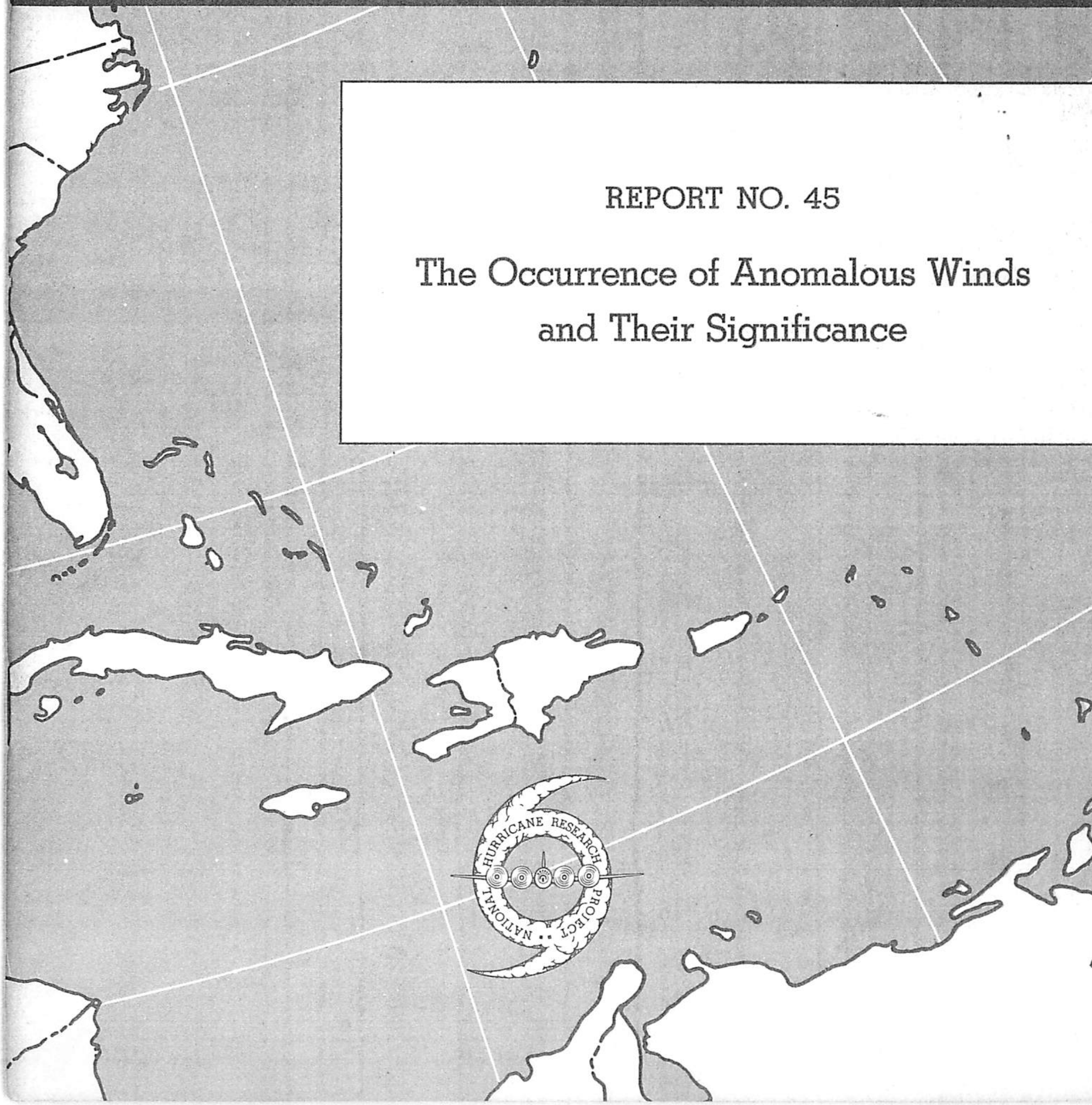
NATIONAL HURRICANE RESEARCH PROJECT

REPORT NO. 45

The Occurrence of Anomalous Winds and Their Significance



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The Occurrence of Anomalous Winds and Their Significance

by

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National Hurricane Research Project, Miami, Fla.

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THE OCCURRENCE OF ANOMALOUS WINDS AND THEIR SIGNIFICANCE

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ABSTRACT

Observational evidence is provided for the occurrence of anomalous winds which represent an anticyclonic rotation in space, and a mechanism for their development is suggested.

The unstable nature of these winds and the role they play in the development of certain types of atmospheric disturbances is then discussed, and it is suggested that anomalous winds provide the dynamic mechanism for triggering hurricane formation and for the observed deepening of troughs downstream from intense pressure ridges.

Finally it is noted that although the observational evidence presented is for the occurrence of anomalous winds over small regions of the atmosphere, their development is dependent on large-scale processes and their effect extends beyond the area where they occur.

1. INTRODUCTION

The gradient wind equation is a quadratic and thus has two solutions. One of these solutions, appropriate to anticyclonic flow, represents a clockwise rotation in space and is, therefore, in the opposite sense to the earth's rotation. Meteorologists have traditionally given little attention to this solution and some even consider it as an algebraic accident with little or no physical significance.

Although there are cogent reasons to believe that winds corresponding to this solution do not occur on a large scale, the contention, ipso facto, that this solution is of no importance is not justified and has probably had far-reaching effects in eliminating from consideration some promising avenues of research in connection with the development of atmospheric disturbances.

In recent years, the importance of meso-scale atmospheric motion has been amply demonstrated. The purpose of this paper is to provide observational evidence that, on this scale, these so-called "anomalous" winds occur more frequently than has been suspected.

In addition, the paper purports to show that the occurrence of these winds is germane to the development of dynamic instability in curved airflow and, as such, is relevant to the development of certain types of atmospheric circulations.

2. ELEMENTARY DYNAMICS OF ANTICYCLONIC MOTION

To introduce the problem, we shall review some elementary aspects of the dynamics of anticyclonic motion.

The gradient wind equation may be written:

$$K_t V^2 + fV = b_n \tag{1}$$

where $K_t = 1/R_t$ = trajectory curvature considered positive for cyclonic motion; V = magnitude of the horizontal wind; f = Coriolis parameter; and $b_n = -\alpha \frac{\partial p}{\partial n}$ = pressure gradient force. From (1)

$$V = -\frac{f}{2K_t}\left(1 \pm \sqrt{1 + \frac{4K_t b_n}{f^2}}\right) \tag{2}$$

Since V must always be positive, the solution with the plus sign has a physical meaning only when K_t is negative; i.e., when the flow is anticyclonic.

We shall confine our discussion to anticyclonic flow and put $K'_t = -K_t$. The two solutions of equation (2) reduce to:

$$V_1 = \frac{f}{2K'_t}\left(1 - \sqrt{1 - \frac{4K'_t b_n}{f^2}}\right) \tag{3}$$

and

$$V_2 = \frac{f}{2K'_t}\left(1 + \sqrt{1 - \frac{4K'_t b_n}{f^2}}\right) \tag{4}$$

From the above equations it can be seen that

$$V_1 \leq \frac{f}{2K'_t} \tag{5}$$

and

$$V_2 \geq \frac{f}{2K'_t} \tag{6}$$

We differentiate equation (1) with respect to V

$$\frac{db_n}{dV} = 2VK_t + f \tag{7}$$

and note that db_n/dV can be zero; i.e., b_n has an extreme value if K_t is negative. This value is reached when

$$V = -\frac{f}{2K_t} = \frac{f}{2K'_t} \tag{8}$$

and represents the maximum value

$$b_n = -\frac{f^2}{4K_t} = \frac{f^2}{4K'_t} \tag{9}$$

which the pressure gradient can attain in anticyclonic motion.

Figure 1 shows the variation of V with b_n in accordance with equation (1). It is seen that the two solutions meet at $f/2K'_t$ when the pressure gradient is at its maximum value $f^2/4K'_t$. From this point, the two solutions V_1 and V_2, corresponding respectively to equations (3) and (4) branch out. V_1 decreases with decreasing pressure gradient and vanishes when the latter becomes zero.

This corresponds to the conditions normally observed in the atmosphere. We shall therefore term V_1 the normal solution of the anticyclonic gradient wind equation. V_2, on the other hand, increases with decreasing pressure gradient and at $b_n = 0$ reaches the value $V_1 = f/K'_t$ and the flow becomes inertial. Following Gustafson [5], we shall term V_2 the anomalous solution.

Figure 2 shows the variation of the gradient anticyclonic wind with trajectory curvature, assuming a constant pressure gradient. It is seen that, in contrast with normal anticyclonic winds which increase with increasing curvature, anomalous winds show a sharp decrease with increasing curvature.

As mentioned in the Introduction, meteorologists in general have attributed little importance to the anomalous solution on the basis that it is seldom, if ever, realized in the atmosphere [4] or that its occurrence is limited to small-scale mechanically produced vortices, or to atmospheric eddies produced by friction [9]. The standard arguments in support of this view are usually some variation of the following:

a. The anomalous solution requires a clockwise rotation in space and, therefore, in the opposite sense to that of the earth. There is no known mechanism capable of producing such a motion on a large scale.

b. By expanding the quantity under the radical sign in equations (3) and (4), we obtain

$$V_1 = \frac{f}{2K'_t}\left[1 - \left(1 - \frac{2K'_t b_n}{f^2} - \frac{2K'^2_t b_n^2}{f^4} + \dots\right)\right] = \frac{b_n}{f} + \frac{K'_t b_n^2}{f^3} + \dots \tag{10}$$

and

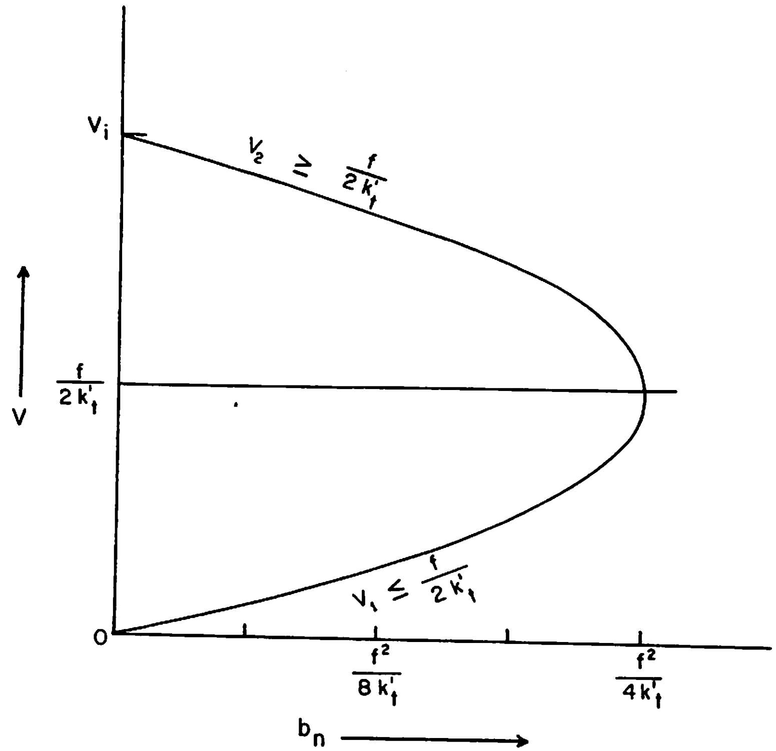

Figure 1. - Variation of gradient anticyclonic wind speed with pressure gradient.

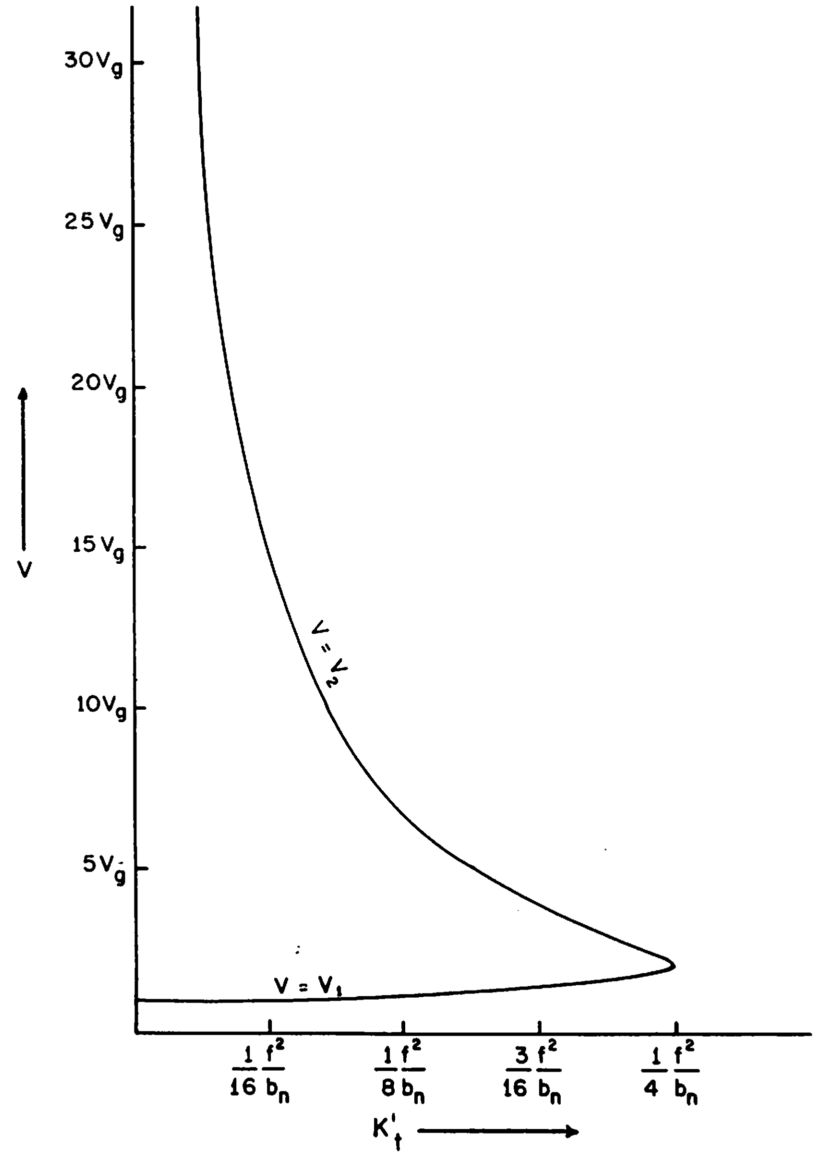

Figure 2. - Variation of gradient anticyclonic wind speed with trajectory curvature. V_g denotes the geostrophic wind.

$$V_2 = \frac{f}{2K'_t}\left[1 + \left(1 - \frac{2K'_t b_n}{f^2} - \frac{2K'^2_t b_n^2}{f^4} + \ldots\right)\right] = \frac{f}{K'_t} - \frac{b_n}{f} - \frac{K'_t b_n^2}{f^3} + \ldots \qquad (11)$$

V_1 is thus continuous when the curvature decreases indefinitely and the isobars are nearly straight. On the other hand, for indefinitely small curvature, V_2 becomes infinitely large and would require an infinite supply of energy, which is not available in the atmosphere [4].

3. OBSERVATIONAL EVIDENCE FOR THE OCCURRENCE OF ANOMALOUS WINDS

While the arguments presented in the preceding section strongly indicate that anomalous winds are not likely to occur on a large scale or in the vicinity of straight isobars, the possibility remains that these winds occur in limited regions where the flow is curved. Gustafson [5] has provided indirect evidence that anomalous winds do occur by comparing observed winds with geostrophic winds and applying the following criteria which follow directly from equations (5) and (6) above:

$$V_1 \leq 2V_g \cos(\psi - \psi_g) \qquad (12)$$

and

$$V_2 \geq 2V_g \cos(\psi - \psi_g) \qquad (13)$$

where V_g is the geostrophic wind speed and ψ and ψ_g denote respectively the direction of the observed and geostrophic wind measured clockwise from due east.

Since the summer of 1956, the National Hurricane Research Project (NHRP), U. S. Weather Bureau, has been operating three specially instrumented airplanes to make detailed observations in and near hurricane cores. A discussion of the characteristics and properties of the instrumentation was given by Hilleary and Christensen [8] and will not be repeated here. Among the most successful missions flown were those in connection with hurricane Daisy which developed near the Bahamas on August 24, 1958. This hurricane had a well-defined and concentrated wind circulation and presented a clear-cut radar configuration which greatly facilitated the location of the storm core. Flight missions were made at different levels on four days from August 25 to August 28, 1958. Among the elements measured were the wind, the temperature, and the radio and pressure altitudes. Quasi-instantaneous values of these parameters were punched on cards at specified intervals ranging from 10 seconds away from the core, to 2 seconds in the core. The punch cards were then evaluated by machine processing and the various parameters were plotted on a coordinate system fixed with respect to the storm center.

Figures 3, 4, and 5 show the wind field obtained by analyzing the observations made in the upper troposphere on August 25, 26, and 27. In making these analyses, it was necessary to regard as quasi-simultaneous, observations

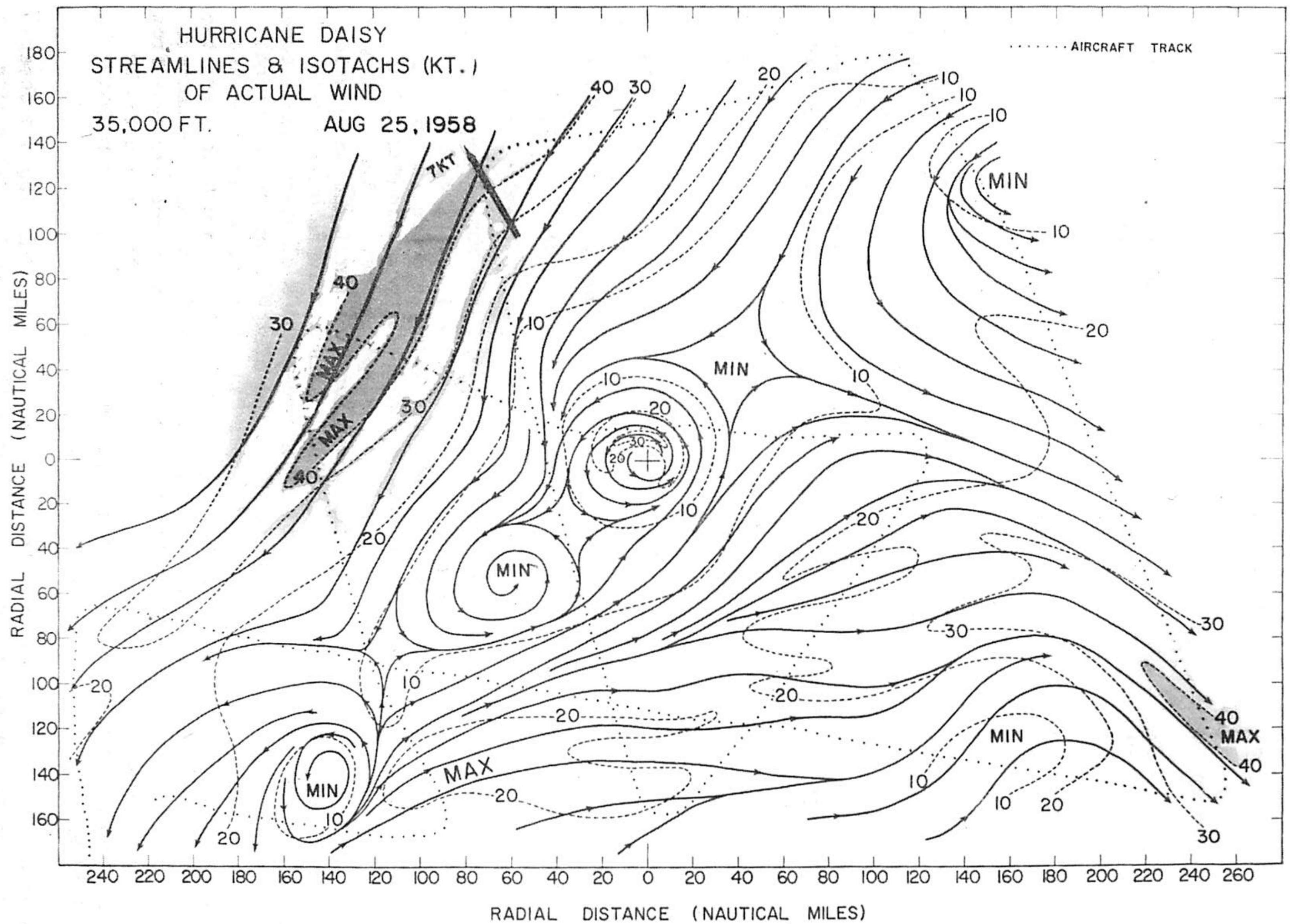

Figure 3. - Wind field at 35,000 feet (pressure altitude, U. S. Standard), around hurricane Daisy on August 25, 1958.

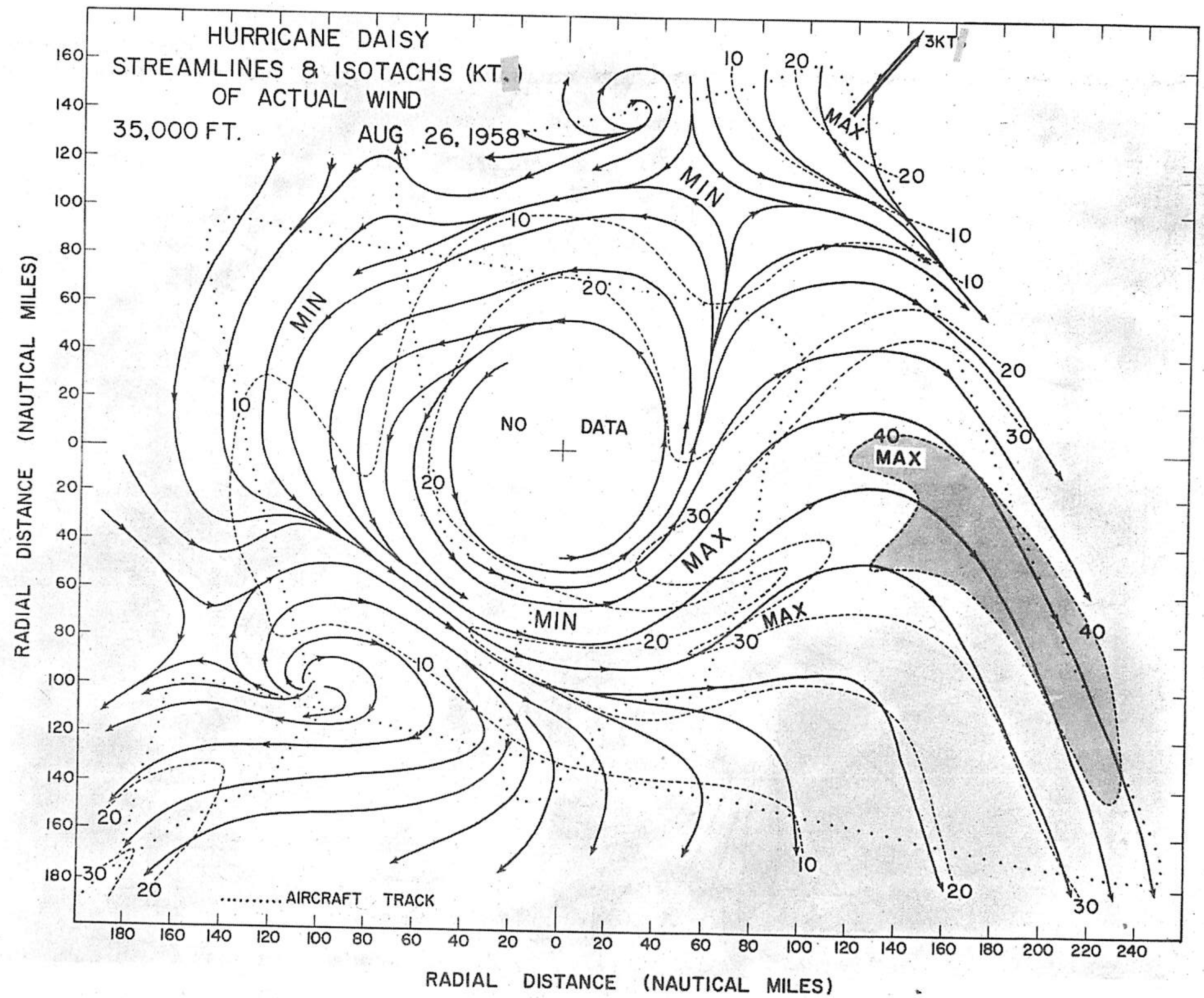

Figure 4. - Wind field at 35,000 feet (pressure altitude, U. S. Standard), around hurricane Daisy on August 26, 1958.

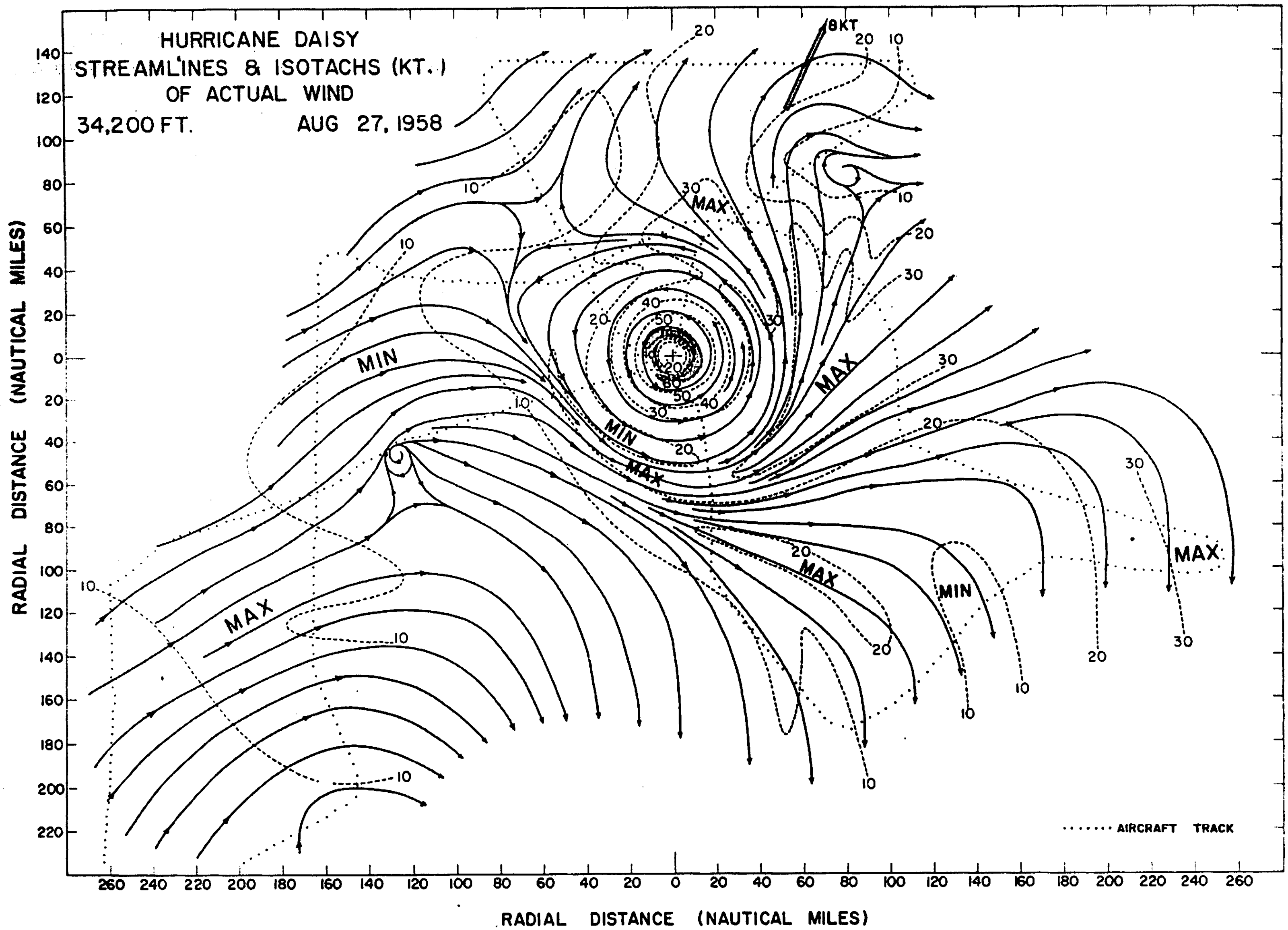

Figure 5. - Wind field at 34,200 feet (pressure altitude, U. S. Standard), around hurricane Daisy on August 27, 1958.

which, in fact, were made over a period of several hours. It is, however, believed that this shortcoming does not invalidate the results obtained.

From the wind fields thus obtained, values of the speed and direction were plotted on a rectangular grid of points 20 nautical miles apart. From these, computations of the quantity $2VK_t$ were made on the IBM 650 Computer. The curvature of the trajectory K_t was computed from the following relations:

If u and v are the westerly and southerly components of the wind, and if we define

$$\psi = \tan^{-1} \frac{u}{v} \tag{14}$$

then

$$\begin{aligned} K_t &= - \frac{d\psi}{ds} = \cos^2 \psi \left(\frac{u}{v^2} \frac{dv}{ds} - \frac{1}{v} \frac{du}{ds} \right) \\ &= \frac{1}{V^2} \left(u\frac{dv}{ds} - v \frac{du}{ds} \right) \\ &= \frac{1}{V^3} \left(u \frac{dv}{dt} - v \frac{du}{dt} \right) \end{aligned} \tag{15}$$

If the motion of the storm is represented by the vector $\vec{c}$ with components c_x and c_y in the east and north directions, respectively, and if we assume steady state conditions,

$$\begin{aligned} K_t &= \frac{1}{V^3} [u(\vec{V} - \vec{c}) \cdot \vec{\nabla} v - v (\vec{V} - \vec{c}) \cdot \vec{\nabla} u] \\ &= \frac{1}{V^3} \left\{ u[(u - c_x) \frac{\partial v}{\partial x} + (v - c_y) \frac{\partial v}{\partial y}] - v[(u - c_x) \frac{\partial u}{\partial x} + (v - c_y) \frac{\partial u}{\partial y}] \right\} \end{aligned} \tag{16}$$

Figures 6, 7, and 8 represent fields of the quantity $2K_t V = 2V/R_t$ corresponding to the wind fields of figures 3, 4, and 5, respectively. Regions where this quantity is negative and numerically greater than the Coriolis parameter, i. e., areas where the winds are anomalous, are shaded. The consistency of the patterns of these areas from day to day attests to their authenticity and demonstrates beyond any reasonable doubt that anomalous winds do occur. It will be noted, however, that the areas covered by these winds represent narrow strips of the order of 1 to 2 degrees of latitude in width, and are, therefore, likely to escape detection by ordinary synoptic analysis. In addition, the vertical extent of these winds is also limited, as can be noted from the fact that they are not found in figure 9 which represents the field of $2V/R_t$ a few thousand feet below that of figure 6. This is perhaps the reason why meteorologists in general have failed to realize both the reality and importance of these winds. The latter stems from the fact that they represent an important mechanism for the development of atmospheric instability as we shall now proceed to show.

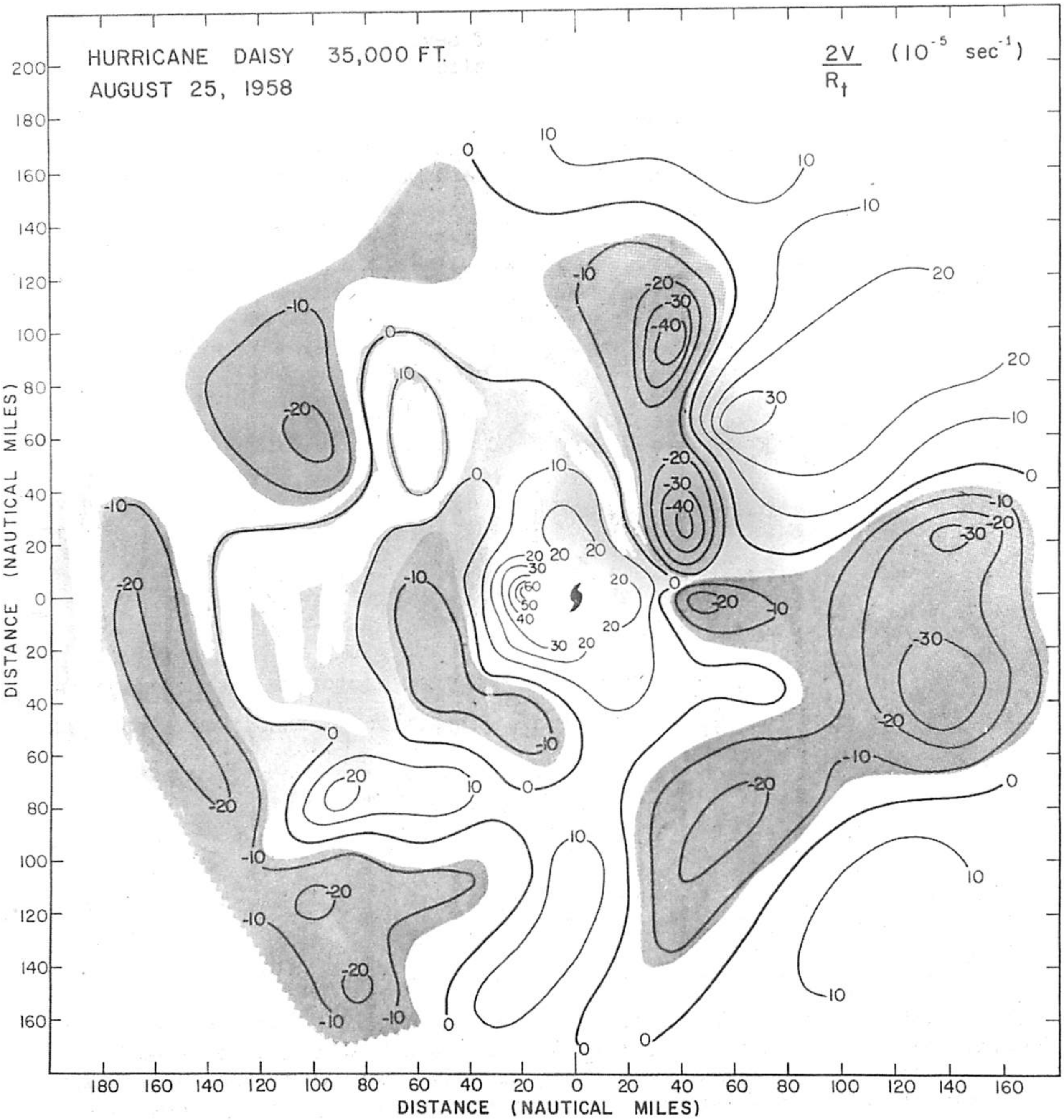

Figure 6. - Field of the quantity $2V/R_t$ at 35,000 feet (pressure altitude, U.S. Standard), around hurricane Daisy on August 25, 1958. Areas with anomalous winds are shaded.

4. THE CONCEPT OF INSTABILITY OF ATMOSPHERIC MOTION

The concept of instability in the atmosphere appeared with the initial developments in the theory of atmospheric disturbances. As early as 1878, Lord Rayleigh [11] investigated conditions under which small displacements at the boundary between two air streams grow into larger disturbances. At about

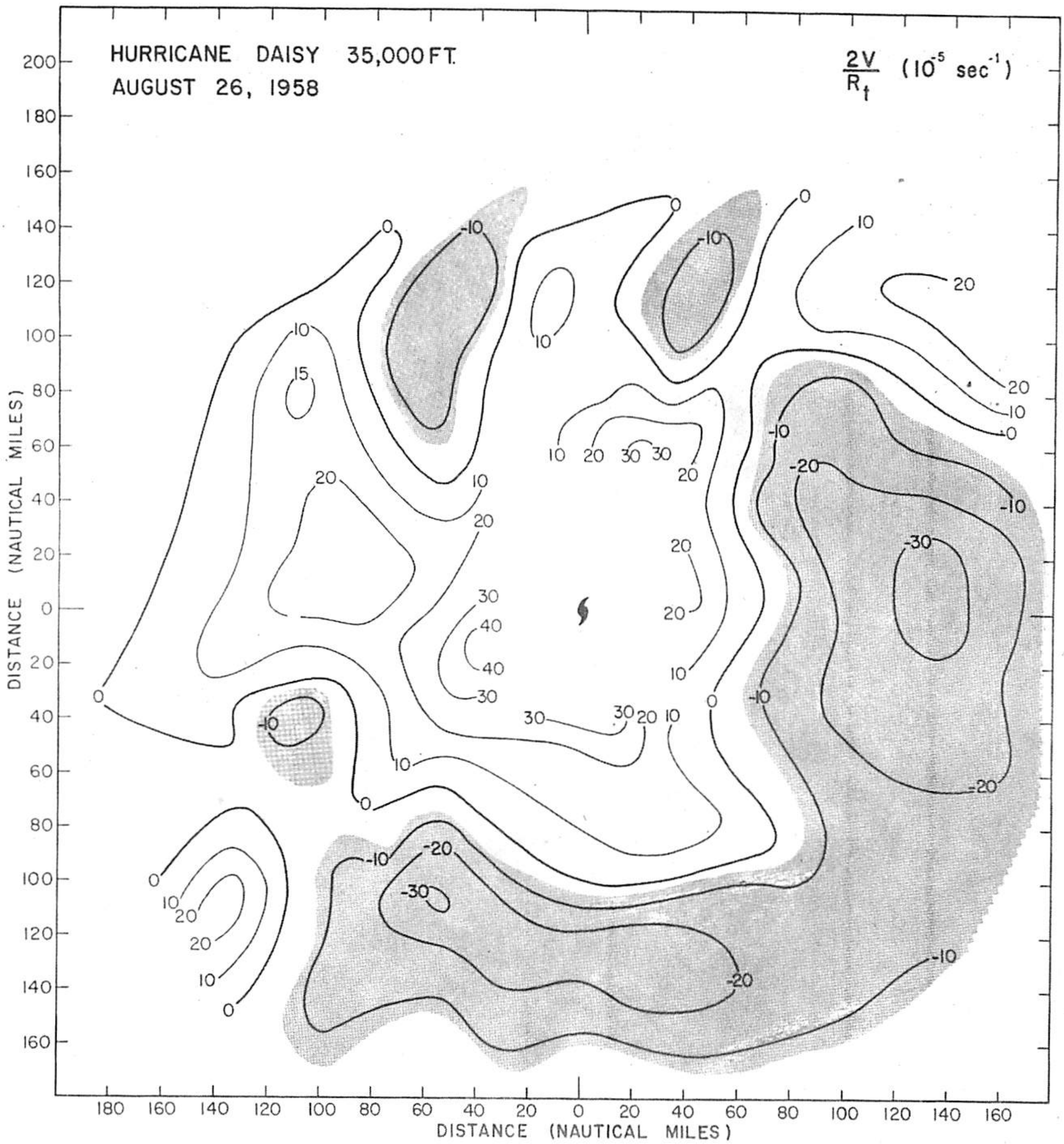

Figure 7. - Same as figure 6, for August 26, 1958.

the same time, Helmholtz [6] found that, under certain conditions, unstable gravity waves developed at the interface between two liquids of different density and velocity. The above studies culminated in the polar front theory of cyclone formation.

Paraelleling the above studies, another series of investigations, beginning with those of Helmholtz [7] and Rayleigh [12] attempted to link instabil-

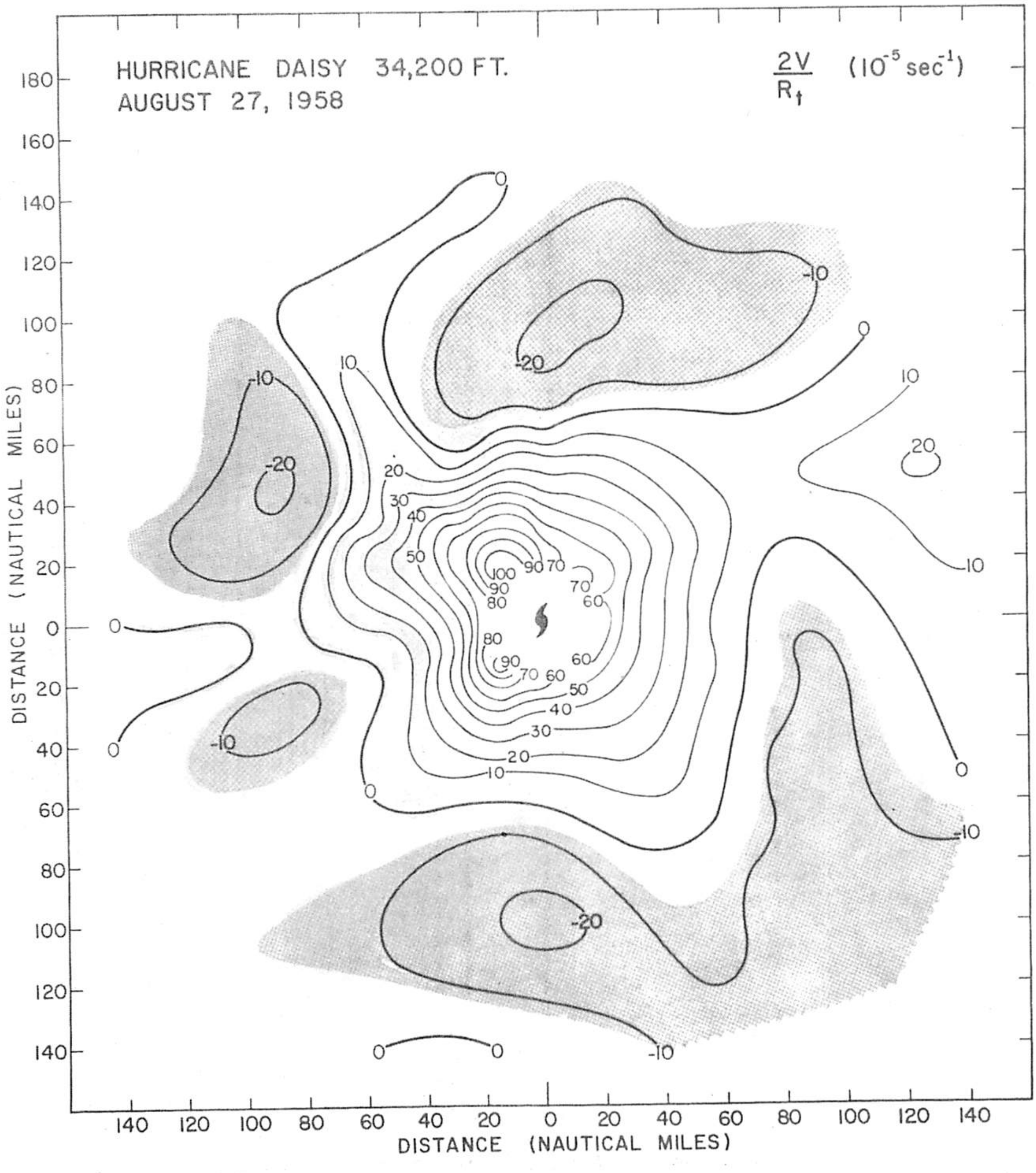

Figure 8. - Same as figure 6, for August 27, 1958, at 34,200 feet (pressure Altitude, U. S. Standard).

ity in air currents with the quasi-horizontal distribution of the kinematic properties of the currents. This type of instability is usually known as dynamic instability.

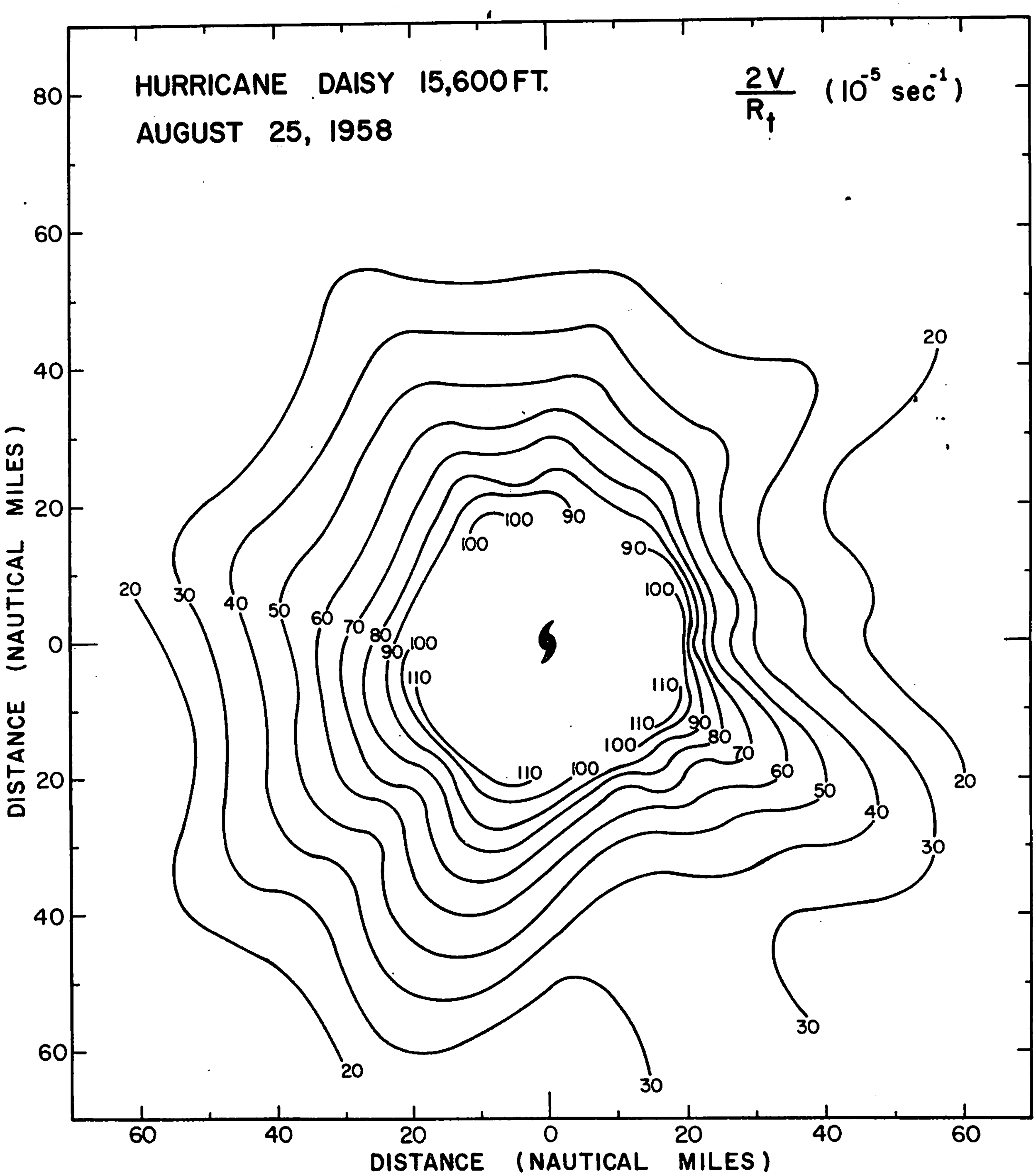

Figure 9. - Field of the quantity $2V/R_t$ at 15,600 feet (pressure altitude, U. S. Standard), around hurricane Daisy on August 25, 1958.

In recent years, attention has been redirected to still another type of instability known as baroclinic instability. This type of instability, the importance of which was first pointed out by Margules [10], occurs by virtue of the vertical wind shear.

There is little doubt that the above three types of instability all play an important role in the atmosphere and there is an obvious need for a unified theory which combines them and brings out their relative importance and their interrelations.

In the absence of such a theory, we must consider the particular type of instability which appears to have the most direct bearing on the problem at hand. In the present case, the importance of anomalous winds is best demonstrated by studying the circumstances attending the development of the so-called dynamic instability, with especial reference to atmospheric currents in curved motion.

5. DYNAMIC INSTABILITY IN CURVED AIRFLOW

In 1936, Solberg [14] investigated the conditions under which dynamic instability occurs in a steady symmetrical polar vortex which is initially in gradient equilibrium. Solberg's approach may be extended to any steady vortex in which the tangential variations of the wind are negligible in comparison with those in the radial direction.

Let us then consider such a vortex in which the angular speed ω is a function of the distance R from the axis of rotation. We shall proceed by the standard method and impose a small perturbation on this steady vortex and find out the circumstances under which the frequency of the total perturbed motion becomes imaginary.

If we disregard friction, we can express the equilibrium of forces acting on the steady vortex by the following equation,

$$\frac{V^2}{R^2}\vec{R} + \frac{2\Omega V}{R}\vec{R} - \theta\,\overrightarrow{\nabla\pi} + \vec{g} = 0 \tag{17}$$

where V is the magnitude of the speed, Ω the earth's rotation, θ the potential temperature, $\vec{g}$ the gravity vector, and

$$\pi = c_p\left(\frac{p}{1000}\right)^{R/c_p}$$

Let us now apply a uniform radial impulse to all the particles at distance R' from the axis so that at the time t = t the particles form another circle with radius R. We stipulate that at any instant, the perturbed particles immediately acquire a pressure equal to that of the points which they are occupying at that instant. We further assume that the perturbed particles, in their displacement over a small distance S from their equilibrium position conserve both their potential temperature θ' and their absolute angular momentum $M'_a = V'R + \Omega R'^2$ so that

$$\left.\begin{aligned} &\theta - \theta' = \vec{S}\cdot\overrightarrow{\nabla\theta} \\ \text{and} \qquad & \\ &M_a - M'_a = \vec{S}\cdot\overrightarrow{\nabla M_a} \end{aligned}\right\} \tag{18}$$

In addition to the forces indicated in equation (17), the perturbed particles sustain an acceleration $d^2 \vec{S}/ dt^2$ so that their motion is determined by the equation

$$\frac{d^2 \vec{S}}{dt^2} + \frac{V'^2}{R^2} \vec{R} + \frac{2\Omega V'}{R} \vec{R} - \theta' \overrightarrow{\nabla\pi} + \vec{g} = 0 \tag{19}$$

From equations (17) and (19), we obtain by subtraction

$$\frac{d^2 \vec{S}}{dt^2} - \frac{V^2 - V'^2}{R^2} \vec{R} - \frac{2\Omega}{R} (V - V') \vec{R} + (\theta - \theta')\overrightarrow{\nabla\pi} = 0 \tag{20}$$

Now

$$V^2 - V'^2 = (V - V') \cdot 2V - (\delta V)^2 \tag{21}$$

where $\delta V = V - V'$ is a perturbation quantity, the square of which may be neglected. Equation (20) thus becomes

$$\frac{d^2 \vec{S}}{dt^2} - [\frac{2}{R} (\omega + \Omega)(V - V')] \vec{R} + (\theta - \theta')\overrightarrow{\nabla\pi} = 0 \tag{22}$$

We now express the above equation in terms of the conservative quantity M_a and make use of the relations in (18); we obtain

$$\frac{d^2 \vec{S}}{dt^2} - [\frac{2}{R^2} (\omega + \Omega)(\vec{S} \cdot \overrightarrow{\nabla M_a}) + \frac{2}{R} (\omega^2 + 3\omega\Omega + 2\Omega^2)\delta R] \vec{R}$$
$$+ (\vec{S} \cdot \overrightarrow{\nabla\theta}) \overrightarrow{\nabla\pi} = 0 \ldots \tag{23}$$

If the perturbation is very small compared to the dimensions of the vortex, i. e. if $R-R'/R \approx 10^{-1}$, the term in δR in the above equation may be neglected and we finally have

$$\frac{d^2 \vec{S}}{dt^2} - [\frac{2}{R^2} (\omega + \Omega)(\vec{S} \cdot \overrightarrow{\nabla M_a})] \vec{R} + (\vec{S} \cdot \overrightarrow{\nabla\theta}) \overrightarrow{\nabla\pi} = 0 \tag{24}$$

Let us assume that the displacement S is proportional to a function of the form $e^{i\gamma t}$ and project equation (24) along two orthogonal directions: one corresponding to the vertical and the other to a radial direction. If the components of S along these directions are given by z and r, respectively, the projections of equation (24) along these directions are:

$$\left.\begin{aligned} &\gamma^2 z - \frac{2\omega_a r}{R} \left(r \frac{\partial M_a}{\partial r} + z \frac{\partial M_a}{\partial z} \right) + \frac{\partial\pi}{\partial z} \left(r \frac{\partial\theta}{\partial r} + z \frac{\partial\theta}{\partial z} \right) = 0 \\ &\text{and} \\ &\gamma^2 r - \frac{2\omega_a z}{R} \left(r \frac{\partial M_a}{\partial r} + z \frac{\partial M_a}{\partial z} \right) + \frac{\partial\pi}{\partial r} \left(r \frac{\partial\theta}{\partial r} + z \frac{\partial\theta}{\partial z} \right) = 0 \end{aligned}\right\} \tag{25}$$

where ω_a denotes the absolute rotation $(\omega + \Omega)$.

For nontrivial values of r and z, the determinant of the above system of linear equations vanishes and we obtain

$$\gamma^4 + \gamma^2 \left[\left(\frac{\partial\pi}{\partial z} \frac{\partial\theta}{\partial z} + \frac{\partial\pi}{\partial r} \frac{\partial\theta}{\partial r} \right) + \left(\frac{2\omega_{az}}{R} \frac{\partial M_a}{\partial r} - \frac{2\omega_{ar}}{R} \frac{\partial M_a}{\partial z} \right) \right]$$

$$- \left(\frac{2\omega_{az}}{R} \frac{\partial\pi}{\partial z} + \frac{2\omega_{ar}}{R} \frac{\partial\pi}{\partial r} \right) \left(\frac{\partial M_a}{\partial z} \frac{\partial\theta}{\partial r} - \frac{\partial M_a}{\partial r} \frac{\partial\theta}{\partial z} \right) = 0 \qquad (26)$$

Examining the order of magnitude of the various terms in equation (26), we find that

$$\left.\begin{aligned} &\frac{\partial\pi}{\partial z} \frac{\partial\theta}{\partial z} + \frac{\partial\pi}{\partial r} \frac{\partial\theta}{\partial r} = \vec{\nabla}\pi \cdot \vec{\nabla}\theta \approx 10^{-4} \\ &\frac{2}{R} \left(\omega_{az} \frac{\partial M_a}{\partial r} - \omega_{ar} \frac{\partial M_a}{\partial z} \right) = -2\vec{\omega} \cdot \vec{\nabla} \times \vec{V}_a \approx 10^{-8} \\ &\frac{2}{R} \left(\omega_{az} \frac{\partial\pi}{\partial z} + \omega_{ar} \frac{\partial\pi}{\partial r} \right) \left(\frac{\partial M_a}{\partial z} \frac{\partial\theta}{\partial r} - \frac{\partial M_a}{\partial r} \frac{\partial\theta}{\partial z} \right) \\ &\qquad = (2\vec{\omega}_a \cdot \vec{\nabla}\pi)(\vec{\nabla}\theta \cdot \vec{\nabla} \times \vec{V}_a) \approx 10^{-12} \end{aligned}\right\} \qquad (27)$$

Solving for γ^2 and neglecting the last two terms of (27) in comparison with the first, we have

$$\gamma_1^2 = - \vec{\nabla}\pi \cdot \vec{\nabla}\theta \qquad (28)$$

The value of this root is determined mainly by the static stability

$$- \frac{\partial\pi}{\partial z} \frac{\partial\theta}{\partial z} = \frac{g}{\theta} \frac{\partial\theta}{\partial z} \qquad (29)$$

The second root γ_2^2 is obtained by dividing the third term of (27) by γ_1^2. Thus,

$$\gamma_2^2 = \frac{\frac{2}{R} \left(\omega_{az} \frac{\partial\pi}{\partial z} + \omega_{ar} \frac{\partial\pi}{\partial r} \right) \left(\frac{\partial M_a}{\partial z} \frac{\partial\theta}{\partial r} - \frac{\partial M_a}{\partial r} \frac{\partial\theta}{\partial z} \right)}{\frac{\partial\pi}{\partial z} \frac{\partial\theta}{\partial z} + \frac{\partial\pi}{\partial r} \frac{\partial\theta}{\partial r}} \qquad (30)$$

The above equation will be greatly simplified if it is projected on an isentropic surface so that $\partial\theta/\partial r$ vanishes. In addition, since the angle between isobaric and isentropic surfaces is generally small, $(\partial\pi/\partial r) \approx 0$ and

$$\gamma_2^2 = [2\omega_{az} (\vec{\nabla} \times \vec{V})_z]_\theta \qquad (31)$$

Equation (31) is identical with Solberg's results for the polar vortex and states that the square of the frequency of the second root is equal to twice the product of the absolute rotation of the vortex and the vertical component of the absolute vorticity, as measured on an isentropic surface.

If we now use a relative frame of reference fixed with respect to the earth, we have

$$\nu_2^2 = [(\frac{2V}{R_t} + f) \; (\zeta + f)]_\theta \tag{32}$$

Since we are considering a steady vortex, the streamlines coincide with the trajectory and $R = R_t$. The frequency ν_2 becomes imaginary and the motion unstable when

$$[(\frac{2V}{R_t} + f) \; (\zeta + f)]_\theta < 0 \tag{33}$$

In other words, dynamic instability occurs with normal winds provided the absolute vorticity, as measured on an isentropic surface, is negative. Alternatively, instability occurs with positive absolute vorticity provided the wind, as measured on an isentropic surface, is anomalous.

The above criterion is identical with that obtained by Van Mieghem [15]. The latter, however, did not envisage the possibility of the occurrence of anomalous winds and, therefore, equated the condition for the release of dynamic instability with the occurrence of negative absolute vorticity.

It should be noted that, according to the theory of equations, the second root may be obtained by subtracting ν_1^2 from the negative value of the coefficient of ν^2 in equation (26). If we then neglect the vertical wind shear $\partial M_a/\partial z = 0$, we have

$$\nu'^2_2 = (\frac{2V}{R_t} + f) \; (\zeta + f) \tag{34}$$

which is similar to equation (32) except that the quantities are now measured on a level surface. Equation (34) is identical with the criterion for dynamic instability obtained by Sawyer [13].

Physically, the operation of the above criterion may be visualized by studying the balance of forces on an anticyclonic vortex which is initially in gradient balance. Let a particle be given an impluse toward lower pressure. If the flow is unstable, the particle tends to continue in the direction of the impulse. There are two cases to consider:

<u>CASE I: Instability with Normal Winds.</u>

If the winds are normal, there is instability if the pressure gradient along the path of the particle increases at such a rate that the speed of the particle remains subgradient. This would require a rapid outward increase in anticyclonic rotation or a rapid decrease in cyclonic rotation expressed by the inequality

$$[\frac{1}{R} \frac{\partial}{\partial r} (VR)]_\theta < - f \tag{35}$$

<u>CASE II: Instability with Anomalous Winds.</u>

From figures 1 and 2, it is seen that if the wind regime is anomalous, the speed increases with decreasing pressure gradient and trajectory curvature. Therefore, if the trajectory curvature of the particle remains constant, the pressure gradient along the trajectory should decrease at a fast rate; otherwise, the speed of the perturbed particle will eventually reach a speed greater than that appropriate for gradient equilibrium and its acceleration will be checked.

Actually, however, the trajectory curvature decreases as the particle moves toward lower pressure. Therefore, instability can occur with a slower rate of pressure gradient decrease or, if the curvature decreases rapidly, instability can occur even with a slow outward increase of the pressure gradient. Equation (33) states that the motion is unstable so long as the combined effect of the change of wind speed and curvature is such that

$$\left[\frac{1}{R} \frac{\partial}{\delta r} (VR) \right]_\theta > -f \qquad (36)$$

6. A MECHANISM FOR THE DEVELOPMENT OF ANOMALOUS WINDS

An insight into the manner in which anomalous winds develop may be gained by considering the circulation theorem of Bjerknes which may be written[9]:

$$\begin{aligned} \frac{dC_a}{dt} &= \frac{dC}{dt} + 2\Omega \frac{d\Sigma}{dt} \\ &= N_{\alpha_1 - p} \end{aligned} \qquad (37)$$

$C_a = \oint_Q \vec{V}_a \cdot \vec{\delta r}$ = absolute circulation around a closed curve Q.

$C = \oint_Q \vec{V} \cdot \vec{\delta r}$ = relative circulation around the same curve.

Σ = the equatorial projection of the area within the curve Q.

$N_{\alpha_1 - p}$ = the number of pressure-volume solenoids enclosed by the curve Q.

A comparison of figures 6, 10, and 11 indicates that there are few solenoids at the level and location where the anomalous winds occur. We may, therefore, set

$$\frac{dC}{dt} + 2\Omega \frac{d\Sigma}{dt} = 0 \qquad (38)$$

or

$$C + 2\Omega\Sigma = \text{constant} \qquad (39)$$

If we approximate the hurricane to a simple circular vortex, the circulation at a distance R from the center is

$$C = 2\pi R^2 \omega \qquad (40)$$

and

$$\Sigma = \pi R^2 \sin \phi \qquad (41)$$

where ϕ is the latitude.

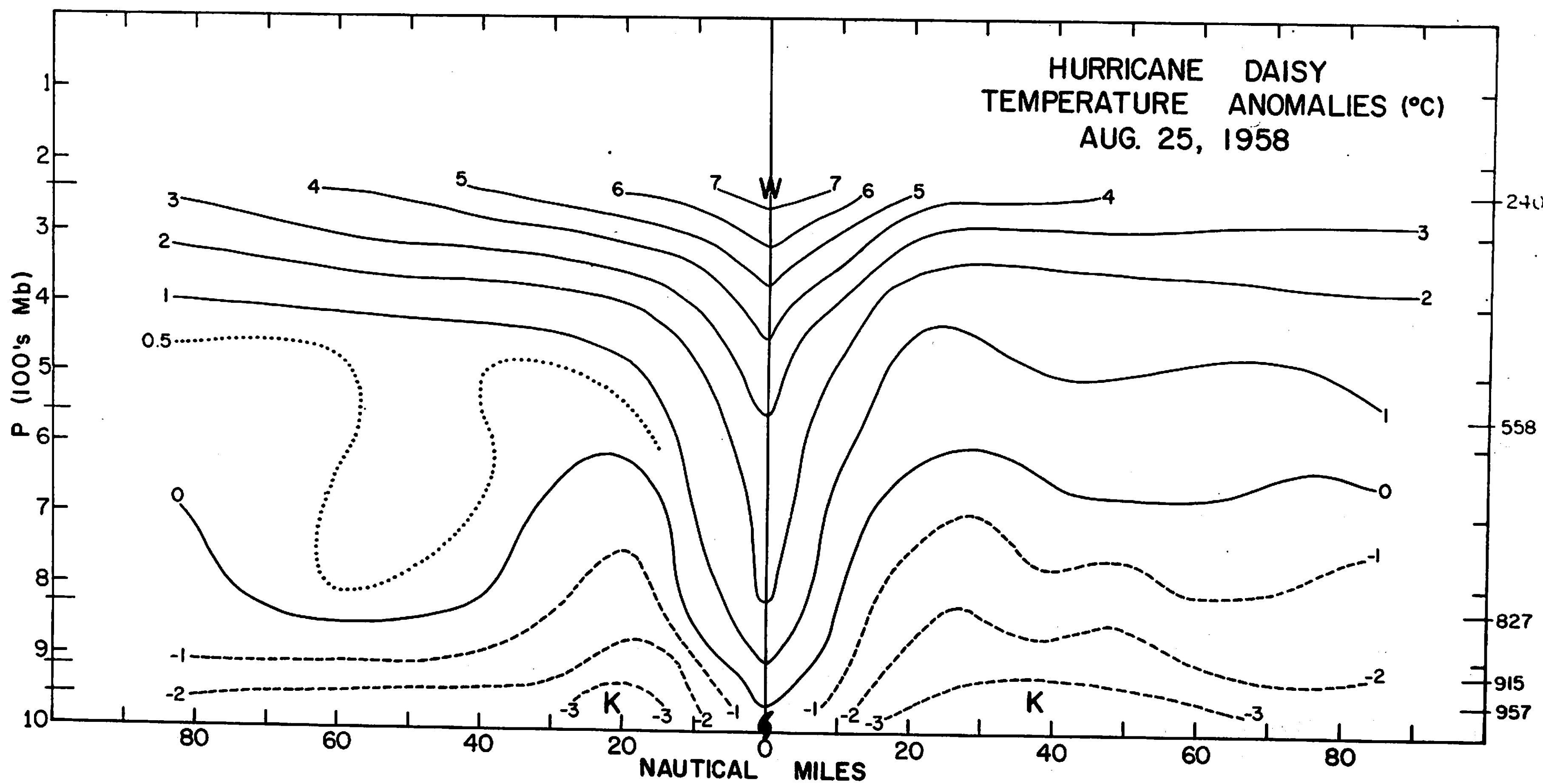

Figure 10. - Vertical cross-section of temperature anomalies from the mean August atmosphere in a direction perpendicular to that of the motion of hurricane Daisy, August 25; 1958. Pressures shown at the right edge indicate the levels at which data were available (analysis by Dr. José A. Colón).

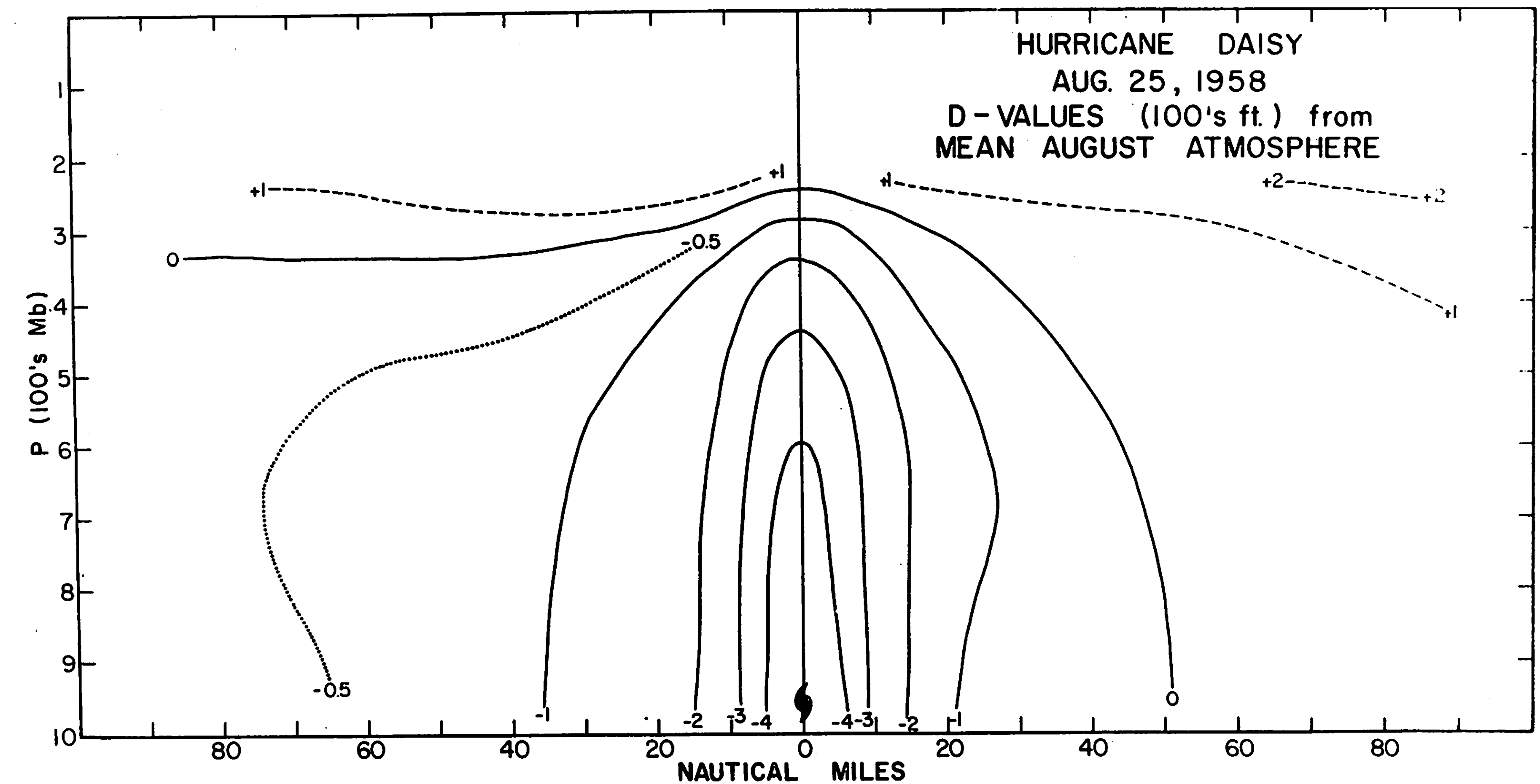

Figure 11. - Vertical cross-section of deviations of the altitudes of pressure surfaces from the mean August atmosphere ("D" values) in a direction perpendicular to that of hurricane Daisy, August 25, 1958. (Analysis by Dr. José A. Colón).

Substituting the above values of C and Σ in (39), we obtain

$$R^2 (\omega + \Omega_z) = \text{constant} \tag{42}$$

whence it can be seen that, in a circular vortex, disregarding the solenoid term in equation (37) is equivalent to assuming conservation of absolute momentum - a concept which is known to be applicable to upper-level hurricane circulation.

According to equation (42), if the vortex has horizontal convergence, the circles contract and ω increases. If, on the other hand, the circles expand as a result of horizontal divergence, ω decreases. In this case, in order that ω should decrease sufficiently so that

$$\left.\begin{aligned} &\omega + \Omega_z \rightarrow 0 \\ \text{i.e.,} \qquad &\frac{2V}{R} + f \rightarrow 0 \end{aligned}\right\} \tag{43}$$

R must approach infinity. Since the expansion cannot proceed so far, we conclude that the anomalous winds in figures 4, 5, and 6 were not produced by a symmetrical horizontal divergence of the air particles which arrived at these higher levels by ascent in the core of the storm. Another mechanism must be sought to account for their occurrence.

From figure 1 it is clear that a change of regime from normal to anomalous winds would be more difficult to accomplish if the pressure gradient force is much lower than the maximum value $f^2/ 4K'_t$, since this would require a big jump in the wind speed. If, however, the pressure gradient is nearly equal to the maximum, a comparatively small impulse would be sufficient to shift an air particle from the lower to the upper branch of the V curve of figure 1.

Let us visualize initial conditions characterized by an anticyclonic air stream in gradient equilibrium. In figure 12, let A be the equilibrium position of a particle in this stream. Now let us visualize that the equilibrium is disturbed by an increase in the pressure gradient brought about, for instance, by an interaction between tropical and extratropical pressure systems. Two cases may be discussed:

<u>CASE A</u>:

The increase of the pressure gradient force is from P to P_1, i. e., the resulting pressure gradient force is well below the maximum

$$f^2/4K'_t.$$

The wind, having become subgradient, the particle turns toward lower pressure and accelerates. Having reached the equilibrium speed B, unless there is considerable damping, the particle may slightly overshoot this position to B_1. It then oscillates with decreasing amplitude about the equilibrium position until balance is finally reached at B.

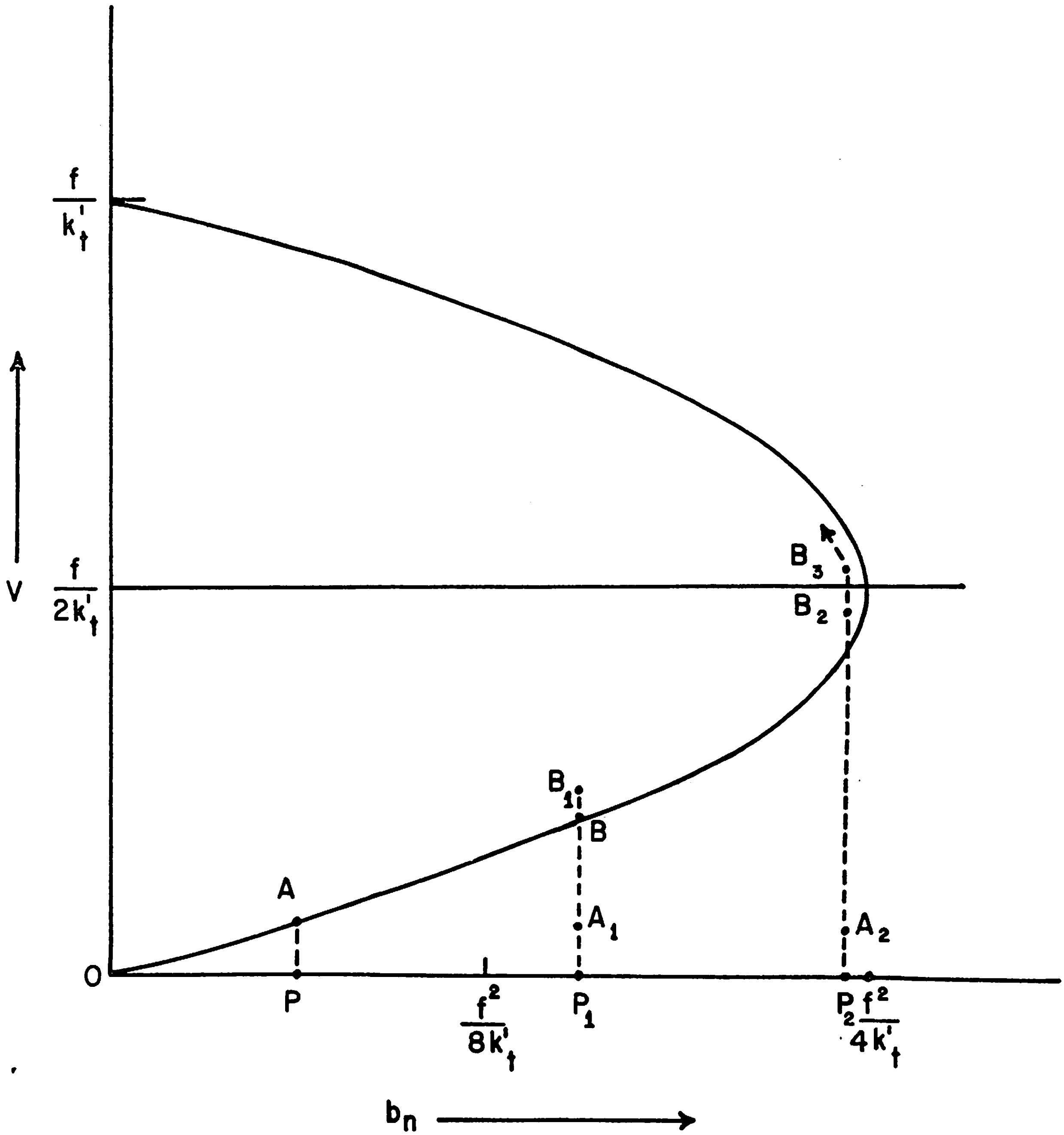

Figure 12. - Illustrating the development of anomalous winds (see text).

CASE B:

The increase of the pressure gradient force is from P to P_2; i.e., the increased pressure gradient force is nearly equal to the maximum value $f^2/4K'_t$.

In this case, the speed of the accelerating particle, by overshooting the equilibrium position, may reach the value B_3 which is in excess of the critical speed $f/2K'_t$. Whereas at B_2 the speed of the particle is supergradient, at B_3 it is subgradient for the same pressure gradient. Thus, by overshooting the critical speed, the particle is henceforth constrained to move toward lower pressure and sustain further acceleration unless the distribution of the wind field along its path is such that the absolute vorticity is negative; in this case the motion will be stabilized in accordance with equation (33).

7. EXAMPLES OF SITUATIONS IN WHICH ANOMALOUS WINDS PLAY A ROLE

In a previous paper [1], the author suggested from theoretical considerations that the occurrence of anomalous winds in the manner described above provides a mechanism for triggering hurricane development. Details concerning how this is brought about and some observational evidence to this effect are given in a separate paper [2] and are beyond the scope of the present study.

Here we would suggest that, in view of their essentially unstable nature, anomalous winds must play an important role not only in hurricane formation but also in other types of atmospheric development, especially those which occur downstream from a pressure ridge which has undergone marked intensification, or those accompanying the superposition of strong westerly winds on the crests of the long waves.

A case in point is that discussed by J. Bjerknes [3] linking the deepening of a wave trough with a strong intensification of the ridge upstream from it. Specifically, Bjerknes suggests that if the ridge intensifies so that the curvature of the contours exceeds a critical value

$$\frac{f^2}{4b_n} = \frac{f}{4V_g} \tag{44}$$

where V_g denotes the geostrophic wind, the air particles cannot follow the contours and must perforce flow toward lower pressure and accelerate. This acceleration results in supergradient winds which then curve back to higher pressure, the net results being a deepening of the trough downstream from the ridge.

Now that the likelihood of the occurrence of anomalous winds in situations analogous to that described above has been demonstrated, the deepening of the trough may be looked at in a new light. Let us suppose that the wind becomes anomalous as a result of the strengthening of the pressure gradient to a value near the maximum possible for anticyclonic flow, and let us follow the trajectory of a particle situated at the Point A in figure 13, where the wind changes regime and becomes subgradient in the manner discussed in the previous section. As a result, the air particle moves to lower pressure and accelerates. In doing so, its curvature decreases, as can be seen from the figure. Unless the pressure gradient along the trajectory increases rapidly, the progressively decreasing curvature requires an increasingly stronger speed for balanced motion (see fig. 2) and the particle, therefore, continues to accelerate. However, if the curvature vanishes so that the particle follows a straight trajectory, the wind instantly becomes supergradient and the particle must curve back to higher pressure. In the mean, the particle must, therefore,

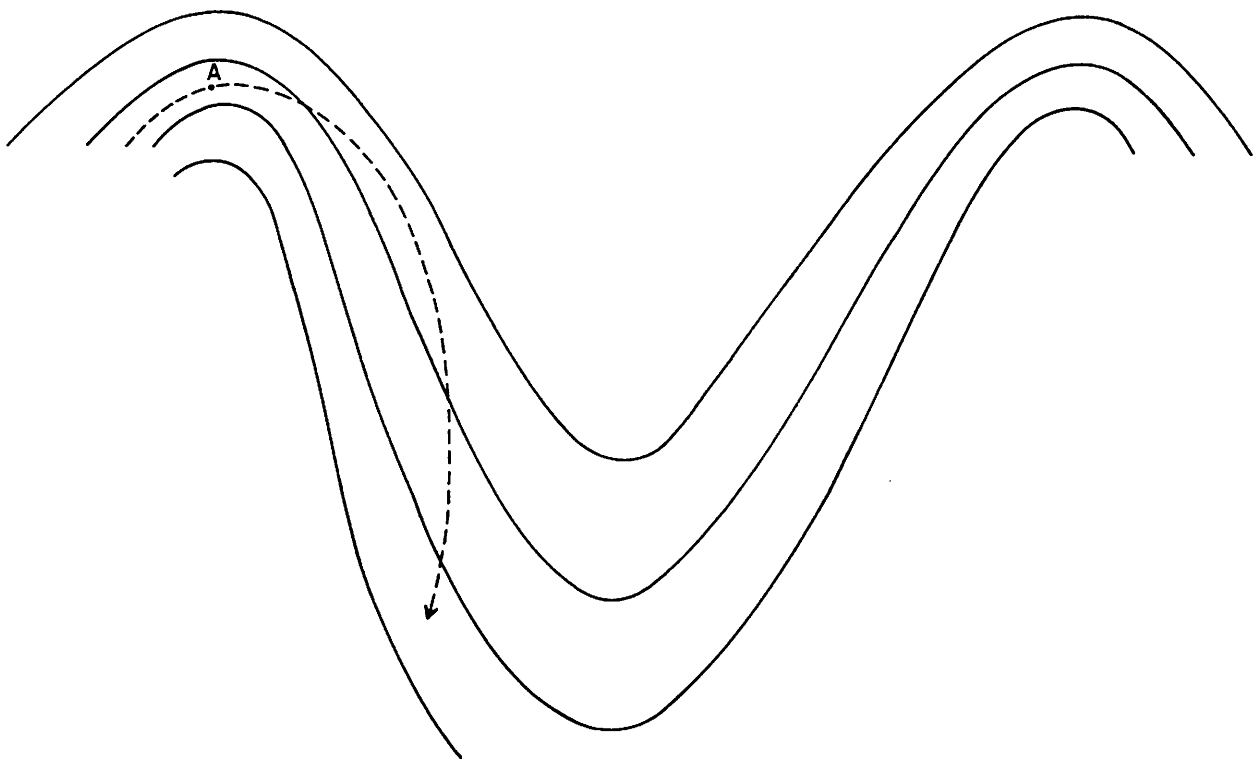

Figure 13. - Illustrating the cyclogenetic effect of anomalous winds. The dashed curve represents the probable trajectory of an air particle after its speed changes to the anomalous regime at the Point A.

follow a slightly anticyclonic trajectory and is constrained by the geometry of the pressure distribution to curve back to higher pressure, as shown by the dashed curve in figure 13. Thus, by stipulating the occurrence of anomalous winds, we arrive at the same ultimate result as Bjerknes did by excluding their occurrence.

8. CONCLUSION

Although the observational evidence presented here is for the occurrence of anomalous winds over small regions of the atmosphere, it is significant to note:

a. That their occurrence may be dependent on large-scale processes, such as those involving the intensification of pressure ridges or the latitudinal shift of the westerlies; and

b. That their effect extends beyond the area where they occur.

The fact that these winds can occur only in anticyclonic motion again demonstrates the vital regulatory role of the high pressure regions in determining the state and motion of the atmosphere.

9. ACKNOWLEDGMENTS

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