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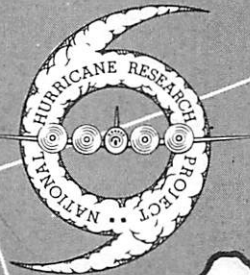
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# NATIONAL HURRICANE RESEARCH PROJECT

REPORT NO. 19

Further Examination of the Balance of  
Angular Momentum in the Mature Hurricane

ATMOSPHERIC SCIENCE  
LABORATORY COLLECTION





U. S. DEPARTMENT OF COMMERCE  
Sinclair Weeks, Secretary  
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Angular Momentum in the Mature Hurricane

by

Richard L. Pfeffer

Air Force Cambridge Research Center

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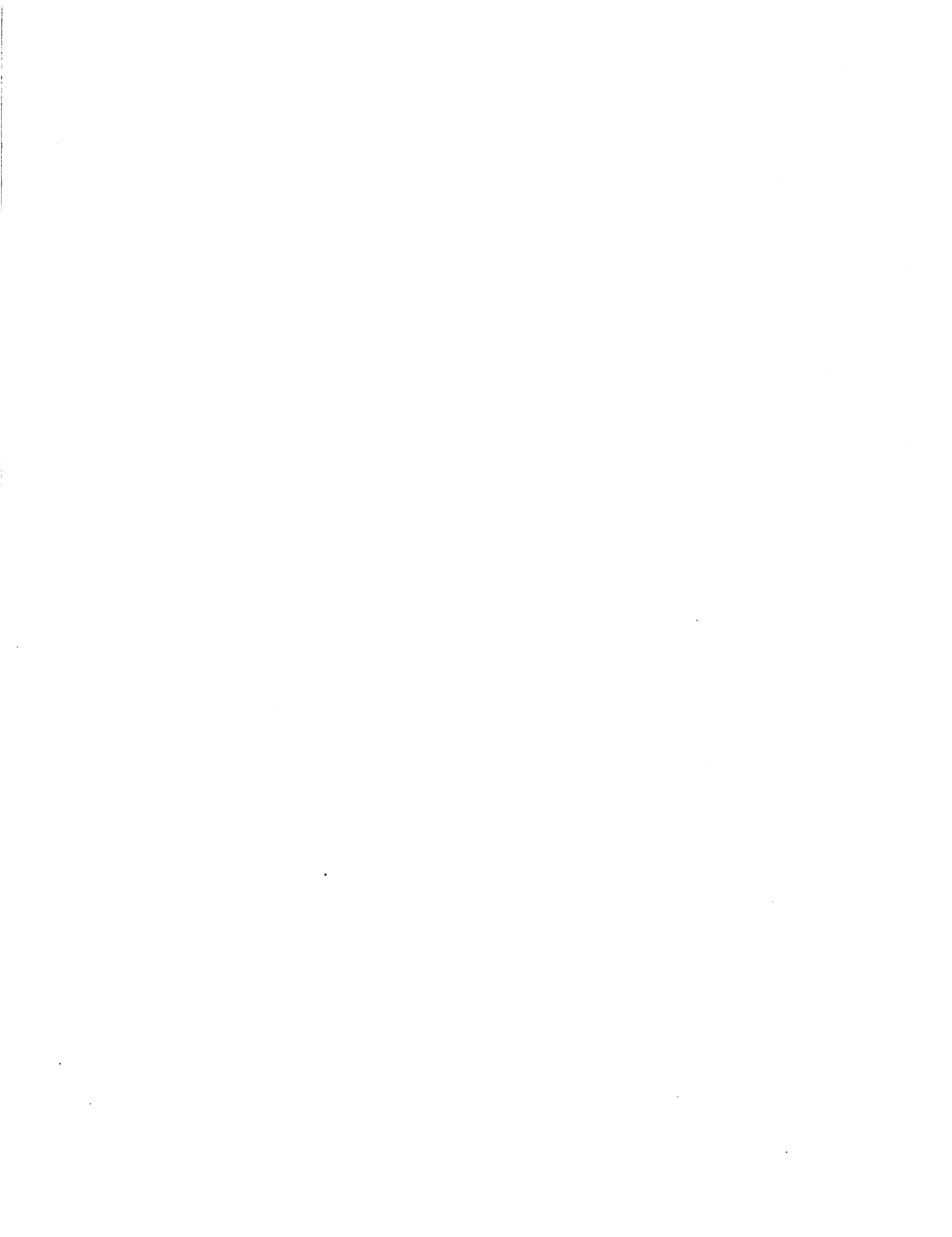
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FURTHER EXAMINATION OF THE BALANCE OF ANGULAR MOMENTUM  
IN THE MATURE HURRICANE

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[Manuscript received January 17, 1958]

ABSTRACT

The Angular momentum equation for an arbitrary rotating system in the atmosphere is examined. It is shown that the Coriolis torque serves to redistribute relative angular momentum without altering to any marked degree the total amount within the system. In particular, where the action of the Coriolis torque tends to develop a cyclone near the ground it must tend to develop an equally intense anticyclone aloft. In order that a rotational system, such as the hurricane, develop through a considerable depth of the atmosphere, the tendency for an upper level anticyclone to be created by the Coriolis torque must be compensated by a transport of angular momentum into the system. An observational study is presented which shows that, in the case of the mature hurricane, both azimuthal-mean motions and horizontal-eddy processes play important roles in accomplishing this transport.

1. INTRODUCTION

In a previous article the writer (Pfeffer [3]) examined the balance of angular momentum about the axis of rotation of the mature hurricane. It was shown there that the frictional drain of cyclonic (positive) angular momentum from the atmosphere into the earth which takes place within the cyclonic core of the hurricane is compensated by a transport of angular momentum into the hurricane from the surrounding atmosphere. The results of the observational study suggest, furthermore, that horizontal-eddy processes (characterized by mass exchanges in horizontal planes) are responsible for the transport of angular momentum from the surrounding atmosphere into the region which borders the high-wind zone of the hurricane, and that vertical-eddy processes (characterized by mass exchanges in vertical planes passing through the axis of rotation) are responsible for the transport of angular momentum from this supply region into the high-wind zone.

The integrals which were measured in [3] appear in the equation for the time rate of change of angular momentum integrated through the vertical depth of the atmosphere. In this equation the term which explicitly measures the

effect of the earth's rotation is negligible in comparison with the other terms since its contribution in the lower portion of the atmosphere is opposite in direction and nearly equal in magnitude to its contribution at higher levels. This equation cannot, therefore, be used to study the role of the earth's rotation in the angular momentum balance of the hurricane.

It is the purpose of the present article to present a more detailed study in which the angular momentum balance of the lower and upper portions of the hurricane are examined separately. The present formulation permits an investigation of the role of the earth's rotation as well as certain other aspects of hurricane dynamics which could not be treated using the vertically integrated form of the angular momentum equation.

## 2. THEORETICAL CONSIDERATIONS

The angular momentum of a parcel of air about an arbitrary vertical axis which is fixed with respect to the earth can vary only as a result of torques exerted on the parcel by external agencies. Adopting a spherical polar coordinate system  $(r, \Delta\phi, \theta)$  with origin at the center of the earth, where  $r$  is linear distance from the origin,  $\Delta\phi$  is angular distance measured from the chosen axis, and  $\theta$  is azimuth measured positive in the counterclockwise sense, we may express this principle without sensible error in the form,

$$\rho \frac{dM}{dt} = - \frac{\partial p}{\partial \theta} - \rho R D \quad (1)$$

where  $M$  is the absolute angular momentum per unit mass about the specified axis,  $R \equiv r \sin \Delta\phi$  is linear distance from this axis,  $p$  is pressure,  $\rho$  is density,  $D$  is the component of the viscous force per unit mass in the  $\theta$ -direction, and  $t$  is time.

We shall let  $V$  represent that portion of a conical volume, with vertex located at the center of the earth, which extends from the surface of the earth to the "top" of the atmosphere. Such a volume is shown schematically in figure 1. In the figure  $a$  is the radius of the earth,  $S$  is area measured along the vertical wall,  $(\Delta\phi)_S$  is the angular distance of the wall of the cone from the axis and  $\phi_0$  is the latitude of the axis. It may be seen from the diagram that the portion of the cone which lies above the surface of the earth closely approximates a cylindrical volume with radius  $R_S = a \sin (\Delta\phi)_S$ . As noted by Starr [5], if the axis of such a cylinder is made to coincide with the axis of rotation of a rotating wind system, and if the size of the cylinder is chosen properly, the angular momentum contained within the volume constitutes a measure of the intensity of the system.

Integrating (1) over the volume,  $V$ , and making use of the continuity equation for mass and Gauss' theorem we may write,

$$\frac{\partial}{\partial t} \int_V \rho M dV = \int_S \rho M C_n dS + \int_{S_0} \rho M v dS_0 - \int_V \frac{\partial p}{\partial \theta} dV - \int_V \rho R D dV, \quad (2)$$



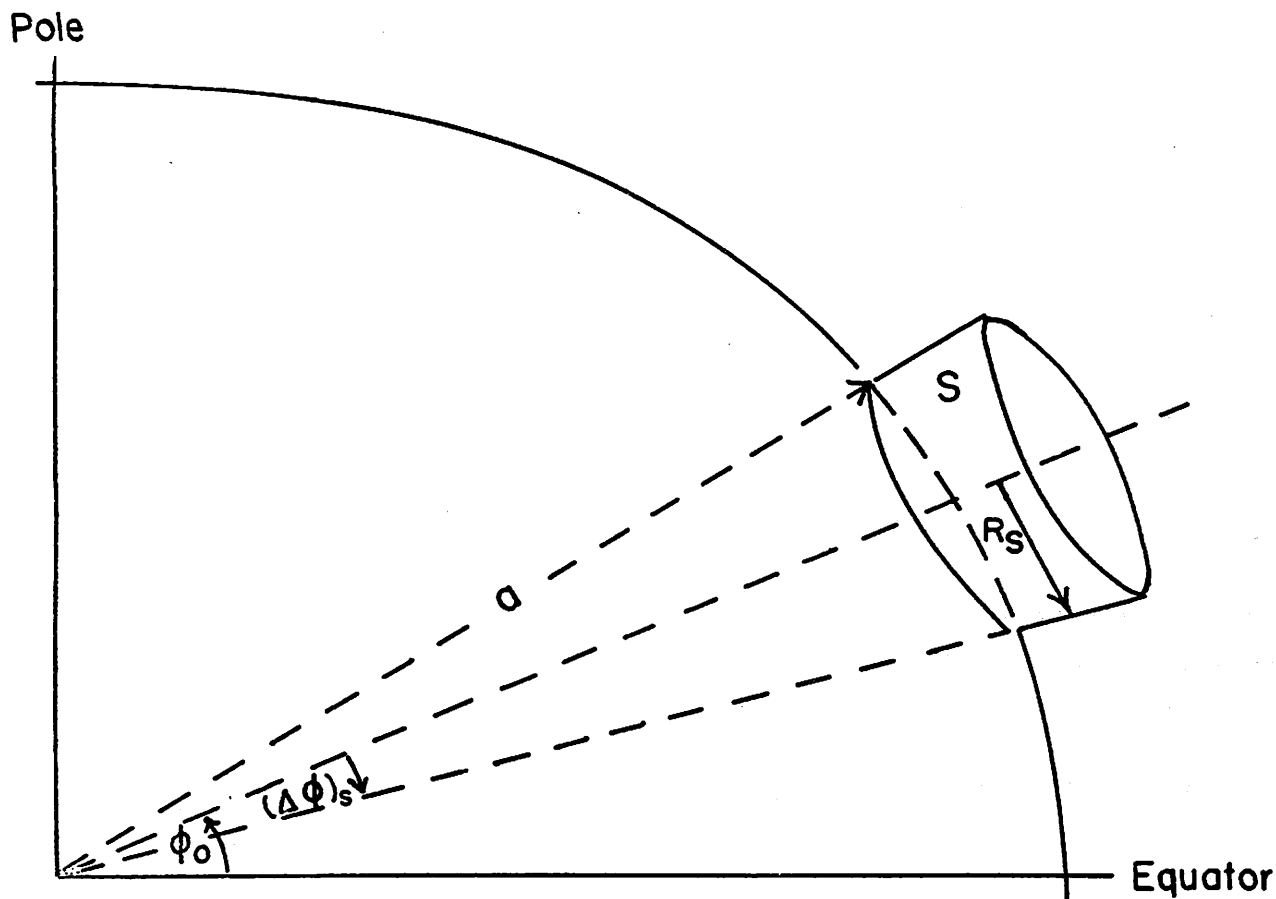


Figure 1. - Schematic picture of conical volume.

where  $S_0$  is the area of the lower boundary of the conical section,  $C_n$  is the component of velocity in the negative  $\Delta\phi$  direction and  $w$  is the vertical component of velocity (positive upward). Equation (2) states that the total angular momentum (about the chosen axis) contained within  $V$  may vary as a result of the flux of angular momentum across the boundaries of this volume and in consequence of torques due to pressure gradients and to viscosity respectively.

Making use of the identity,

$$M \equiv M_R + M_\omega = RC_T + R^2 \omega \sin \phi_0,$$

where  $C_T$  is the component of velocity in the direction of  $\theta$ , and  $M_R \equiv RC_T$  is

the component of angular momentum associated with the motion of the atmosphere relative to the surface of the earth, and  $M_\omega \equiv R^2 \omega \sin \phi_0$  (where  $\omega$  is the angular velocity of the earth) is the component of angular momentum associated with the rotation of the atmosphere with the earth, we may rewrite (2) in the form,

$$\frac{\partial}{\partial t} \int_V \rho M_R dV + \frac{\partial}{\partial t} \int_V \rho M_\omega dV = \int_S \rho M_R C_n dS + \int_S \rho M_\omega C_n dS - \int_V \frac{\partial p}{\partial \theta} dV + \int_{S_0} \rho M_R \omega dS_0 + \int_{S_0} \rho M_\omega \omega dS_0 - \int_V \rho R D dV. \quad (3)$$

Now, it can be shown that,

$$-\frac{\partial}{\partial t} \int_V \rho M_\omega dV + \int_S \rho M_\omega C_n dS + \int_{S_0} \rho M_\omega \omega dS_0 \equiv \int_V \rho f_n R C_n dV - \int_V \rho f_w R \omega dV, \quad (4)$$

where  $f_n = 2(\omega \sin \phi_0) \cos \Delta \phi$  and  $f_w = 2(\omega \sin \phi_0) \sin \Delta \phi$ .

Thus, the difference between the inward transport of  $M_\omega$  and the time rate of change of  $M_\omega$  is identically the "Coriolis torque" on the fluid within the volume. Introducing (4) into (3) we obtain

$$\frac{\partial}{\partial t} \int_V \rho M_R dV = \int_S \rho M_R C_n dS + \int_{S_0} \rho M_R \omega dS_0 + \int_V \rho f_n R C_n dV - \int_V \rho f_w R \omega dV - \int_V \frac{\partial p}{\partial \theta} dV - \int_V \rho R D dV. \quad (5)$$

It may be noted that the pressure torque term,

$$-\int_V \frac{\partial p}{\partial \theta} dV \equiv - \int_0^\infty \int_0^{\Delta \phi} \left( f \frac{\partial p}{\partial \theta} d\theta \right) r^2 \sin \Delta \phi d(\Delta \phi) dr,$$

will give a contribution only if the path of integration in the  $\theta$ -direction is obstructed by a topographical barrier along which there exists a horizontal pressure gradient. The magnitude of this torque depends on the structure of the irregularity and the magnitude of the pressure gradient. This so-called "mountain torque" has been shown to be of some importance in the general circulation problem (White [7]) and may be of importance also in connection with the formation of cyclonic disturbances to the lee of high mountain barriers. This term vanishes identically, however, if no topographical barriers are present. It cannot, therefore, be of importance in connection with systems, such as the hurricane, which spend most of their life cycles over the ocean or over flat land. In such cases the term  $\int_{S_0} \rho M_R \omega dV$  vanishes also, and (5) takes the form,

$$\frac{\partial}{\partial t} \int_V \rho M_R dV = \int_S \rho M_R C_n dS + \int_V \rho f_n RC_n dV - \int_V \rho f_w RwdV - \int_V \rho RDdV. \quad (6)$$

With the aid of the hydrostatic equation we may express the Coriolis torque as follows:

$$\int_V \rho f_n RC_n dV - \int_V \rho f_w RwdV = \omega \sin \phi_0 \int_V (R_S^2 - R^2) \frac{\partial \rho}{\partial t} dV =$$

$$\frac{\omega \sin \phi_0}{g} \int_{S_0} (R_S^2 - R^2) \frac{\partial p_0}{\partial t} dS_0, \quad (7)$$

where  $p_0$  is the pressure at the ground,  $g$  is the acceleration of gravity, and  $R_S$  is the distance of the conical wall from the axis. When  $\frac{\partial p_0}{\partial t}$  is positive this term tends to increase the angular momentum within the volume, and vice versa. This is exactly opposite to the observed behavior of the atmosphere. Furthermore, plausible estimates of the magnitude of surface pressure changes in the hurricane indicate that this term is negligible in comparison with the other terms on the right hand side of (6). Equation (6) therefore reduces to,

$$\frac{\partial}{\partial t} \int_V \rho M_R dV = \int_S \rho M_R C_n dS - \int_V \rho RDdV. \quad (8)$$

It might appear, therefore, that the earth's rotation has a negligible effect on the rate of change of relative angular momentum within the volume. This is not, however, the case as our intuition leads us to believe. The immediate role of the Coriolis torque is to redistribute relative angular momentum without altering to any measureable degree the sum total of the relative angular momentum within the volume. That is, although the Coriolis integral is negligible when taken over the entire volume of the conical section which lies between the surface of the earth and the top of the atmosphere, the integrand takes on significant non-zero values on a point-for-point basis. Thus, at individual points, inward motion generates cyclonic angular momentum and outward motion generates anticyclonic angular momentum. The accumulation of relative angular momentum in the conical volume then takes place as a result of the selective transport into the volume of fluid elements which acquire the same sign of relative rotation due to the Coriolis torque.

To determine the way in which the motions of the atmosphere are organized to bring about this transport of angular momentum, we may make use of the identity,

$$g \equiv [\bar{q}] + [q]'' + q', \quad (9a)$$

where  $q$  may be any quantity and,

$$[q] = \frac{1}{2\pi} \oint q d\theta ; q' = q - [q]$$

$$\bar{q} = \frac{1}{p_0} \int_0^{p_0} q dp ; q'' = q - \bar{q} .$$

When  $q = C_n$ , equation (9a) reduces to,

$$C_n \approx [C_n]'' + C_n' , \quad (9b)$$

since  $[C_n]$  is negligible in comparison with the other quantities in (9b). The quantity  $[C_n]''$  is associated with mass exchanges in vertical planes which pass through the axis of the conical volume, and the quantity  $C_n'$  is associated with mass exchanges which take place in horizontal planes. In accordance with the terminology adopted in [3], such mass exchanges will be called "vertical-eddy" processes and "horizontal-eddy" processes, respectively.

With the use of (9a) and (9b) the first term on the right hand side of (8) may be expressed in the form,

$$\int_S \rho M_R C_n dS \approx \frac{2\pi R^2 p_0}{g} \left\{ \overline{[C_T]'' [C_n]''} + \overline{[C_T]' C_n'} \right\} . \quad (10)$$

The vertical and horizontal-eddy transports of angular momentum which appear in (10) were evaluated in [3].

The last term in (8) measures the total torque exerted on the air within the volume due to friction. Two factors may contribute to this integral. The first is the torque exerted by molecular and small-scale viscosity along the boundaries of the volume, and the second is the torque exerted by the underlying surface (i.e. skin friction). In all probability the effects of molecular and small-scale eddy viscosity are of a much lower order of magnitude than the surface wind stress and may in fact be neglected in comparison with it. In this case the friction term may be expressed in the form of an integral over the surface of intersection between the volume and the earth, as follows,

$$-\int_V \rho R D dV = -2\pi \rho_0 K r^3 \int_0^{\Delta\phi} \sin^2 \Delta\phi [ |C| C_T ]_0 d(\Delta\phi) , \quad (11)$$

where  $K$  is the coefficient of skin friction,  $|C|$  is the absolute wind speed, and the subscript zero refers to quantities measured at the ground. In the derivation of (11), use was made of the relationship,

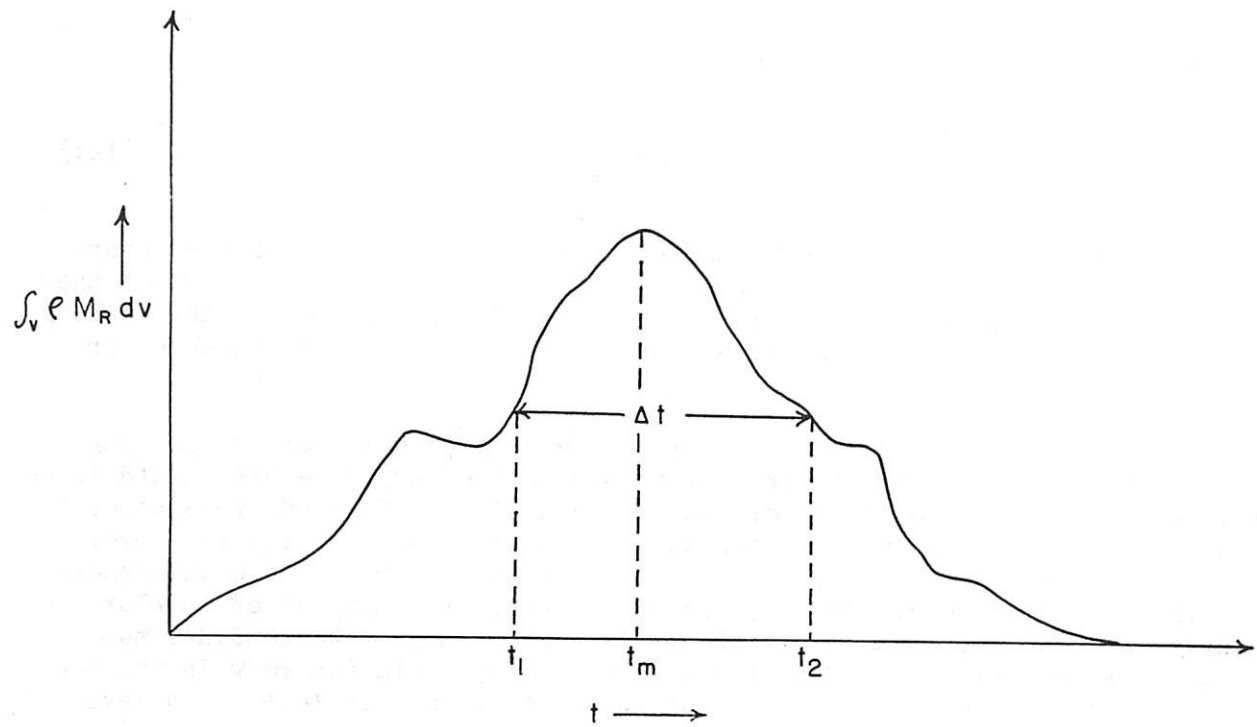


Figure 2. - Schematic picture showing time variation of relative angular momentum within a conical volume during the passage of a hurricane.

$$\tau_0 = - \rho_0 K (|C| C_T)_0 ,$$

where  $\tau_0$  is the surface stress (see Stanton [4] and Taylor [6]).

We next consider the time variation of the relative angular momentum within the volume as a hurricane passes through it. During the approach of the storm the relative angular momentum increases (perhaps sporadically, since the intensity of a hurricane may undergo large fluctuations even in the mature stages of its life cycle). It attains its maximum value at about the time the eye of the hurricane reaches the axis of the conical volume, and then decreases progressively, as shown schematically in figure 2. We now select a time interval,  $\Delta t = t_2 - t_1$ , (fig. 2) such that, (1) The maximum value of the integral  $\int_V \rho M_R dV$  lies within this interval, and (2) The magnitude of the integral  $\int_V \rho M_R dV$  is the same at the end of this interval as it was at the beginning. If we integrate (8) over  $\Delta t$  the left hand member vanishes and we have,

$$0 = \int_{t_1}^{t_2} \int_S \rho M_R C_n dS dt - \int_{t_1}^{t_2} \int_V \rho R D dV dt . \tag{12a}$$

Alternately, at  $t = t_m$  we could write,

$$0 = \int_S \rho M_R C_n dS - \int_V \rho R D dV. \quad (t = t_m) \quad (12b)$$

Equation (12b), together with (10) and (11), formed the basis for the observational study presented in [3]. As noted earlier, however, this form of the angular momentum equation is not suited for a study of the role of the earth's rotation. We shall now put the equations into a form which is suitable for such a study.

Perhaps the most obvious effect of the earth's rotation occurs in connection with the fact that the radial motions in the hurricane are inward in the lower portion of the atmosphere and outward aloft. To study this aspect of the earth's rotation we may integrate the angular momentum equation over a conical volume which extends through only a limited depth of the atmosphere. For this problem the Coriolis torque and the vertical transport of angular momentum into the volume do not vanish. In particular, if we consider two such volumes, one extending from the surface of the earth (where  $w$  is assumed to be zero) to a level near 600 mb. and the other from this level to a level near the tropopause (where  $w$  is again assumed to be zero), equation (12b) would be replaced by

$$0 = \int_{S_u} \rho M_R C_n dS_u + \int_{S_0} \rho M_R w dS_0 + \int_{V_u} \rho f_n R C_n dV, \quad (t = t_m) \quad (13a)$$

$$0 = \int_{S_L} \rho M_R C_n dS_L - \int_{S_0} \rho M_R w dS_0 + \int_{V_L} \rho f_n R C_n dV - \int_{V_L} \rho R D dV, \quad (t = t_m) \quad (13b)$$

and equation (10) would be replaced by

$$\int_{S_u} \rho M_R C_n dS_u = \frac{2\pi R^2}{g} \int_0^{p_1} \left\{ [C_T] [C_n]'' + [C_T]'' [C_n]'' + [C_T' C_n'] \right\} dp, \quad (14a)$$

$$\int_{S_L} \rho M_R C_n dS_L = \frac{2\pi R^2}{g} \int_{p_1}^{p_0} \left\{ [C_T] [C_n]'' + [C_T]'' [C_n]'' + [C_T' C_n'] \right\} dp, \quad (14b)$$

where the subscripts  $u$  and  $L$  refer to the upper and lower volumes, respectively, and  $p_1$  is the pressure at the boundary between the two volumes. In (13a, b) we have neglected the integral  $\int_V \rho f_w R w dV$  in comparison with

$\int_V \rho f_n R C_n dV$ , and we have assumed that the frictional torque acts only in the

lower portion of the atmosphere.

Since the first term on the right hand side of (14a, b) vanishes when the integration is taken over the entire depth of the atmosphere, this term can serve only to redistribute angular momentum between the lower and upper volume. It will be convenient to combine this term with the second term on the right to obtain,

$$\int_{S_u} \rho M_R C_n dS_u = \frac{2\pi R^2}{g} \int_0^{P_1} \left\{ [C_T] [C_n]'' + [C_T' C_n'] \right\} dp \quad (14c)$$

$$\int_{S_L} \rho M_R C_n dS_L = \frac{2\pi R^2}{g} \int_{P_1}^{P_0} \left\{ [C_T] [C_n]'' + [C_T' C_n'] \right\} dp. \quad (14d)$$

The first term on the right hand side of (14c,d) measures the contribution of azimuthally averaged motions to the horizontal transport of angular momentum into each volume, and the second term measures the contribution of horizontal-eddy processes to this transport.

The vertical transport of angular momentum across the boundary between the lower and upper volumes may be expressed in the form,

$$\int_{S_0} \rho M_R w dS_0 = 2\pi r^3 \int_{\Delta\phi} \rho \sin^2 \Delta\phi \left\{ [C_T] [w] + [C_T' w'] \right\} d(\Delta\phi), \quad (15)$$

and the Coriolis torque may be written as follows:

$$\int_{V_u} \rho f_n R C_n dV_u = \frac{4\pi}{g} (\omega \sin \phi_0) r^3 \int_0^{P_1} \int_{\Delta\phi} (\cos \Delta\phi) (\sin^2 \Delta\phi) [C_n]'' d(\Delta\phi) dp \quad (16a)$$

$$\int_{V_L} \rho f_n R C_n dV_L = \frac{4\pi}{g} (\omega \sin \phi_0) r^3 \int_{P_1}^{P_0} \int_{\Delta\phi} (\cos \Delta\phi) (\sin^2 \Delta\phi) [C_n]'' d(\Delta\phi) dp. \quad (16b)$$

The remainder of this article is devoted to a quantitative evaluation of the angular momentum balance of the lower and upper portions of the atmosphere, as described in equations (13a,b). Use will be made of equations (14c,d), (15), (16a,b) and (11) in evaluating each of the terms in (13a,b).

### 3. DATA AND COMPUTATIONAL PROCEDURE

The data employed in the present investigation were derived from composite wind charts of the mature hurricane prepared by Jordan [2] and Hughes [1]. It should be made clear at the outset that these charts do not capture all features of the flow patterns found in individual hurricanes. In particular, those features which do not have the same relative position in one hurricane that they have in another tend to be suppressed. When processes which depend on such features are evaluated from composite wind charts, therefore, their magnitudes are underestimated. We consider, for example, the horizontal-eddy transport of angular momentum. Since this transport depends largely on the shape and location of each of the surrounding systems, which differ from one hurricane to another, the evaluations presented in [3] and in the present study probably underestimate its magnitude. On the other hand, the horizontal transport of angular momentum by azimuthal-mean motions should be less subject to such errors since it depends on a feature which has the same relative location in all hurricanes, namely a circulation with inflow near the ground (where the maximum cyclonic angular momentum is found) and outflow aloft (where the minimum cyclonic angular momentum is found). These limitations in the data should be kept in mind when evaluating the results of the present study.

The composite wind data were used to study the balance of angular momentum in each of two volumes, one bounded by the conical walls  $\Delta\phi = 2^\circ$  and  $\Delta\phi = 4^\circ$ , and the other by  $\Delta\phi = 4^\circ$  and  $\Delta\phi = 6^\circ$ . Each volume was, in turn, divided into a lower and an upper portion, the former bounded by  $p = 1000$  mb. and  $p = 600$  mb. and the latter by  $p = 600$  mb. and  $p = 125$  mb., and the balance of angular momentum in each was examined separately.

The computational procedures used to evaluate the terms in equation (14c,d) were those employed in [3], and the values of friction were taken directly from [3]. In regard to the remaining terms it may be noted that both the Coriolis torque and the vertical transport of angular momentum by azimuthal-mean motions are sensitive to violations in mass continuity due to errors in the data. Although there are many ways in which such violations might be corrected for, it seems reasonable to alter the data as little as possible in attempting to arrive at estimates of such sensitive effects. One completely objective method of adjusting for mass continuity consists in using the approximate expression (9b) systematically throughout the development, or equivalently, replacing  $[C_n]$  by  $[C_n]''$  wherever it appears in the equations. Accordingly,  $[C_n]''$  was used in the continuity equation in the computation of the vertical velocity at 600 mb. averaged over each of two annular rings, namely  $2^\circ < \Delta\phi < 4^\circ$  and  $4^\circ < \Delta\phi < 6^\circ$ . In this computation, local changes of density were assumed to be zero, and the vertical motion was assumed to vanish at the ground and at 125 mb. The values of the vertical motion obtained in this way were used in the evaluation of the vertical transport of angular momentum by azimuthal-mean motions, and the quantity  $[C_n]''$  was used in the evaluation of the Coriolis torque.

Since vertical motion is not measured directly it would be necessary to



use a somewhat analogous procedure in order to evaluate the second term in (15). Instead of computing the mass flux into a volume which encloses the center of the hurricane, one would compute the mass flux into a small volume enclosing each grid point on the conical wall and use the continuity equation to determine the vertical motion at that point. Since  $w$  is a sensitive quantity to compute, and since the main deviations of  $w$  from its azimuthal-mean value are associated with the spiral bands of the hurricane, which do not show up on the composite wind charts, the evaluation of the quantity

$[C_T w]$  from these charts would yield results of questionable significance.

For this reason, and also because a great deal of computation has to be performed in evaluating it, it was decided not to compute this term at the present time. It is recognized, however, that to establish the complete momentum cycle it will be necessary ultimately to estimate the magnitude of this term.

#### 4. RESULTS

The balance of angular momentum in the volumes bounded by the conical walls  $\Delta\phi = 2^\circ$  and  $\Delta\phi = 4^\circ$ , and by  $\Delta\phi = 4^\circ$  and  $\Delta\phi = 6^\circ$ , is presented in table 1. In evaluating these results it should be realized that they are more sensitive to error than those in [3] since the convergence of angular momentum into annular rings is generally a small difference between individually large fluxes. Thus, only the most gross aspects of these results may be considered reliable.

Table 1. Convergence of the angular momentum flux and frictional drain within the volumes  $2^\circ < \Delta\phi < 4^\circ$  and  $4^\circ < \Delta\phi < 6^\circ$ , based on the composite wind charts. 1 unit =  $10^{23}$  gm. cm.<sup>2</sup> sec.<sup>-2</sup>.

$\Delta\phi$	$2^\circ - 4^\circ$	$4^\circ - 6^\circ$
Horizontal transport of angular momentum by azimuthal-mean motions	+6.8	-1.2
Horizontal transport of angular momentum by horizontal eddies	+2.8	+3.9
Total transport of angular momentum	+9.6	+2.7
Frictional drain of angular momentum	+8.5	+12.3

Comparison of the last two rows in table 1 reveals that there is a large discrepancy between the estimated angular momentum transport and the estimated frictional drain of angular momentum within the volume  $4^\circ < \Delta\phi < 6^\circ$ . This is not, however, the case in the volume  $2^\circ < \Delta\phi < 4^\circ$ , where the two estimates show good agreement. Focusing attention on rows 1 and 2 of column 1 we find that azimuthal-mean motions accomplish most of the transport of angular

momentum into the inner volume. Although the magnitude of the horizontal-eddy transport into this volume might be under-estimated by the present data, the errors involved would represent a comparatively small percentage of the total transport of angular momentum into this volume. In the outer volume, however, the contribution of azimuthal-mean motions to the net transport of angular momentum is slightly negative. Since the magnitude of this process is comparatively small in this volume, errors in the estimate of the horizontal-eddy transport would comprise a large fraction of the total transport of angular momentum into this volume. Owing to the fact that the horizontal-eddy transport is more readily affected by the averaging that is done in the construction of composite wind charts, it is reasonable to expect, therefore, that the deficiencies in the outer volume are made up in the real atmosphere by this process. The horizontal-eddy transport of angular momentum could, in fact, be substantially more important than the present measurements indicate.

A further breakdown of the results is shown in tables 2 and 3, where the angular momentum balance of the lower and upper layer of each volume is presented. Equation (13a,b) provides the basis for this analysis. As noted earlier, this equation includes contributions due to the Coriolis torque and also to vertical transports of angular momentum. Table 2 gives the balance of angular momentum in the lower layer. According to the results, the Coriolis torque contributes most to the total within both the inner and outer volumes. In addition, the horizontal transport of angular momentum into the inner volume is accomplished mainly by azimuthal-mean motions, whereas in the outer volume this transport is due primarily to horizontal eddies. The vertical transport of angular momentum is apparently quite small, with an upward transport indicated in the inner ring and a downward transport in the outer ring. Comparison of the last two rows shows good agreement between the total of these effects and the frictional drain.

Table 2. Angular momentum balance of the lower portion of the atmosphere (1000 mb. to 600 mb.), based on the composite wind charts. 1 unit =  $10^{23}$  gm. cm. <sup>2</sup> sec. <sup>-2</sup>.

$\Delta\phi$	2° - 4°	4° - 6°
Horizontal transport of angular momentum by azimuthal-mean motions	+2.2	-0.1
Horizontal transport of angular momentum by horizontal eddies	+0.8	+2.6
Coriolis torque	+5.4	+7.8
Vertical transport of angular momentum by azimuthal-mean motions	-0.5	+0.4
Total	+7.9	+10.7
Frictional drain of angular momentum	+8.5	+12.3

Table 3. Angular momentum balance of the upper portion of the atmosphere (600 mb to 125 Mb), based on the composite wind charts. 1 unit =  $10^{23}$  gm. cm.<sup>2</sup> sec.<sup>-2</sup>.

$\Delta\phi$	2° - 4°	4° - 6°
Horizontal transport of angular momentum by azimuthal-mean motions	+4.6	-1.1
Horizontal transport of angular momentum by horizontal eddies	+2.0	+1.3
Coriolis torque	-5.4	-7.8
Vertical transport of angular momentum by azimuthal-mean motions	+0.5	-0.4
Total	+1.7	-8.0
Frictional drain of angular momentum	0	0

Since the Coriolis torque provides a positive contribution in the lower portion of the atmosphere, it must provide an equal and opposite negative contribution in the upper portion. Turning to table 3 we find that this term accounts for a large supply of anticyclonic angular momentum at upper levels in both the inner and outer volumes. In the inner volume both horizontal eddies and azimuthal-mean motions tend to fill this up with positive angular momentum, mainly through horizontal transports. In the outer ring, however, the horizontal transport of angular momentum by azimuthal-mean motions serves to drain angular momentum from the upper layer. The only positive contribution here comes from the horizontal-eddy transport of angular momentum. According to the present estimates this process is not sufficient to balance the depletion due to other processes, as a comparison of the last two rows indicates. It will be noted that the discrepancy in this case turns out to be greater than the largest single contribution from any term in the table. Furthermore, it is not likely that this discrepancy is made up by the neglected vertical transport, since this would drain angular momentum from the lower layer, which, according to table 2, is already slightly deficient. If our earlier speculation is correct, namely that the present measurements underestimate the horizontal transport of angular momentum by horizontal-eddy processes, the conclusion to be reached from the above results is that horizontal-eddy processes serve to make good the deficit of angular momentum which azimuthal-mean motions (including the Coriolis torque) tend to create in the upper portions of the outer volume.

## 5. CONCLUSIONS

Rotating wind systems, such as the hurricane, generally extend through great depths of the atmosphere. Although the Coriolis torque must play a vital role in the development of such systems, it cannot account for their great depth. Where the action of the Coriolis torque sets up a cyclone in the lower portion of the atmosphere it must set up an equally intense anti-cyclone in the upper portion. In order that a cyclonic system develop through a considerable depth of the atmosphere there must, therefore, be a transport of cyclonic angular momentum into the volume at upper levels. In the mature hurricane, within the volume  $2^\circ < \Delta \phi < 4^\circ$ , and probably also within  $0^\circ < \Delta \phi < 2^\circ$ , azimuthal-mean motions transport inward most of the angular momentum necessary to compensate for the Coriolis torque at upper levels. On the other hand, azimuthal-mean motions do not transport enough angular momentum into the belt  $4^\circ < \Delta \phi < 6^\circ$  to compensate for the Coriolis torque there. In all probability horizontal-eddy processes make up the balance, although data taken from the composite wind charts are evidently not adequate to capture such effects.

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