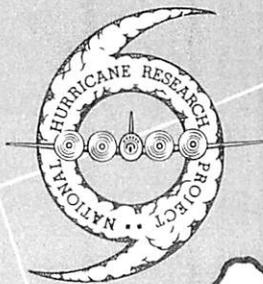


# NATIONAL HURRICANE RESEARCH PROJECT

REPORT NO. 56

A Theoretical Analysis of the Field of  
Motion in the Hurricane Boundary Layer



U. S. DEPARTMENT OF COMMERCE  
Luther H. Hodges, Secretary  
WEATHER BUREAU  
F. W. Reichelderfer, Chief

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in the Hurricane Boundary Layer

by

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NATIONAL HURRICANE RESEARCH PROJECT REPORTS

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A THEORETICAL ANALYSIS OF THE  
FIELD OF MOTION IN THE HURRICANE BOUNDARY LAYER

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ABSTRACT

Radial motions (and, hence, divergence and vertical motions) in a steady state, symmetrical vortex can exist only in the presence of tangential friction. The field of vertical motion in a mature hurricane must, therefore, be closely related to the frictional force field.

In the present paper, the hurricane inflow layer is treated as a generalized Ekman problem. The theoretical development is similar to a derivation presented in 1935 by B. Haurwitz in which the classical Ekman theory was modified to include centrifugal effects and momentum advection. Here, however, a more realistic pressure field is utilized.

When centrifugal and shearing effects are included, the Ekman layer equations are non-linear. To obtain solutions, the field of motion is considered as the sum of the gradient wind and a steady, frictionally produced perturbation. The usual perturbation techniques are used to linearize the equations and the field of motion is obtained for a model pressure field which resembles that of the lower layers of hurricanes. The solutions are discussed in detail.

1. INTRODUCTION

Penetration of hurricanes by meteorologically instrumented aircraft has led to vast progress in the description of these weather systems (see, for example, [ 1, 2, 11, 12, 13, 20]). Despite this, several aspects of hurricane structure remain largely speculative. Among these is the distribution of radial motion in the hurricane-inflow layer. Aircraft observations of radial velocity are of questionable reliability because this velocity component has a relatively small magnitude. Furthermore, the vertical variation of radial motion in the inflow layer cannot be measured because simultaneous penetration by two or more aircraft in the lowest 2-3 km. is hazardous.

From thermodynamic considerations, Palmén and Riehl [17] concluded that inflow must be restricted to the lower 10,000 feet of the storm. This conclusion is consistent with the mean hurricanes composited from standard data networks by E. S. Jordan [8] and Miller [16]. Aircraft penetration has also, in some storms [11, 12], indicated that net inflow is restricted to the lowest 10,000 feet. On the other hand, some hurricanes have shown net inflow at, and above, 500 mb. [ 20, 3, 19]. Riehl [3, 19] indicates that net inflow in the middle and higher troposphere is only to be found in the periphery of the storm. This would indicate that the depth of the inflow layer decreases as one moves inward from the outskirts of the storm. In another paper, however, Malkus and Riehl ([14] see p. 6) suggest that the depth of the inflow layer may increase by a factor of two as one progresses toward the storm center.

The research reported on below was performed for the purpose of exploring the structure of the inflow layer from a theoretical point of view. It was our hope that the theoretical results would clarify some of the disagreement produced by the various observational studies. As is frequently the case, however, the theoretical work replaces one problem with another. In this case, the structure of the inflow layer is strongly dependent upon the austausch coefficient for momentum; certain aspects of our results appear to be quite unrealistic due to improper formulation of the austausch mechanism.

It should also be pointed out that the essence of this model was first proposed by Haurwitz [5,6]. Our interpretation of the model, however, is quite different from that of Haurwitz.

## 2. THEORY

For the sake of simplicity, we will limit ourselves to steady state conditions. The model is then intended to represent mature, slowly moving storms. We will employ cylindrical coordinates,  $r, \theta, z$ ;  $r$  is radial distance from the origin (measured in a horizontal plane),  $\theta$  is the azimuth angle, and  $z$  is height above mean sea level. The storm center is at  $r = 0$ . The model is further simplified through neglect of derivatives with respect to azimuthal distance ( $\frac{1}{r} \frac{\partial}{\partial \theta}$ ). Experience with hurricane data indicates that these derivatives are usually small compared to derivatives with respect to radial distance.

In view of the above, our equations will be written for steady, symmetrical flow. The component of the equation of horizontal motion in  $\theta$ -direction is then

$$v_r \zeta_a + w \frac{\partial v_\theta}{\partial z} = F_\theta \quad (1)$$

$v_r, v_\theta,$  and  $w$  are, respectively, the radial, tangential and vertical components of the velocity.  $F_\theta$  is the component of the viscous force in the  $\theta$ -direction and  $\zeta_a$  is the absolute vorticity,

$$\zeta_a = \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} + f \quad (2)$$

( $f$  is the Coriolis parameter). In the hurricane-inflow layer,  $w(\partial v_\theta)/(\partial z)$  is small compared to  $v_r \zeta_a$  [14, 21] and

$$v_r \zeta_a \approx F_\theta \quad (3)$$

Equation (3) shows that no radial motions and, from mass continuity, no vertical motions can exist in the absence of tangential friction (except in the trivial case where  $\zeta_a \equiv 0$ ).

For steady, symmetrical flow, the radial component of the equation of horizontal motion is, with sufficient accuracy [21],

$$-\frac{v_\theta^2}{r} - f v_\theta = -\frac{1}{\rho} \frac{\partial p}{\partial r} + F_r \quad (4)$$

$F_r$  is the radial component of the frictional force per unit mass;  $\rho$  is the density and  $p$  is the pressure. If austausch assumptions are made for  $F_r$  and  $F_\theta$ , equations (3) and (4) contain the four dependent variables  $v_\theta$ ,  $v_r$ ,  $\rho$  and  $p$ . After Haurwitz [5,6], the system (3) and (4) will be treated as a generalized Ekman problem. That is,  $\frac{1}{\rho} \frac{\partial p}{\partial r}$  will be specified as a function of radius alone and solutions for  $v_\theta$  and  $v_r$  will be obtained. In view of the fact that observations indicate virtually no horizontal gradient of temperature in the lower layers of hurricanes, the assumption regarding the constancy of  $\frac{1}{\rho} \frac{\partial p}{\partial r}$  with height appears to be fairly realistic for the inflow layer.

As noted earlier, we plan to utilize austausch formulations for  $F_\theta$  and  $F_r$ . Haurwitz [5] has shown that closed solutions, which include both lateral and vertical austausch effects, are only attainable for very special pressure fields. Since these pressure fields are not at all representative of hurricane conditions, lateral mixing will be neglected so that a realistic pressure profile can be employed. For  $F_\theta$  and  $F_r$ , we write

$$F_\theta = K_1 \frac{\partial^2 v_\theta}{\partial z^2} \quad (5)$$

$$F_r = K_1 \frac{\partial^2 v_r}{\partial z^2} \quad (6)$$

Equations (5) and (6) imply the further restriction that  $K_1$  (the kinematic coefficient of eddy viscosity for vertical mixing) is independent of height. From (3), (4), (5), and (6), we obtain the system

$$K_1 \frac{\partial^2 v_\theta}{\partial z^2} = v_r \zeta_a \quad (7)$$

$$K_1 \frac{\partial^2 v_r}{\partial z^2} = \frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{v_\theta^2}{r} - f v_\theta \quad (8)$$

which is non-linear.

As suggested by Haurwitz [5,6], we will linearize these equations by perturbation techniques. We first note, however, that Haurwitz [5] was able to show that the solutions to the linearized equations closely approximate the solutions to the non-linear equations for the special case where

$$\frac{1}{\rho} \frac{\partial p}{\partial r} \propto r.$$

We assume

$$v_\theta = v_{\theta g} + v'_\theta \quad (9)$$

$$v_r = v'_r \quad (10)$$

where  $v_{\theta g}$  is the gradient wind for steady, symmetrical, non-anomalous flow (i.e.)

$$\frac{v_{\theta g}^2}{r} + f v_{\theta g} = \frac{1}{\rho} \frac{\partial p}{\partial r} \quad (11a)$$

$$v_{\theta g} = -\frac{rf}{2} + \sqrt{\left(\frac{rf}{2}\right)^2 + \frac{r}{\rho} \frac{\partial p}{\partial r}} \quad (11b)$$

$v'_\theta$  and  $v'_r$  are perturbation quantities. Substitution of (9) and (10) into (7) and (8), neglect of second order quantities, and utilization of (11a) leads to

$$K_1 \frac{\partial^2 v'_r}{\partial z^2} = -\left(f + \frac{2v_{\theta g}}{r}\right) v'_\theta \quad (12)$$

$$K_1 \frac{\partial^2 v'_\theta}{\partial z^2} = \zeta_{ag} v'_r \quad (13)$$

where

$$\zeta_{ag} = \frac{\partial v_{\theta g}}{\partial r} + \frac{v_{\theta g}}{r} + f. \quad (14)$$

Elimination of  $v'_r$  between (13) and (12) gives

$$K_1^2 \frac{\partial^4 v'_\theta}{\partial z^4} = -\zeta_{ag} \left(f + \frac{2v_{\theta g}}{r}\right) v'_\theta \quad (15)$$

We define

$$\lambda = \lambda(r) = \left\{ \frac{\zeta_{ag} \left(f + \frac{2v_{\theta g}}{r}\right)}{4 K_1^2} \right\}^{\frac{1}{4}} \quad (16)$$

The general solution of (15) is given by

$$\begin{aligned} v'_\theta = e^{-\lambda z} [F_1(r) \cos \lambda z + F_2(r) \sin \lambda z] \\ + e^{\lambda z} [F_3(r) \cos \lambda z + F_4(r) \sin \lambda z] \end{aligned} \quad (17)$$

where  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_4$  are arbitrary functions of  $r$ .

We impose the usual requirement that  $v'_\theta$  remain finite as  $z$  increases.

This implies that  $F_3 = F_4 = 0$ . From (13), we obtain

$$v'_r = v_r = -\frac{2\lambda^2 K_1}{\zeta_{ag}} e^{-\lambda z} [F_2 \cos \lambda z - F_1 \sin \lambda z]. \quad (18)$$

By use of (9), we have

$$v_{\theta} = v_{\theta g} + e^{-\lambda z} [F_1 \cos \lambda z + F_2 \sin \lambda z] \quad (19)$$

The austausch formulation gives the stress components at  $z = 0$  (a few meters above the sea surface) as

$$\begin{aligned} \tau_{rz}(0) &= \rho_0 K_1 \left( \frac{\partial v_r}{\partial z} \right)_{z=0} \\ \tau_{\theta z}(0) &= \rho_0 K_1 \left( \frac{\partial v_{\theta}}{\partial z} \right)_{z=0} \end{aligned} \quad (20)$$

$\rho_0$  is the  $z = 0$  density. The well known empirical formulations for  $\tau_{rz}(0)$  and  $\tau_{\theta z}(0)$  are

$$\begin{aligned} \tau_{rz}(0) &= \rho_0 K_F V_0 v_r(0) \\ \tau_{\theta z}(0) &= \rho_0 K_F V_0 v_{\theta}(0) \end{aligned} \quad (21)$$

where  $V_0$  is the wind speed at  $z = 0$  and  $K_F$  is the drag coefficient. Replacement of  $V_0$  by  $v_{\theta g}$  and elimination of the stress components between (20) and (21) gives

$$\left( \frac{\partial v_r}{\partial z} \right)_{z=0} = \frac{K_F}{K_1} v_{\theta g} v_r(0) \quad (22a)$$

$$\left( \frac{\partial v_{\theta}}{\partial z} \right)_{z=0} = \frac{K_F}{K_1} v_{\theta g} v_{\theta}(0) \quad (22b)$$

Equations (22a) and (22b) are now used to evaluate  $F_2$  and  $F_1$ . This yields

$$F_1 = - \frac{(\lambda + \chi v_{\theta g}) \chi v_{\theta g}^2}{[(\lambda + \chi v_{\theta g})^2 + \lambda^2]} \quad (23)$$

and

$$F_2 = \frac{\lambda \chi v_{\theta g}^2}{[(\lambda + \chi v_{\theta g})^2 + \lambda^2]} \quad (24)$$

where

$$\chi = \frac{K_F}{K_1} \quad (25)$$

From (18), (23) and (24), the top of the inflow layer is given by the smallest value of  $h$  which will satisfy

$$h = \frac{1}{\lambda} \arctan \left[ - \frac{\lambda}{\lambda + \chi v_{\theta g}} \right] \quad (26)$$

As pointed out earlier, the solution (as represented by (18), (19), (23), (24)) is similar to that obtained by Haurwitz [5,6]. The difference between his solution and ours stems from a different formulation of the lower boundary condition. Haurwitz's lower boundary condition can be obtained by setting

$\frac{K_F}{K_L} v_{\theta g}$  in (22a) and (22b) equal to a constant. Because of this difference,

we have taken the liberty of reproducing our derivation.

The computations, discussed in the next section, utilize a functional form for  $\partial p / \partial r$  which has been found to be quite representative of the hurricane-pressure field (see, for example, [4]). This expression, for the  $z = 0$  level, is

$$\frac{\partial p}{\partial r} = \frac{(p_N - p_0)}{r^2} R e^{-R/r} \quad (27)$$

$p_0$  is the central pressure,  $p_N$  is the pressure which is approached as  $r \rightarrow \infty$  and  $R$  is twice the radius of maximum  $\partial p / \partial r$ . In the model,  $\frac{1}{\rho} \frac{\partial p}{\partial r}$ , not  $\frac{\partial p}{\partial r}$  is assumed independent of height. To achieve this, we write

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \left( \frac{p_N - p_0}{\rho} \right) \left( \frac{R}{r^2} \right) e^{-R/r} \quad (28)$$

where  $\left( \frac{p_N - p_0}{\rho} \right)$  is assumed independent of height. For computational purposes, we selected  $\left( \frac{p_N - p_0}{\rho} \right) = 6 \times 10^3 \text{ cb. ton}^{-1} \text{ m.}^3$ . This is equivalent to  $p_N - p_0$  of 60 mb. at  $\rho = 1 \times 10^{-3} \text{ ton m.}^3$ .  $R$  was chosen to be 30 km.

### 3. DISCUSSION

When  $K_L$  is independent of  $r$ , we find, from (16), that  $\lambda$  increases as the fourth root of  $\zeta_{ag} \left( f + \frac{2 v_{\theta g}}{r} \right)$ . From (26), we find that  $h$  decreases with increasing  $\lambda$ . Figure 1 shows the  $\zeta_{ag}$  profile computed from (28) and (11b) ( $f = 5 \times 10^{-5} \text{ sec.}^{-1}$  (20° N. lat.)).  $\zeta_{ag}$  increases from the storm periphery toward the center and then, very close to the center, decreases sharply. Since  $v_{\theta g} / r$  follows a similar pattern,  $\lambda$  will increase as the storm center is approached and then, close to the center,  $\lambda$  will decrease as the radius becomes

still smaller. Therefore, within the framework of the model,  $h$  will decrease inward from the outskirts of the storm until the maximum  $\lambda$  is reached. At still smaller radii,  $h$  will increase.

Haurwitz, in 1936 [15], contended that this behavior of  $h$  (decreasing  $h$  with increasing  $\lambda$ ) could partially explain the hurricane eye. Haurwitz argued that if  $h$  reaches zero, the "central core of warmer air will not be lifted up by the air inflowing from the outside". From this, Haurwitz reasoned that ascent could not take place in the storm center and that the clear air of the eye is understandable in terms of the lowering of the inflow layer due to the increase of  $\zeta_{ag}$  and  $v_{\theta g}/r$ . The Haurwitz effect appears to be consistent with the description of  $h$  given by Gangopadhyay and Riehl in [3].

However, a point not discussed by Haurwitz has to do with the fact that the model requires  $h$  to increase inward of the radius of maximum  $\lambda$ . Our feeling is that this aspect of the solution is mathematical fiction produced by the assumption of a radially constant  $K_1$ .

The suggestion by Malkus and Riehl [14], that  $h$  increases by a factor of two between the outer periphery and the core of a hurricane, is in direct conflict with the Haurwitz effect. Within the framework of the present model, an increase in  $h$  between the outskirts and the storm core would require  $K_1$  to increase more rapidly (percentagewise) than  $\zeta_{ag} (f + \frac{2v_{\theta g}}{r})$ .

We now turn to a quantitative assessment of equations (18) and (19) for the case where  $K_1 = 50 \text{ m.}^2/\text{sec}$ . This value of  $K_1$  is of the order of magnitude suggested by Kasahara [9] based on budget studies of hurricanes by Kasahara [10] and Syono [23]. Haurwitz [15] also used  $K_1 = 50 \text{ m.}^2/\text{sec}$ .  $K_F$  was set equal to  $3 \times 10^{-3}$ . This value seems to be appropriate for strong winds over water [9,22,18]. Budget studies conducted by Riehl and Malkus [20], Riehl [19], and Miller [15] have confirmed that  $K_F \sim 10^{-3}$  in actual hurricanes.

Figure 2 shows  $h$  as a function of  $r$ . We note a decrease from  $h \approx 2.5 \text{ km}$ . at  $r = 250 \text{ km}$ . to  $h \approx 320 \text{ m}$ . at  $r = 10 \text{ km}$ . Figure 3 shows the  $v_{\theta g}$  distribution. Here we see the more-or-less typical radial profile of tangential

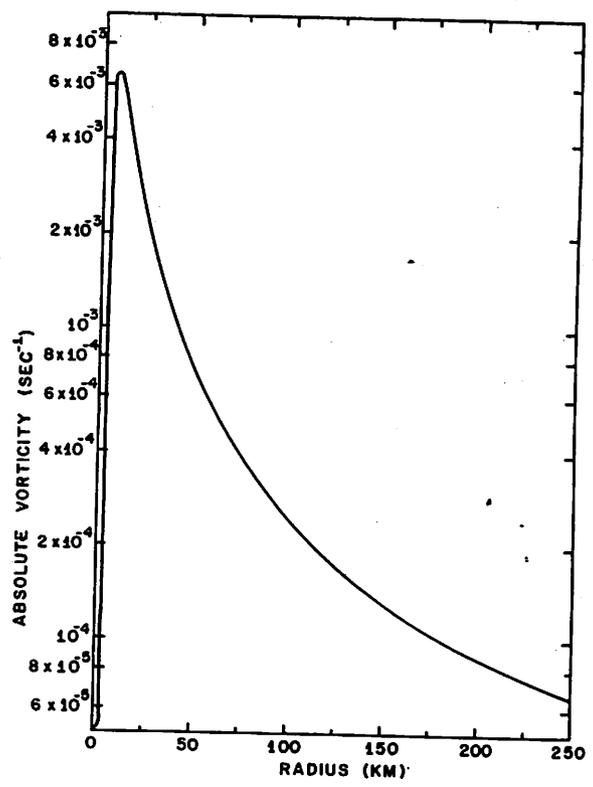


Figure 1. - Radial profile of  $\zeta_{ag} = \frac{\partial v_{\theta g}}{\partial r} + \frac{v_{\theta g}}{r} + f$ .

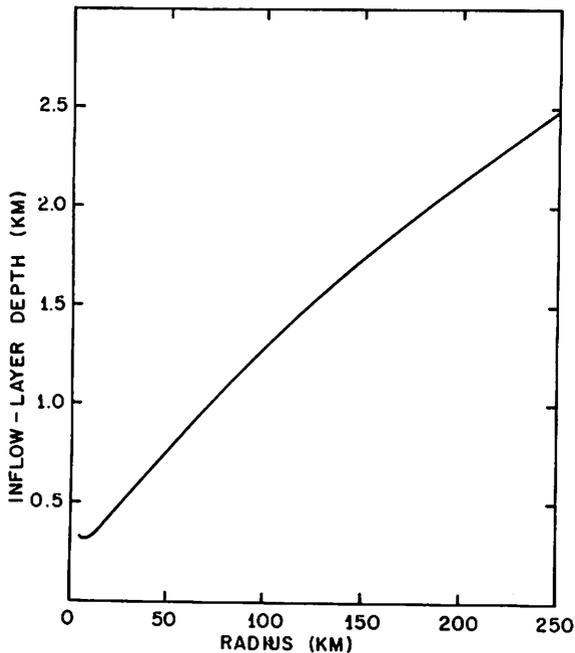


Figure 2. - Depth of the inflow layer as a function of radius.

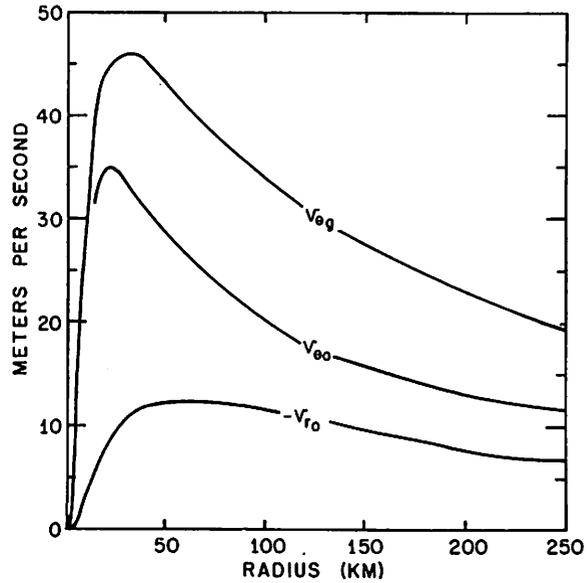


Figure 3. - Radial profiles of gradient wind ( $v_{eg}$ ), surface tangential wind ( $v_{e0}$ ) and surface radial wind ( $v_{r0}$ ).

winds associated with moderate hurricanes. Maximum winds are  $46 \text{ m. sec.}^{-1}$  at about  $r = 30 \text{ km}$ . The surface tangential ( $v_{e0}$ ) and radial ( $v_{r0}$ ) winds are also entered in figure 3. We note that the vertical shear in  $v_e$  between  $z = 0$  and  $z = h$  is quite large and, over much of the storm, exceeds  $10 \text{ m. sec.}^{-1}$ . Although a certain amount of vertical shear must be present in a real hurricane, the available observational material is not sufficient to ascertain whether or not the computed shears are excessively large. Jordan [7] has shown that vertical-wind shears over islands can be quite large in typhoon situations. Whether or not this is also true over the oceans is an open question.

The radial variation of  $v_{r0}$  appears to be consistent with that found in other hurricane models [14,21], in that maximum  $v_{r0}$  occurs at a considerably larger  $r$  than does maximum  $v_{e0}$ . Again, observational materials are not adequate for assessing the reality of the  $v_{r0}$  profile except for indirect checks such as interpreting theoretical profiles of divergence and vertical motion in terms of cloud patterns. To a limited extent, this will be considered later.

Figure 4 shows the surface-inflow angle as a function of radius. The shape of the profile agrees quite well with the inflow-angle profiles assumed by Malkus and Riehl [14] and Rosenthal [21]; the angle has little radial variation at radii greater than 100 km. but decreases rapidly with decreasing radius inward of  $r = 100 \text{ km}$ . We note that the maximum-inflow angle exceeds  $30^\circ$ .

This is much larger than would be expected from the Malkus-Riehl model [14] or from the model previously examined by the author [21]. We must, therefore, conclude that [14] and [21] significantly underestimated the frictional force or that the present model overestimates this force.

The model-radial velocities vary with height. Over much of the storm, the surface values are significantly smaller than the values at the height of maximum  $v_r$ . To illustrate this, we have prepared figure 5 which shows a radial profile of the maximum  $v_r$ .

(These values occur at different heights. Therefore, this curve is not representative of any particular level). Figure 5 shows radial motions whose magnitudes exceed  $14 \text{ m. sec.}^{-1}$ . This appears to be quite large for a hurricane of moderate intensity.

Figure 6 shows the radial profile of maximum convergence and the height at which this maximum convergence takes place. The maximum convergence is quite strong in the core of the storm where it approaches values of  $10^{-3} \text{ sec.}^{-1}$ . The vertical motion at  $z = h$  (fig. 7) is strongest close to the region of strongest  $v_{\theta g}$  but at a considerably larger radius than that of the maximum convergence. This, of course, reflects the fact that  $h$  decreases as  $r$  decreases.

The most important aspect of figures 6 and 7 is the result that vertical motions are quite weak despite the rather strong convergences. This would seem to indicate that the  $h$  values in the storm core are much too small. To obtain realistic vertical motions in the wall-cloud region,  $h$  must decrease inward much more slowly than indicated by figure 2. Indeed the Malkus-Riehl [14] suggestion that  $h$  increases from the storm periphery to the storm core may very well be correct.

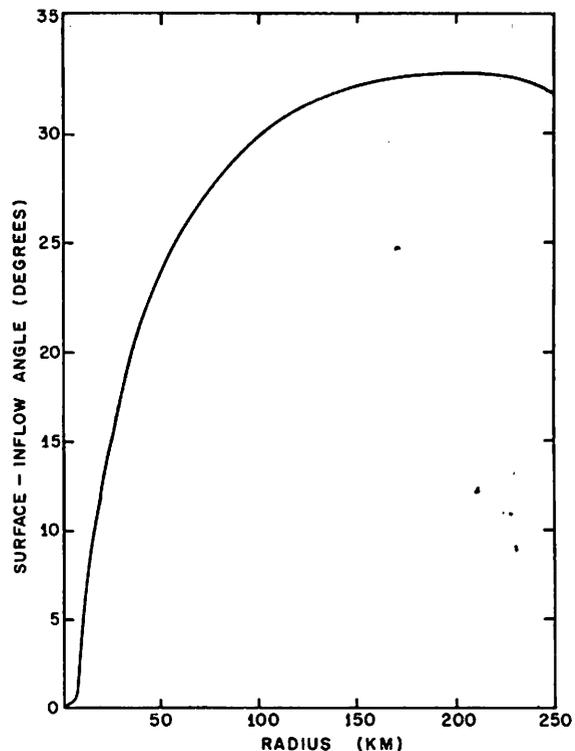


Figure 4. - Surface-inflow angle as a function of radius.

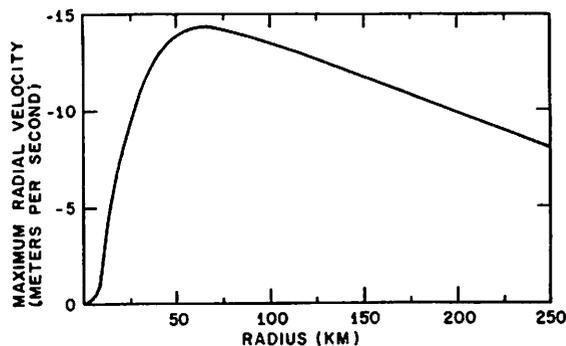


Figure 5. - Radial profile of radial velocity along the surface of maximum radial motion.

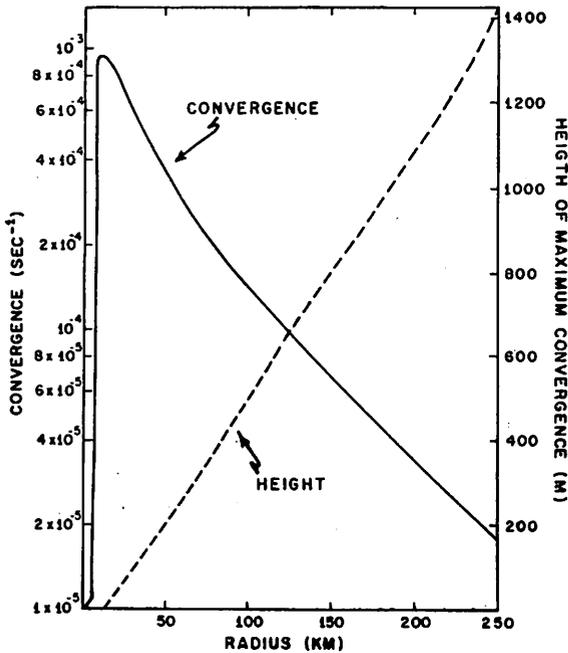


Figure 6. - Radial profiles of: (1) convergence along the surface of maximum convergence, and (2) the height of the surface of maximum convergence.

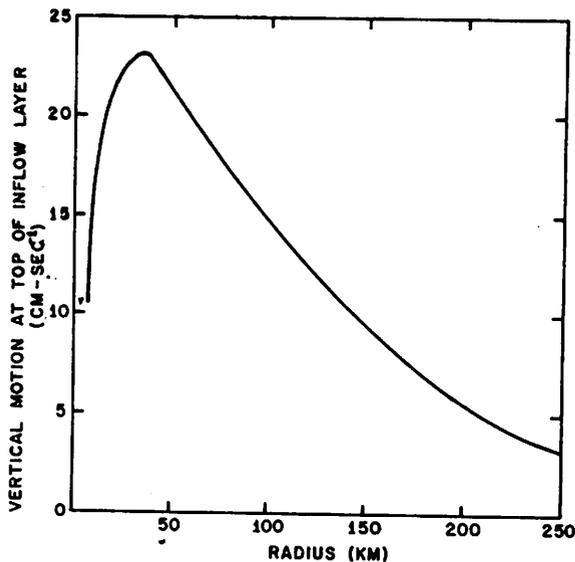


Figure 7. - Radial profile of the vertical motion at  $z = h$ .

As noted earlier, this type of variation in  $h$  could be incorporated into the present model by allowing  $K_1$  to increase inward from the outskirts of the storm. To produce this effect, in a manner which would be physically attractive,  $K_1$  should be related to the velocity field. However, since there is little in the way of theoretical or observational material to guide such a formulation, it does not appear to be prudent to attempt such a generalization of the model at this time.

#### 4. CONCLUSIONS

A radially constant Austausch coefficient produces a very shallow inflow layer in the vicinity of the storm core. As a result, even strong convergences produce rather weak vertical motions in the wall-cloud region. These results indicate that the Austausch coefficient must be allowed to increase as the storm core is approached. However, the proper formulation for a varying Austausch coefficient is unknown.

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