Improved Initial Pump Equation

for the MRV Systems ALTO^{TM} Profiling Float

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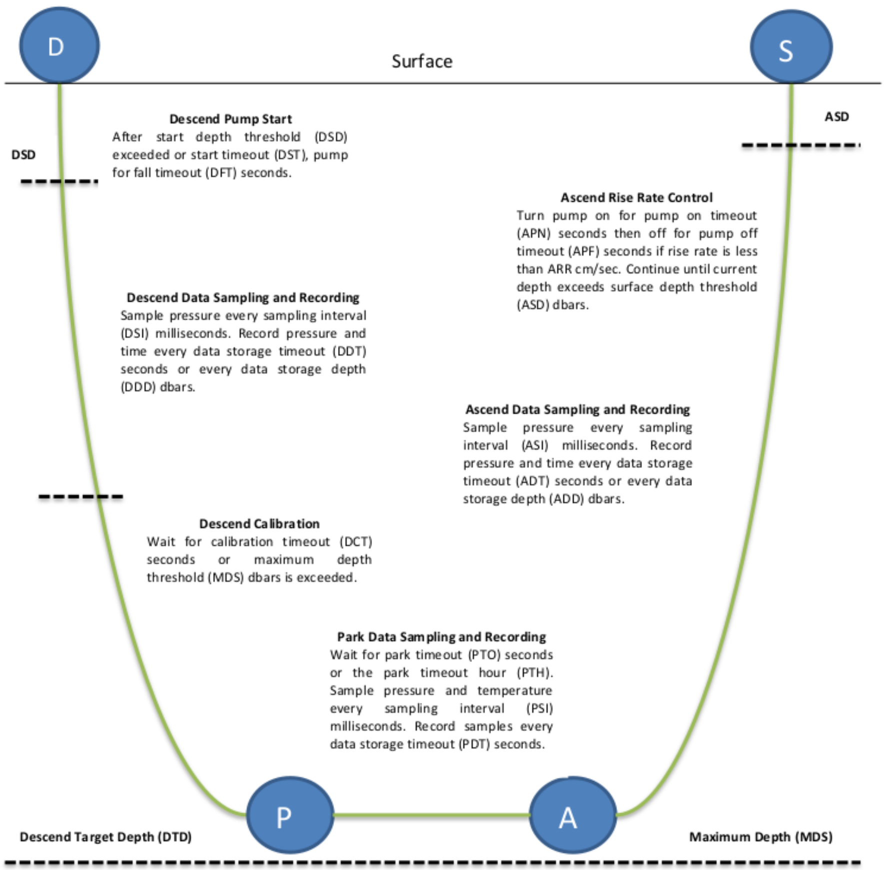

Figure 1: Here is a diagram showing some of the parameters which control the float's profile

1 Motivation And Objective

In an effort to streamline the production process and reduce costs for customers, MRV does not ballast each ALTO^{TM}. Instead, the floats are ballasted in mass to be *heavy* at their target depths.

In other words, ALTOs will not be neutrally buoyant at the target depth specified by the customer with all of the oil pulled inside.

Measuring the volume of each float would add days onto the delivery schedule and costs would be carried to the end costumer. By making each float heavier, the floats pump more oil (spending more energy) to reach their target depth.

This document serves to highlight and explain a new equation for piloting ALTOs which are ballasted *heavy*.

2 The Equation

As to not bury the lede, the initial pump for ALTOs is determined by two parameters: **DSD** and **DFT**.

DSD is the Descent Start Depth. This parameter determines at what depth the float will start the initial pump.

DFT is the Descent Fall Timeout. This parameter determines how long the float will pump for the initial pump.

Set the **DFT** as follows:

$$DFT = \frac{(\frac{M}{\rho} - V_0(1 - \alpha[T_0 - T] + \beta[P_0 - P])) * 30000}{10000 - DSD} \tag{1}$$

where M is the mass of the float, ρ is the target density of the water, V_0 is the volume of the float at the surface with all of the oil sucked in, α is a material constant which accounts for thermal contraction of the hull, T_0 is the temperature where the float's surface volume V_0 was calculated, T is the temperature at depth, β is a material constant which accounts for compression of the hull, P_0 is the pressure where the float's surface volume V_0 was calculated, P is the pressure at the target.

Let's unpack this slightly daunting equation.

The pilot wants the float to come to rest at a target depth P which has a target density ρ. The float's mass is constant – and MRV ballasts all ALTOs for a region at the same mass. The values of α and β are treated as constants in these calculations, since they don't vary much from float to float. That leaves one variable, V_0 which is the float's volume at the surface with all the oil sucked in. Because MRV does not ballast each float, we do not know this number exactly, however, for an initial guess we can use the average V_0 for all of MRV's ALTOs.

So the equation becomes:

$$DFT = \frac{(\frac{19500}{\rho} - 18666(1 - (7.29 * 10^{-5})[19.5 - T] - (2.43 * 10^{-6})P)) * 30000}{10000 - DSD} \tag{2}$$

Where the pilot inputs the target water density ρ, target water temperature T, target water pressure P, and what depth the pilot would like the float to start pumping **DSD**.

3 The Process

Let the mass of the float be M_{float}. At the surface, with all of the oil pulled in, the float will begin to sink. The float's volume here is V_0.

We want the float to reach a target depth as fast as possible, but also balance the descent rate such that we do not overshoot the target depth. Because the float is ballasted heavy, it needs to pump out oil – effectively increasing its volume so it can come to rest near the target depth. How much does the volume need to be for the float to be neutrally buoyant at the target depth?

The target depth has a density ρ. For the float to rest at that density, it would need to have the same density as the target density ρ.

The float's density is a function of its mass and volume:

$$\rho_{\text{float}} = \frac{M_{\text{float}}}{V_{\text{float}}} \tag{3}$$

Each float is ballasted to weigh roughly the same amount, so the masses between floats are roughly the same. The more challenging number to find out is V_{float}. But we at least have a starting point: V_0.

As the float descends into the depths, the water becomes colder and the pressure continues to climb. These two environmental effects serve to thermally contract and compress the float's hull. Both of these effects squeeze the hull by small, but not insignificant amounts.

We can account for these effects as

$$V_{\text{float}} = V_0 - \Delta V_\alpha - \Delta V_\beta \tag{4}$$

where ΔV_α is the change in volume due to the thermal contraction and ΔV_β is the change in volume due to the compression.

Aside from the environmental effects on the volume, which serve to decrease the volume and make the float less buoyant, the float can pump oil to regain volume and become more buoyant!

The floats pump for durations of time, and this initial pump offers the starting point for the float to attempt on its first dive. Now we have a more complete picture of how the float's volume changes dynamically throughout the water column

$$V_{\text{float}} = V_0 - \Delta V_\alpha - \Delta V_\beta + \Delta V_{\text{pumping}} \tag{5}$$

where $\Delta V_{\text{pumping}} = R\Delta t$ with R the pump rate and Δt the pump time parameter (initial pump guess).

The pump rate R is a function of depth because the pump is fighting against more sea pressure as it descends. Let's assume R is linear with depth. From testing and manufacturer datasheets, we know the pump will move 20 cc's of oil per minute at the surface and 16 cc's of oil per minute at 2000m (3000 psi).

With these two points, we can write a linear equation to calculate the pump rate for a given depth (pressure) P.

$$R = \frac{(20-16)\ \frac{\text{cc}}{\text{min}}}{(0-2000)\ \text{dBar}} * P + 20\frac{\text{cc}}{\text{min}} \tag{6}$$

$$R = (20 - \frac{P}{500})\frac{\text{cc}}{\text{min}} \tag{7}$$

We can then calculate the float's volume V_{float} as

$$V = V_{\text{float}} = V_0 - \Delta V_\alpha - \Delta V_\beta + (20 - \frac{P}{500})\Delta t \tag{8}$$

We calculate the environmental factor of thermal contraction ΔV_α as

$$\Delta V_\alpha = \alpha V_0 \Delta T \tag{9}$$

And the environmental factor of compression ΔV_β as

$$\Delta V_\beta = -\beta V_0 \Delta P \tag{10}$$

where α and β are material constants (functions of the float's construction).

So now we have an even better picture of the volume of the float as a function of temperature T, pressure P, and pumping time Δt

$$V_{\text{float}} = V_0 - \alpha V_0(T_{\text{calculated}} - T_{\text{target}}) - (-\beta V_0(P_{\text{calculated}} - P_{\text{target}})) + (20 - \frac{P}{500})\Delta t \tag{11}$$

According to Archimedes (and lots of experimental confirmation), if the float displaces a mass of water more than the float's own mass, the float will rise. If the displaced water mass is the same, the float is neutrally buoyant. And if the mass of water displaced by the float is less than the float's mass, the float will sink.

We calculate the mass of the water displaced by the float as

$$M_{\text{water}} = \rho V_{\text{float}} \tag{12}$$

To make the float neutral at some target density, the left hand side of the equation can be set to the float's own mass. Then we can solve for the pump time Δt (which is controlled by **DFT**) to achive V_{float}.

4 Example Calculation

Let's try an example calculation. An ALTO pilot wants to set a parking depth of 1200m. The ALTO has a few key parameters to achieve this goal, **DFT** (the initial pump time) and **DSD** (what depth the pump should turn on).

The pilot decides to pump right after the surface, around 80 dBar (see ALTO S/N 10083).

The target water conditions are for the South China Sea. With a target temperature of $T = 4$ C, salinity $S = 34.51$ and density $\rho = 1.0329175$ g/cm^3, at a target depth of $P = 1200$ dBar.

To be neutral at ρ, the float will need to displace a mass of water equal to its own mass:

$$M = \rho V \tag{13}$$

Rearrange the equation to find the volume:

$$V = \frac{M}{\rho} \tag{14}$$

These ALAMOs have been ballasted at 19275 g, so the volume of the water displaced is

$$V = \frac{19275 \text{ g}}{1.0329175 \ \frac{\text{g}}{\text{cm}^3}} = 18660.7 \text{ cm}^3 \tag{15}$$

The minimum displacement at the surface (with all the oil inside) for MRV's ALTOs is 18642 cm^3. We need to apply thermal contraction and compression effects (see Eqs.(4), (9), and (10)).

$$V_{\text{float}} = V_0 - \alpha V_0 (T_{\text{calculated}} - T_{\text{target}}) - (-\beta V_0 (P_{\text{calculated}} - P_{\text{target}})) \tag{16}$$

With a thermal contraction constant of $\alpha = 7.29 * 10^{-5}$ C^{-1}, and a compression constant of $\beta = 2.43 * 10^{-6}$ dBar^{-1}, we can plug and chug to find that the float displaces

$$V_{\text{float}} = 18566.6 \text{ cm}^3 \tag{17}$$

The float naturally will displace 18566.6 cm^3 when it is brought to 1200m. However for the float to be neutrally buoyant it needs to be displacing 18660.7 cm^3.

This difference of 94.1 cm^3 can be made up for in pumping

$$\Delta V_{\text{pumping}} = (18660.7 - 18566.6) \text{ cm}^3 = 94.1 \text{ cm}^3 \tag{18}$$

The pumping goes as

$$\Delta V_{\text{pumping}} = (20 - \frac{P}{500}) \frac{\text{cc}}{\text{min}} (\frac{\text{min}}{60 \text{ seconds}}) \Delta t \tag{19}$$

If the pilot chooses to start pumping at $P = 80$ dBar, then the time to pump follows

$$\Delta t = \frac{\Delta V_{\text{pumping}}(30000)}{10000 - P} \tag{20}$$

Let's use the piloting parameters now, and substitute in our values (remember 1 cm^3 is conveniently 1 cc)

$$\mathbf{DFT} = \frac{\Delta V_{\text{pumping}}(30000)}{10000 - \mathbf{DSD}} \frac{\text{s}}{\text{cc}} \tag{21}$$

$$\mathbf{DFT} = \frac{94.1\ \text{cm}^3(30000)}{10000 - 80} \frac{\text{s}}{\text{cc}} \tag{22}$$

$$\mathbf{DFT} = 285 \text{ seconds} \tag{23}$$