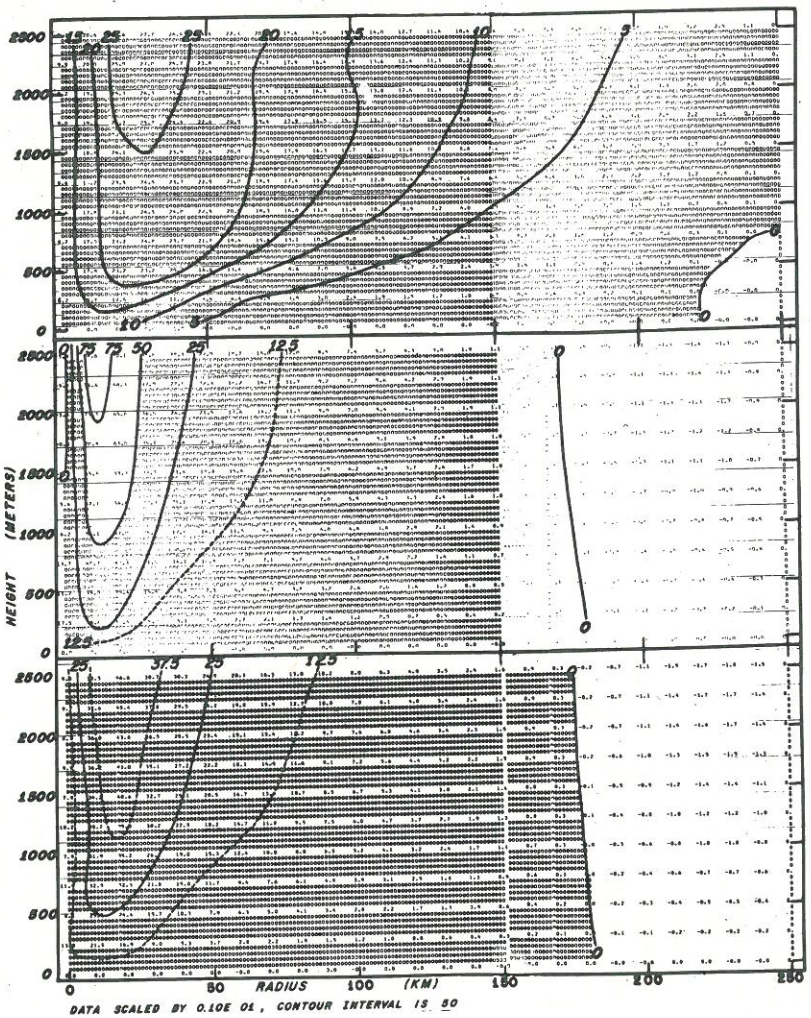

Figure 7. - Experiment I, vertical motion (m. sec.$^{-1}$), (a) Initial field (b) after 11 hr., (c) 3 hr. after (b) and with K_h = 0.

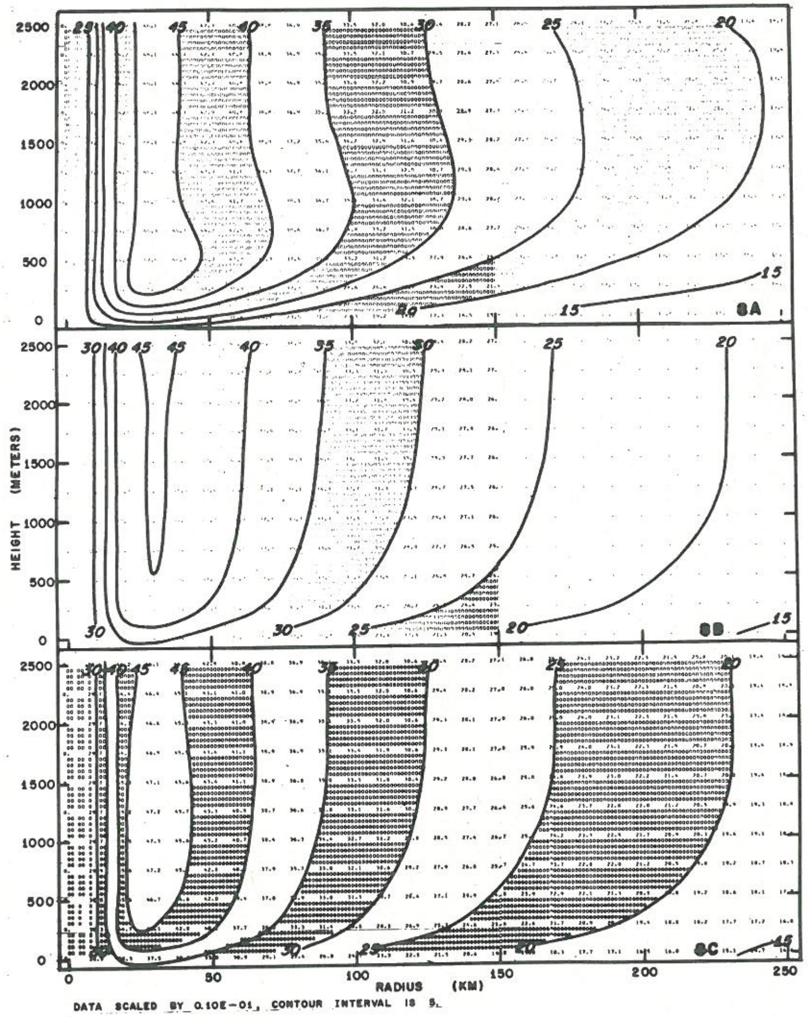

Figure 8. - Experiment I, tangential motion (m. sec.$^{-1}$), (a) Initial field (b) after 11 hr., (c) 3 hr. after (b) and with K_h = 0.

was small. At the outer boundary, the radial inflow had decreased substantially, reflecting the influence of reduced surface drag. The point of maximum radial inflow had moved inward from about 70 km. to 30 km., which is just outside the radius of maximum tangential winds. In figure 7b the maximum vertical motion at 2,500 m. was about 78 cm. sec.$^{-1}$, at a radius of 10 km. There was some downward motion at the center, but this did not extend to the surface. Outside a radius of 40 km., the vertical motion had decreased, and outside a radius of 170 km. there was slight downward motion. This was apparently caused by the reduction of the radial inflow at the outer boundary because of reduced surface drag. There were no pronounced changes in the tangential winds, (fig. 8b). The surface winds increased slightly and the vertical shear between the surface and the first level (250 m.) decreased. The winds at r = 10 km. became slightly super-gradient at heights from 250 to 2,250 m. This seemed to be caused by lateral diffusion of the tangential wind.

At the end of 11 hr., K_h (the coefficient of eddy viscosity for lateral mixing) was set equal to 0, and the forecasts were continued for an additional 3 hr. (fig. 5c-8c). The downward motion at the center disappeared; the point of maximum upward motion at 2,500 m. moved outward from 10 km. to 20 km., and decreased from 78 cm. sec.$^{-1}$ to 46 cm. sec.$^{-1}$. The tangential winds at r = 10 km. decreased to very nearly gradient balance, but winds increased at some interior grid points for radii 20-40 km. This resulted in a new area of slightly super-gradient winds.

The model has succeeded in simulating the two important features of the hurricane inflow layer as planned. These are the maintenance of a very nearly constant depth of the inflow layer and the concentration of the maximum vertical motion near the region of maximum winds. However, the latter occurs inside the radius of maximum winds. This is probably unrealistic, although it is also a feature of other models (Estoque, [3]; Barrientos, [1]). Reconnaissance aircraft data would indicate that the maximum vertical motion is very near the region of maximum winds (Gray, [4]). It is also apparent that figure 4 gives surface drag coefficients which are too small for low wind speeds. We will consider these points further after a discussion of Experiment II.

In Experiment II, K_m was determined from equation (23) and figure 2 as in the previous case. The surface drag coefficient ranged from 1.9 x 10^{-3} at 250 km. to 4.2 x 10^{-3} at 30 km. In this experiment we attempted to duplicate some of the features of hurricane Donna (1960) on the days of its maximum intensity just before it moved inland over Florida. The actual radial pressure profile from Donna was used to define the pressure gradient in lieu of equation (3), but some minor adjustments in the profile as given by Miller [9] proved to be necessary in order to insure that the gradient wind was everywhere greater than the observed surface wind.

The results are shown in figures 9-12. Steady state was very nearly realized after 5 hr. The initial radial wind field (fig. 10a) shows an inflow depth of approximately 3,000 m., strong convergence between 30 and 40 km., and the maximum radial motion was concentrated near the region of maximum winds, the values at 3,000 m. being 145-176 cm. sec.$^{-1}$ at 30-40 km. There was some upward motion at the center. Maximum tangential surface winds were

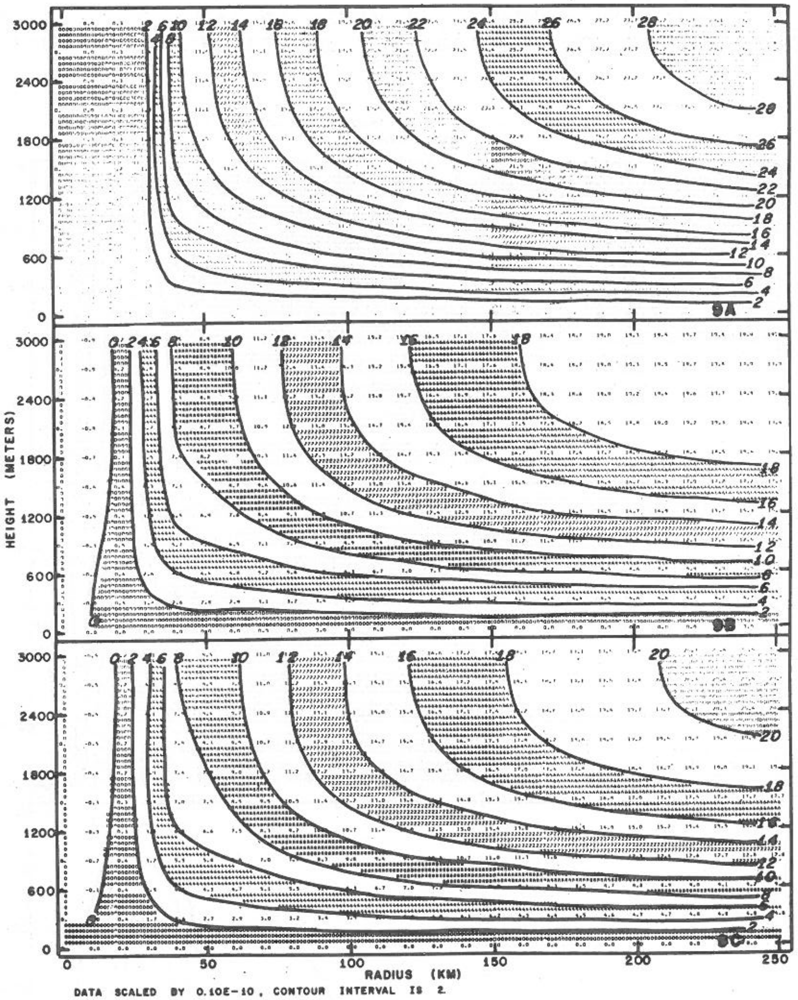

Figure 9. - Experiment II, meridional streamlines, (a) Initial field, (b) after 5 hr., (c) 2 hr. after (b) and with $K_h = 1.5 \times 10^8$ cm.^{2} sec.$^{-1}$.

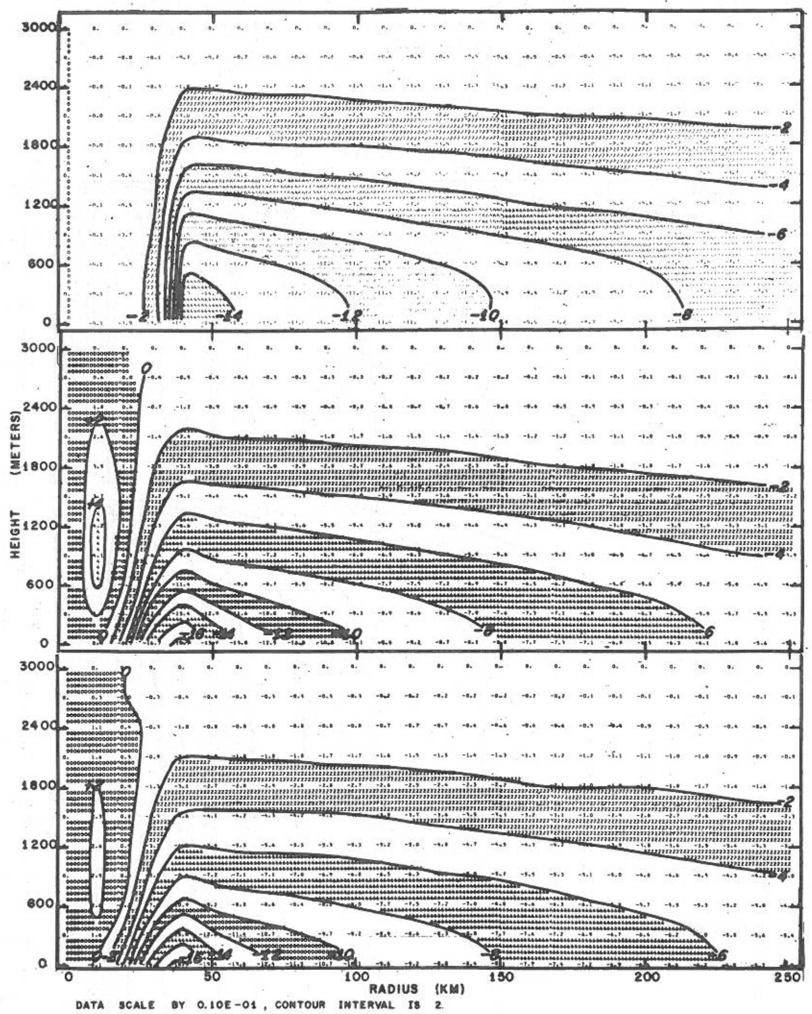

Figure 10. - Experiment II, radial motion (m. sec.$^{-1}$), (a) Initial field (b) after 5 hr., (c) 2 hr. after (b) and with $K_h = 1.5 \times 10^8$ cm.^{2} sec.$^{-1}$.

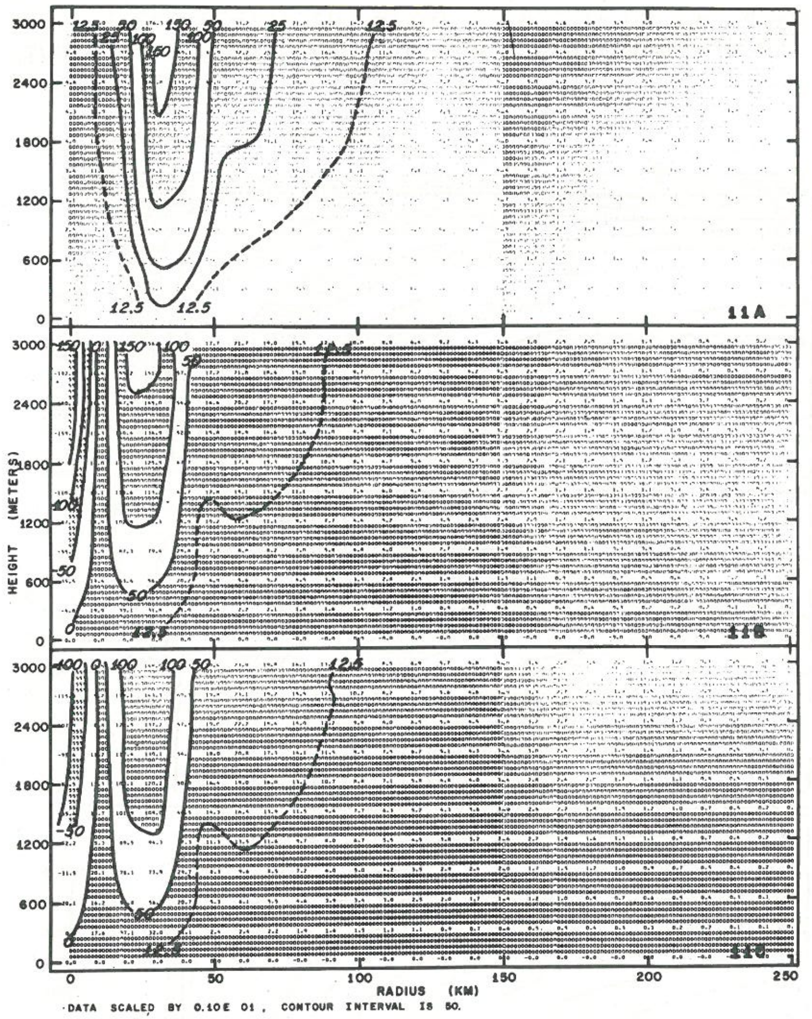

Figure 11. - Experiment II, vertical motion (cm. sec sec.$^{-1}$), (a) Initial field (b) after 5 hr., (c) 2 hr. after (b) and with $K_h = 1.5 \times 10^8$ cm.^{2} sec.$^{-1}$.

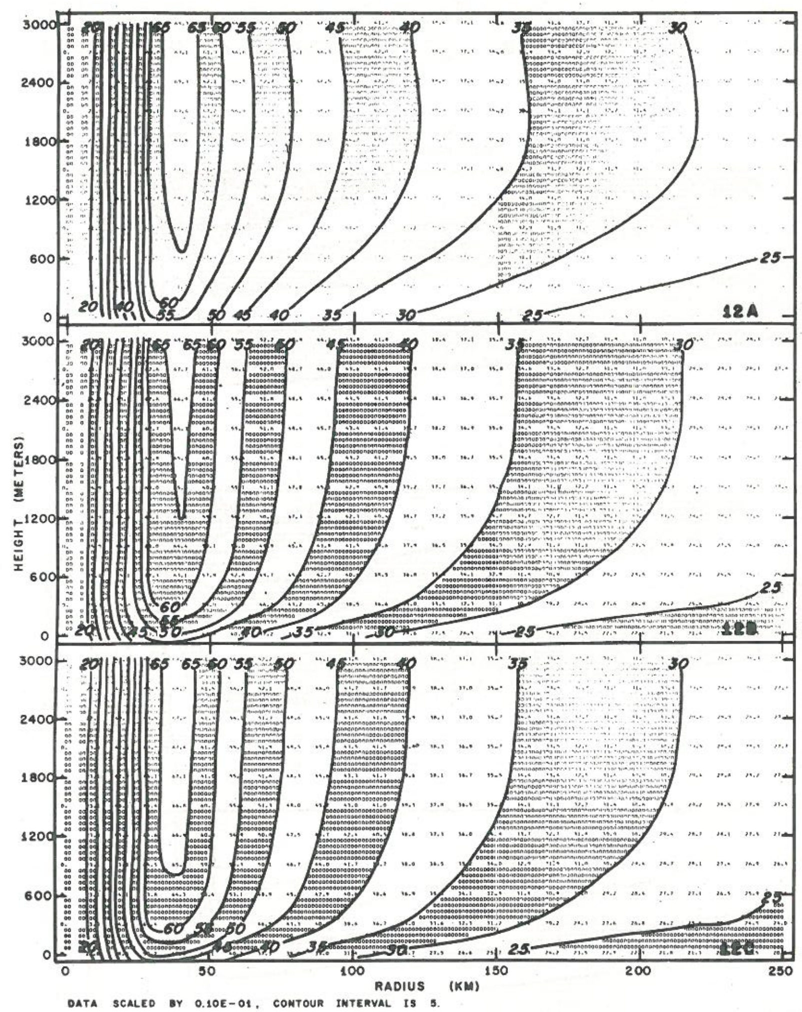

Figure 12. - Experiment II, tangential motion (m. sec.$^{-1}$), (a) Initial field (b) after 5 hr., (c) 2 hr. after (b) and with $K_h = 1.5 \times 10^8$ cm.^{2} sec.$^{-1}$.

59 m.p.s., at 30 km., compared to a maximum of 56 m.p.s. observed in Donna at a slightly smaller radius.

After 5 hr. the main features of the radial wind field had been preserved (fig. 10b). The net radial inflow at the outer boundary had decreased, although not as greatly as in Experiment I. The maximum radial winds at 40 km. increased from 15.0 m.p.s. to 18.7 m.p.s. Some outflow had developed at radii of 10 and 20 km. At other radii the depth of the inflow remained constant, although there was very little inflow above 2,000 m. Significant changes in the vertical motion field (fig. 11b) were observed. Strong downward motion was present at the center, with a value of almost 2 m.p.s. being present at 3,000 m. This is almost certainly too large, particularly when it is recalled that in a more complete model(Estoque [3])evaporational cooling in the center should intensify the downward motion. The maximum upward motion had shifted inward from 30 km. to 20 km. At other radii the vertical motion at 3,000 m. had decreased, but no downward motion occurred at outer radii as was the case in Experiment I. Outside a radius of about 150 km., there were no big changes in the tangential winds (fig. 12b). From 30 to 150 km. the surface winds decreased, and strong vertical shears developed between the surface and 300 m. The total shear between 300 and 3,000 m. was small, which seems to be characteristic of the hurricane. At radii of 10 and 20 km. winds at the interior grid points became super-gradient, presumably as a result of lateral diffusion and lateral advection.

After 5 hr., we again changed the lateral mixing, and continued the forecasts, this time for 2 hr. K_h was reduced from 3.0×10^8 to 1.5×10^8 cm.^{2} sec.$^{-1}$. Two interesting results occurred. First, the vertical descent in the center decreased, the point of maximum upward motion shifted outward, to coincide with the point of maximum tangential winds. Upward motion increased slightly from 40 km. to 80 km. Estoque [3] chose a value of 10^9 for K_h and Barrientos [1] suggested 3.0×10^8; in both models the maximum upward motion was inside the region of maximum tangential winds. The choice of 1.5×10^8 cm.^{2} sec.$^{-1}$ resulted in coincidence between the maximum upward motion and the maximum tangential winds, and may suggest that the other values are somewhat too high. The reduction of the lateral mixing also resulted in a decrease in the tangential winds at interior grid points at radii of 10 and 20 km.; these winds were still super-gradient but were more nearly in gradient balance than before the lateral mixing was reduced.

Some of the results from Experiment II can be compared with similar calculations made for hurricane Donna by Miller [9]. Figure 13 shows the surface stress as a function of radius. The stresses for Donna were determined by vertical integration of the tangential equation of motion between sea level and 700 mb., neglecting lateral mixing and $w(\partial v/\partial z)$. The two curves have almost identical shapes, but the model consistently underestimates the stress; we don't really know which is the more accurate. In figure 14 we have plotted the radial profile of the vertical motion at 3,000 m. From this curve we have computed the areal means for three radial intervals, corresponding to the intervals for which vertical motions at 700 mb. were available (in hurricane Donna). The two sets of means are superimposed upon figure 14 for comparative purposes. The similarity is considered satisfactory although the model consistently underestimates the vertical motion; this is due partly to the restriction that the model be non-divergent at the lateral boundary. Figure 15

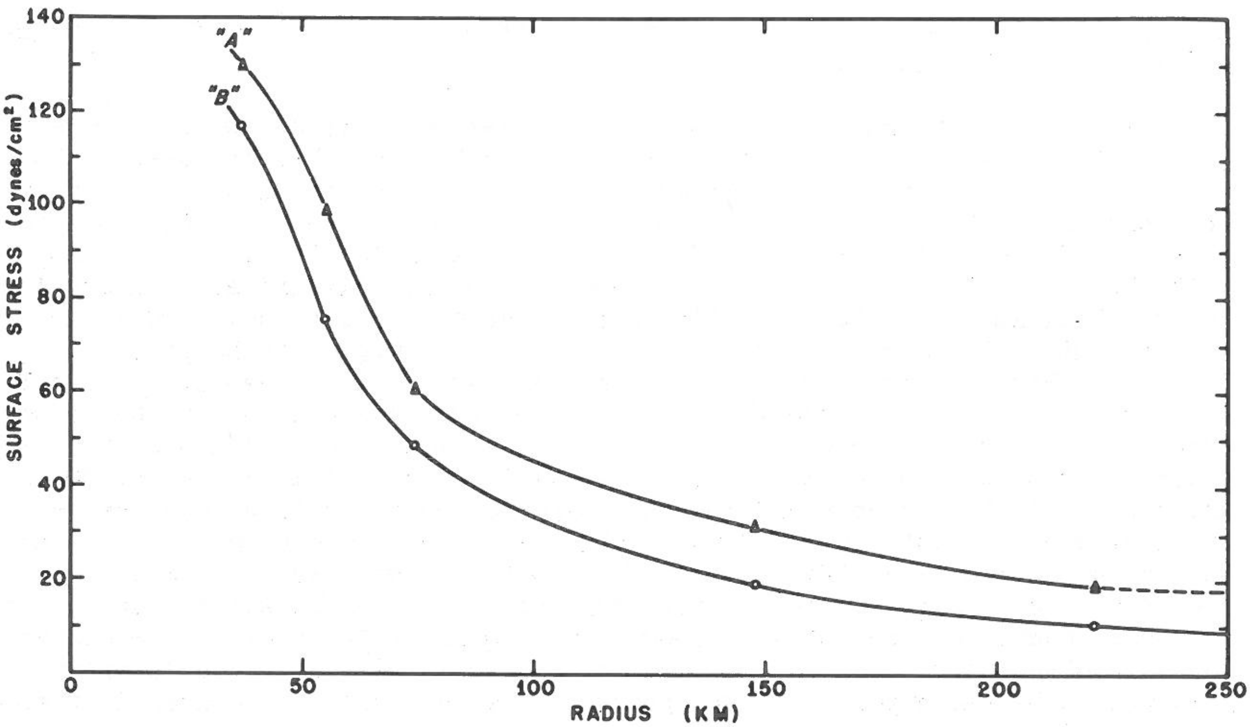

Figure 13. - Surface stress as a function of radius. Curve A (hurricane Donna, September 10, 1960), curve B (Experiment II).

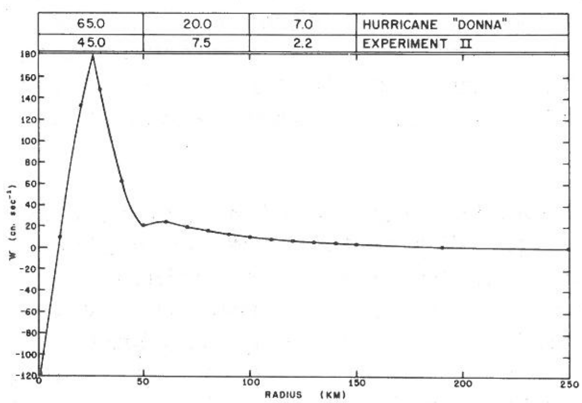

Figure 14. - Radial profile of vertical motion (cm. sec.$^{-1}$), Experiment II, at 3,000 m. with superimposed areal means compared with hurricane Donna at 700 mb. (September 10).

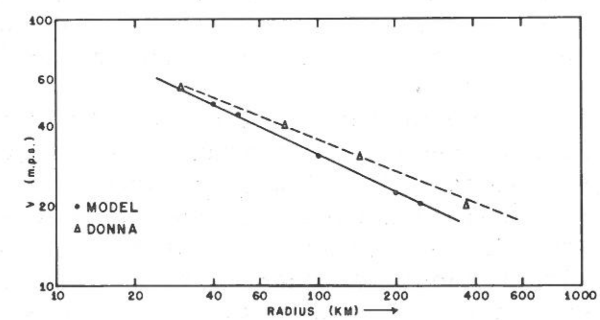

Figure 15. - Radial profile of tangential wind with similar profile for Donna (September 10).

shows a comparison between the radial profiles of the wind for the surface to 900-mb. level in Donna and the tangential wind at z = 0 in the model. The agreement between the two is excellent. The Donna wind profile could be approximated by a $V r^{0.48}$ = constant vortex, while the model is best fitted by a $V r^{0.52}$ = constant.

5. SUMMARY AND CONCLUSIONS

The formulation of a linearized model of the hurricane inflow layer, with K_m and C_d chosen to be radially constant, results in a model with the depth of the inflow layer decreasing radially inward. This decrease in depth causes the vertical motion at the top of the inflow layer to be too small. It is a simple matter to determine the radial variations of K_m and C_d necessary to make the depth of the inflow layer constant along the radius. By choosing K_m as a linear function of the pressure gradient force, C_d as a linear function of the surface wind speed, and an appropriate value for K_h (the coefficient for eddy viscosity for lateral mixing), a model can be constructed with a radially constant inflow depth, strong convergence just outside the radius of maximum winds, with the maximum upward motion at about the radius of maximum winds, and descending motion in the center. All these features seem to be supported by aircraft data, although the evidence is not conclusive, because of the difficulties in measuring radial and vertical winds. A comparison between the model and hurricane Donna (1962), upon which the model is based, indicates that the agreement between the model and the observations is satisfactory. Further comparisons between observations and models such as this one must await a more detailed measurement of the turbulent structure of the tropical cyclone by aircraft.

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