Ray

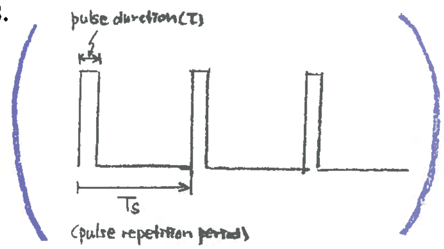
1 (out of 35)

* Several radar related questions

MET5411 (Radar MET): Ray (45 minutes) *

• Assume a Doppler radar transmits a 10 cm wavelength pulse for 4×10^{-6} s every 10^{-3} s. The peak power is a megawatt. The radar also "listens" for 4×10^{-6} s.

a. What is the PRF?
b. What is the duty cycle?
c. What is the average power transmitted?
d. What is the unambiguous range?
e. What is the Nyquist co-interval?
f. What is the "Doppler Dilemma"?
g. What is the means for overcoming the "Doppler Dilemma"?



Sol)

We are given a radar with the following characteristics:

$\lambda = 10cm$ wavelength
$\tau = 4\times10^{-6}$ s pulse duration
$T_s = 10^{-3}$ s pulse repetition period
$P_T = 10^6$ W peak transmitted power

(a) PRF = pulse repetition frequency

By definition $PRF = T_s^{-1}$

So here $PRF = (10^{-3}s)^{-1} = 1000$ Hz

(b) The duty cycle is the PRF times the pulse duration or equivalently the pulse duration divided by the pulse repetition period. It is a unitless quantity. If we use the definition in the form

$$\boxed{\text{duty cycle} = \frac{\text{pulse duration}}{\text{pulse repetition period}}}$$

We see that the duty cycle represents the fraction of the time the radar is transmitting a pulse. For our case,

$$\text{duty cycle} = \frac{4\times10^{-6}s}{10^{-3}s} = 4\times10^{-3}$$

→ no units.

(c) The average power transmitted is

$$\boxed{\bar{P}_t = \frac{P_T\,\tau}{T_s}} = \frac{(10^6W)(4\times10^{-6}s)}{10^{-3}s} = 4kW = 4000W$$

→ peak transmitted power * duty cycle

(d) The unambiguous max range is

$$r_{max} = \frac{cT_s}{2} = \frac{(3\times10^8 m/s)(10^{-3}s)}{2} = \cancel{300 km}$$

?

→ 150km

(e) The Nyquist co-interval is

$$f_{NY} = \pm\frac{1}{2T_s} = \pm\frac{1}{2(10^{-3}s)} = \pm 500 \text{ Hz}$$

Since $f = \pm\frac{2V}{\lambda}$ ⟶ ∵ $\left\{\begin{array}{l} \phi = \phi_0 + \frac{2r}{\lambda}2\pi \\ \frac{d\phi}{dt} = 0 + \frac{4\pi}{\lambda}\frac{dr}{dt} \\ 2\pi f = \frac{4\pi}{\lambda}V \\ f = \frac{2V}{\lambda} \end{array}\right.$ ⟵ phase of returned signal

So

$$V_{NY} = \pm\frac{f_{NY}\lambda}{2}$$

→ $\pm\frac{500 Hz\cdot(0.1m)}{2}$

∴ $f_{NY} = \pm 500$ Hz, $V_{NY} = \pm 25$ m/s

(f) Doppler Dilemma

$$|r_{max}V_{max}| = \frac{cT_s}{2}\cdot\frac{1}{2T_s}\frac{\lambda}{2} = \frac{c\lambda}{8}$$

→ can not, in general increase both.

→ See other notes

(g) → " " "

g.

• partial solution: Increase radar λ. However this requires larger antenna → expensive //

Also, this increases ground clutter, and reduces resolution.

• another solution is to vary the PRF: Real echoes will not move, whereas aliased ones will shift depending on whether the PRF is increased or decreased. Automated transmission schemes would be complicated but possible.

• Another solution could be post processing. For velocity folding, algorithms which look for velocity disc. with range + reducing them by Nyquist vel. → vel. unfolding

• phase/Polarization: Vary phase or polarization → can distinguish echoes from spec. pul

Also, take smaller samples for reflectivity ($T_{s\text{-ref}} = 4T_{s\text{-vel}}$) → extend max unambiguous range for ref. by factor of 4. Then use range folding tech. to extend range for vel. field (eg – vel. detected at 150 km where there is no ref. however there is a measured ref at 300km; fold vel. out to 300km to match vel. echo.)

f. The Doppler dilemma refers to the contradiction that arises between optimizing the max velocity that can be detected and the max range that can be detected as a function of the PRF.

* pulsed Doppler radar

MET5411 (Radar MET): Ray (?)

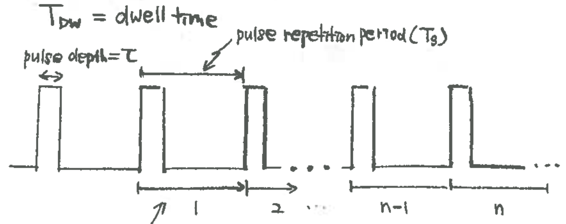
- For pulsed Doppler radar systems, what is the advantage and disadvantage of different PRF's and dwell times? Be as complete as you can, of course.

Sol)

Let τ = pulse duration

T_s = time between successive pulses → pulse repetition period = $(PRF)^{-1}$

T_{DW} = dwell time

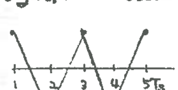


we composite the signal from n consecutive samples.

Due to the statistical nature of weather echoes we average several consecutive returns. If we take a sample of size M, each observation being independent of all other observations the variance of sample decreases by a factor of M. The length of time over which we collect M samples is the dwell time

✓ $T_{DW} = M T_s$

The highest resolvable frequency is the Nyquist frequency. This corresponds to a high-low-high-low ··· signal. Thus oscillation has a frequency of $f_{NY} = \frac{1}{2T_s} = \frac{PRF}{2}$



In a sample of M observations, each T_s apart in time the lowest freq., longest period we can resolve is

fundamental period MT_s

" frequency $f_o = \frac{1}{MT_s}$

The resolved harmonic frequencies are

$$f_n = \frac{n}{MT_s} \quad ; n = 1, 2, \cdots, \frac{M}{2}$$

Thus the frequency resolution is

$$\Delta f = \frac{1}{MT_s}$$

The Doppler radar transmits a pulse with phase ϕ_o. This pulse travels a distance r and scatters off a target. The phase of the returned signal is

$$\boxed{\phi = \phi_o + \frac{2r}{\lambda} 2\pi}$$

The time rate change in phase, the angular frequency, is

$$\frac{d\phi}{dt} = \overset{0}{\cancel{\frac{d\phi_o}{dt}}} + \frac{4\pi}{\lambda}\frac{dr}{dt}$$

$$\omega = 2\pi f = \frac{4\pi V}{\lambda} \leftarrow \text{radial velocity}$$

$$\boxed{f = \frac{2V}{\lambda}}$$

This is the fundamental Doppler formula relating frequency (doppler phase shift/time) to the doppler velocity.

Given a frequency resolution of $\Delta f = \frac{1}{MT_s}$, the corresponding velocity resolution is

$$\Delta f = \frac{2\Delta V}{\lambda}$$

$$\Delta V = \frac{\lambda}{2}\Delta f = \frac{\lambda}{2MT_s}$$

The lowest resolved frequency is $f_o = \frac{1}{MT_s}$. This yields a velocity of $V = \frac{\lambda f_o}{2} = \frac{\lambda}{2MT_s}$. The "harmonic" (resolved) velocities are

$V_n = \frac{\lambda n f_o}{2} = \frac{\lambda n}{2MT_s}$, $n = 1, 2, \cdots, \frac{M}{2}$. The highest resolved velocity is

$$V_{max} = \frac{\lambda f_{NY}}{2} = \frac{\lambda}{2}\frac{1}{2T_s} = \frac{\lambda}{4T_s} = \frac{\lambda \cdot PRF}{4}$$

So summarizing,

Make M consecutive samples, each T_s apart in time. This reduces the variance in estimates as compared to the variance in a single observation.

$\Delta f = \frac{1}{MT_s} = f_o$; the fundamental freq → lowest resolved freq. → longest resolved period.

$f_n = \frac{n}{MT_s}$; $n = 1, 2, \cdots, \frac{M}{2}$ → resolved harmonic frequencies

$f_{NY} = \frac{1}{2T_s}$ ⇒ Nyquist freq. → highest resolved freq. → shortest resolved period

Via the Doppler relation

$$f = \frac{2V}{\lambda}$$

$\Delta V = \frac{\lambda}{2MT_s}$ ⟶ velocity resolution.

$V_n = \frac{\lambda n}{2MT_s}$ ⟶ resolved harmonics of velocity

$\boxed{V_{max} = \frac{\lambda}{4T_s} = \frac{\lambda \cdot PRF}{4}}$ → max. unambiguous velocity.

The max unambiguous range for a single pulse

$$\boxed{r_{max} = \frac{CT_s}{2} = \frac{C}{2 \cdot PRF}}$$

However, if we listen over M pulses the max range ⇒ corresponding to the first transmitted pulse is

$$R_{max} = \frac{CT_{DW}}{2} \simeq \frac{MT_sC}{2}$$

This is wrong. We listen to M consecutive pulse. Each has

$$r_{max} = \frac{CT_s}{2} = \frac{C}{2PRF}$$

Sol)

First, let's define PRF + dwell times

PRF = Pulse Repetition Frequency : This is the frequency with which the pulses are emitted. Usually expressed as # pulses per second

Dwell time is the length of time spent at a particular range gate. That is, the amount of time allowed for the calculation of the Doppler spectra at a specific "location" (antenna direction and/or a specified range gate).

Advantages / disadvantages of different PRFs :

The maximum Doppler shift freq. that can be detected (Nyquist freq) = $\frac{1}{2}$(PRF).

The max unambiguous doppler velocity (V_{max}) is that which produces a phase shift of π radians (1/2 cycle)

$$V_{max} = \frac{1}{2} f_{max} \lambda = \frac{1}{4}(PRF)\lambda$$

The max unambiguous range (R_{max}) of the radar in that range where the pulses make a single round trip

$$R_{max} = \frac{c}{2 \cdot PRF}$$

So the advantage of Large PRF is that it increases max detectable velocity (V_{max}). The advantage of small PRF is that it increases max unambiguous range (R_{max}).

→ There is a trade-off of R_{max} & V_{max} ("Doppler Dilemma")

$$* \begin{cases} V_{max} = \frac{\lambda \cdot PRF}{4} \\ r_{max} = \frac{c}{2 \cdot PRF} \end{cases}$$

Adv. / Disadv. of different Dwell times :

Dwell Time = MT_s , M = # of samples , T_s = 1/PRF

Increased dwell time → more "listening" → longer range (R_{max} ↑), less second trip echoes, etc ; trade-off → decreased resolution

Decreased dwell time → decreased range , increased resolution

* effective area + max detection range

MET5411 (Radar MET): Ray (~~20~~ 10^5 minutes)

• (a) If the antenna gain is 10,000 and the wavelength is 5 cm, find the effective area. (b) Let the target cross section be 1 m^2, elevation angle 0°, the propagation be in space, and the peak transmitted power be 1 MW. If the minimum detectable power is 10^{-12} W, what is the range of the target at which detection will be lost?

The formula for effective area, A_e, is: $A_e = \frac{G\lambda^2}{4\pi}$

(a) The formula for effective area is $Ae = \frac{G\lambda^2}{4\pi}$.

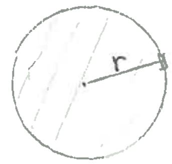
We are given $G = 10^5$ and $\lambda = 0.05\,m$

Thus $Ae = \frac{10^5 (0.05m)^2}{4\pi} = 19.89\ m^2$

(b)

If there is a target at range r with cross sectional area At, it will intercept an amount of power

$$\boxed{P_\sigma = \underbrace{\frac{P_t G}{4\pi r^2}}_{\text{transmitted power focused along beam axis/area (power density)}} \cdot \underbrace{At}_{\text{cross sectional area of target}}}$$



Assume the target does not absorb any power but re-radiates its all isotropically. The power intercepted by the radar antenna is

$$P_r = \frac{P_\sigma}{4\pi r^2} \cdot Ae = \frac{P_t G}{(4\pi r^2)^2} \cdot At \cdot Ae$$

We are given $Ae = \frac{G\lambda^2}{4\pi}$

So $$P_r = \frac{P_t G At G \lambda^2}{(4\pi)^3 r^4} = \frac{P_t G^2 \lambda^2 At}{(4\pi)^3 r^4}$$

We want r such that $P_r = P_{min}$ given $P_t = P_{max}, G, \lambda, At$.

Solve for r

$$r^4 = \frac{P_t G^2 \lambda^2 At}{(4\pi)^3 P_r}$$

So at $r = r_{max}$ where $P_r = P_{min}$ + $P_t = P_{max}$

$$r_{max} = \left(\frac{P_{max} G^2 \lambda^2 At}{(4\pi)^3 P_{min}}\right)^{1/4}$$

Sub. known values

$$r_{max} = \left[\frac{10^6\,W \cdot (10^5)^2 (0.05m)^2 (1m)^2}{(4\pi)^3 \cdot 10^{-12}\,W}\right]^{1/4}$$

≈ 335 km.

MET5411 (Radar MET): Ray (20 minutes)

- A 10 cm wavelength radar radiates a peak power of $P_t = 10^6$ watts, with a pulse length of 1 μs, and has an antenna gain of 50 dB. A spherical raindrop of 8.4 mm in diameter is located at $r = 20$ km.
  a. What is the power density at the drop?
  b. What is the power density returned to the antenna?

Rayleigh approx. → hint: $\sigma_b = \frac{\pi^5}{\lambda^4}|K|^2 D^6$ and $|K|^2 \approx .93$

a. How much power per unit area is there at $r = 20$km?

$\boxed{S = \frac{P_t}{4\pi r^2} \cdot G}$ ← power density has units of W/m²

Where $P_t = 10^6$ W is the transmitted power

$r = 20$km is the range to the target

$G = 50$ dB $= 10^5$ is the gain of the antenna

∗ Z dB $= 10 \log_{10} G$

50 dB $= 10 \log_{10} G$

$5 = \log_{10} G$

$G = 10^5$

∗ Z dB $= 10 \log_{10} G$

Sub. known values

$$S = \frac{P_t G}{4\pi r^2} = \frac{10^6\text{ W} \cdot 10^5}{4\pi (20\times10^3\text{ m})^2}$$

$\boxed{S = 19.9\text{ W/m}^2}$

b. We will assume the drop re-radiates intercepted power isotropically. Let σ_b denote the backscatter cross section of the drop. The electrical size of the drop is $\alpha = \frac{\pi d}{\lambda} = \frac{\pi(8.4\times10^{-3}\text{m})}{0.1\text{m}} = 0.26 < 1$ so we apply the Rayleigh approx. and take

$$\sigma_b = \frac{\pi^5}{\lambda^4}|K|^2 d^6 \quad \text{where } |K|^2 = 0.93 \text{ for water at } \lambda = 10\text{cm}$$

So the power intercepted by the drop is

$$P_\sigma = S\sigma = \frac{P_t G}{4\pi r^2}(\sigma_b) \leftarrow \frac{\pi^5}{(0.1\text{m})^4}(0.93)(8.4\times10^{-3}\text{ m})^6$$

$$P_\sigma = (19.9\text{ W/m}^2)(9.998\times10^{-7}\text{ m}^2)$$

$$P_\sigma \approx 1.9896\times10^{-5}\text{ W}$$

Built into the definition of σ_b is the fact that it is defined to be the cross section area of the scatterer such that it would back scatter isotropically. What this means is that the power density received at the radar is

$$\boxed{S_r = \frac{P_\sigma}{4\pi r^2} \approx 3.958\times10^{-15}\text{ W/m}^2}$$

∗ also see the next question! & the previous question!

* radar related Qs

MET5411 (Radar MET): Ray (1 hour) *

- Question

A) A 10 cm radar radiates a peak power of $P_t = 10^6$ watts and has an antenna gain of 50 dB. A spherical rain drop of diameter 8.4 mm is located at r = 20 km. $G = 10^5$

The back scatter cross section $\longrightarrow$ $\sigma_b = \frac{\pi^5}{p^4}|Kw|^2 D^6 \Rightarrow \sigma_b = \frac{\pi^5}{\lambda^4}|K|^2 D^6$, $|K|^2 = 0.93$

a) What is the level of power density at the drop? (W/m²)

b) What is the level of power density returned to the antenna? (W/m²)

c) How much echo power is delivered to the receiver?

d) The reflectivity factor (in dBZ) of randomly distributed drops, all of 8.4 mm diameter, is 40 dBZ. Assume that the radar's resolution volume is $0.1 \times 0.5 \times 0.5\ \text{km}^3$. How many drops are contained in this volume?

B) What does the Doppler signature look like for rotation, divergence, the combination? How can you estimate vertical velocity from this? Explain your answer fully.

C) What all can you tell from a VAD analysis? Velocity Azimuth Display

D) Define and explain the "Doppler Dilemma".

A)

(a)

$$S_{drop} = \frac{P_t G}{4\pi r^2} = \frac{10^6\ W \cdot 10^5}{4\pi (20 \times 10^3 m)^2} \cong 19.894\ W/m^2$$

$\boxed{S_{drop} = 19.894\ W/m^2}$ $\leftarrow$ power density at drop

(b)

$$S_{radar} = \underbrace{\frac{P_t G}{4\pi r^2}}_{\text{power density at drop}} \sigma_b \cdot \frac{1}{4\pi r^2}$$

power intercepted by drop

$$S_{radar} = (19.894\ W/m^2)\left[\frac{\pi^5}{(0.1 m)^4}(0.93)(8.4 \times 10^{-3} m)^6\right] \cdot \frac{1}{4\pi (20 \times 10^3 m)^2}$$

$\boxed{S_{radar} = 3.957 \times 10^{-15}\ W/m^2}$ $\leftarrow$ returned power density at radar

(c)

$$P_r = \underbrace{\frac{P_t G}{4\pi r^2}\sigma_b \frac{1}{4\pi r^2}}_{S_{radar}} \cdot A_e$$

effective area of antenna $A_e = \frac{G\lambda^2}{4\pi}$

$$P_r = S_{radar} \cdot \frac{G\lambda^2}{4\pi} = (3.957 \times 10^{-15}\ W/m^2)\left(\frac{10^5 \cdot (0.1 m)^2}{4\pi}\right)$$

$\boxed{P_r = 3.149 \times 10^{-13}\ W}$ $\rightarrow$ returned power at radar.

(d)

The radar reflectivity factor Z is the quantity

$$Z = \frac{1}{VOL}\sum_{vol} D_i^6 \longrightarrow \text{units of } \frac{mm^6}{m^3}$$

where VOL is the radar resolution volume

We have 40 dBZ $\Rightarrow 10^4$

$$10^4 \frac{mm^6}{m^3} = \left(\frac{1}{VOL}\right) N (8.6 mm)^6$$

$$= \frac{1}{0.025\ km^3} \times \left(\frac{1 km}{1000 m}\right)^3 \times N \times (8.4 mm)^6$$

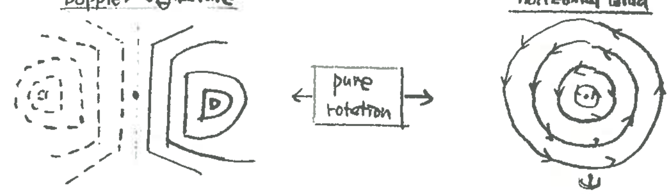
$$= 0.01405 \frac{mm^6}{m^3} \times N$$

$N = 7.12 \times 10^5$ droplets ??

B)

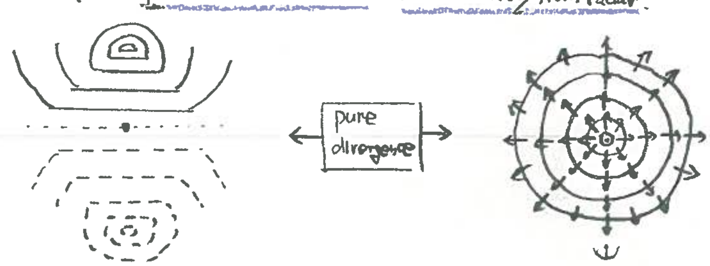
Below are hypothetical Doppler signatures and actual horizontal wind fields some distance from the radar $\longrightarrow$ radar south of flow field

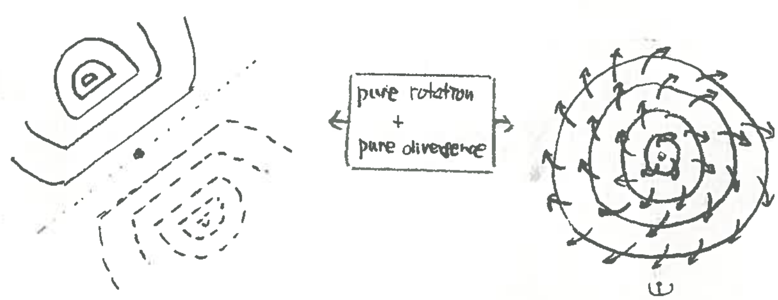
Doppler signature — horizontal wind



where ---- negative doppler velocity $\rightarrow$ radially toward radar

—— positive " " $\rightarrow$ " away from radar.





We can decompose a wind field into a rotational + divergent component

$$\underline{V}_H = \underline{V}_\psi + \underline{V}_\chi$$

$\hookrightarrow \nabla\chi \longrightarrow$ divergent part $\nabla \cdot \underline{V}_\chi = \nabla^2\chi$

$\hookrightarrow \mathbb{k} \times \nabla\psi \longrightarrow$ rotational part $\nabla^2\psi = \mathbb{k} \cdot \nabla \times \underline{V}_\psi$

We have just illustrated doppler signatures for purely rotational flow, purely divergent flow, and a mixture of the two. Generally the wind will be a mixture of both rotational + divergent components. If we can decompose the doppler signature into a rotational + divergent component and we repeat this for many elevation angles, then we can get an vertical profiles of the divergent + rotational components of the wind. Using the Kinematic method we can obtain an estimate of the vertical velocity

$$W(Z_2) = W(Z_1) - \int_{Z_1}^{Z_2} \nabla \cdot V_x \, dZ$$

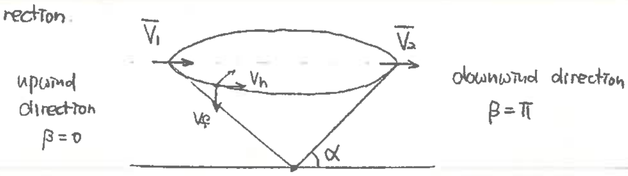
We may integrate ↑ or ↓. As a B.C. we may take $w = 0$. Since errors accumulate as you integrate ↑ or ↓ it is better to integrate down from Z_{top} since our estimates of $\nabla \cdot V_x$ should be (slightly) less in error than those obtained at low elevation angle.

C)

Lhermitte + Atlas (1961) proposed a wind measuring method which is very useful when a doppler radar is surrounded by scatterers having uniform motion. The technique, called Velocity · Azimuth Display, employs an azimuthally scanning beam at a constant elevation angle α. The Doppler velocity measured at height h when the antenna is pointing at an elevation angle α depends on the fall velocity of the particles, V_f, and on the horizontal wind speed V_h. Let β be the azimuth angle of the antenna w.r.t. the upwind direction. The Doppler velocity $\bar{V}$ (actually a spectrum of doppler velocities) is given by

$$\bar{V} = V_h \cos\alpha \cos\beta + V_f \sin\alpha$$

If the wind field + the particle fall velocities are uniform over the region being observed, $\bar{V}$ will vary sinusoidally with a max $\bar{V}_1$ occurring when the beam azimuth passes the upwind ($\beta = 0$) direction and a min $\bar{V}_2$ when the beam azimuth passes the downwind ($\beta = \pi$) direction.



We have 2 eqs in 2 unknowns (V_h + V_f)

$$\beta = 0 : \quad \bar{V}_1 = V_f \sin\alpha + V_h \cos\alpha$$

$$\beta = \pi : \quad \bar{V}_2 = +V_f \sin\alpha - V_h \cos\alpha$$

We can solve for V_f + V_h

$$V_h = \frac{\bar{V}_1 - \bar{V}_2}{2\cos\alpha} \quad ; \quad V_f = \frac{\bar{V}_1 + \bar{V}_2}{2\sin\alpha}$$

Divergence is obtained from the magnitude of the 0^{th} harmonic of VAD. The translational part of the wind is obtained from the amplitude + phase of the 1^{st} harmonic. The orientation of the axis of dilatation + resultant deformation are obtained from the amplitude + phase of the 2^{nd} harmonic.

Vorticity can not be determined from a VAD.

In practice, the coefficients of the 0^{th}, 1^{st}, 2^{nd} harmonics are fitted to the data using a least squares technique. One can also adjust the divergence to fit the kinematically computed vertical velocity. Scans in which a complete circle have not been computed may also be used.

D)

The term "Doppler Dilemma" refers to the fact that one can not have both a large unambiguous maximum range and a large unambiguous maximum Doppler velocity.

The Doppler radar transmits pulses at a frequency called the pulse repetition frequency (PRF). The maximum time available between pulses is $(PRF)^{-1}$. In this time the pulse can travel out and back from the maximum observable range, R_{max}. The distance depends inversely on the PRF.

$$\boxed{R_{max} = \frac{c}{2\,PRF}} \qquad (1)$$

An echo at the range $R_{max} + n\left(\frac{c}{2PRF}\right)$ (any integer) will be received at the radar at the same time as one from range R_{max}. Thus echoes beyond R_{max} are folded into the range $0 \rightarrow R_{max}$. Numerous techniques have been devised to deal with this problem. One obvious solution is to decrease the PRF. ⇒ This, however conflicts with the desire for a large unambiguous Doppler velocity, as we will see. A fundamental sampling theorem states that to measure a frequency f_d it is necessary to sample at a freq. of at least $2f_d$. The sampling rate is the PRF, thus

$$2f_d = PRF \;\Rightarrow\; f_{Ny} = \frac{PRF}{2}$$

The Doppler theorem states that the observed velocity is related to the freq. shift f by

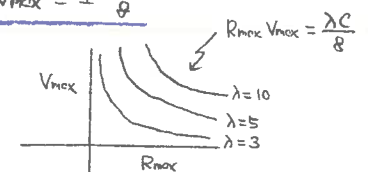
$$V = \frac{f\lambda}{2} \qquad \because \left(\begin{array}{l} \phi = \phi_0 + \frac{4\pi r}{\lambda} \\ \frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dr}{dt} \end{array} \right) \rightarrow \left(\begin{array}{l} 2\pi f = 2\pi \frac{2V}{\lambda} \\ \therefore V = \frac{f\lambda}{2} \end{array} \right)$$

It follows that the max unambiguous Doppler velocity (which defines the Nyquist interval) is given by

$$\boxed{V_{max} = \pm \frac{(PRF)\lambda}{4}} \qquad (2)$$

To extend this range we must increase the PRF or use a longer wavelength λ. If we multiply (1) + (2) we obtain

$$R_{max} V_{max} = \pm \frac{\lambda c}{8}$$



$$V = \frac{f\lambda}{2} \qquad r_{max} = \frac{c}{2PRF} \qquad V_{max} = \frac{PRF \cdot \lambda}{4}$$

We now see what is meant by the Doppler Dilemma.

- To obtain a large R_{max} → decrease PRF but this decreases V_{max}
- To " " " V_{max} → increase " " " " R_{max}

⇒ We want both large V_{max} + R_{max}

One alternative is to increase the wavelength λ

advantages
- reduction in attenuation effects
- larger R_{max} + V_{max}

disadv.
- reduced ability to detect small particles → reduced resolution.
- greater ground clutter problems.
- larger antenna + more costly radar system.

It should be noted that there are ways to distinguish range folded echoes.

① Interlaced PRFs
② Use of multiple polarization
③ phase diversity.

* radar concepts

MET5411 (Radar concepts): Ray (?)

• Question

a) Distinguish between reflectivity, reflectivity factor and equivalent reflectivity factor.

b) What are some methods for reducing range ambiguity? → $R_{max} = \frac{c}{2\,PRF}$

c) Why are radar returns less meaningful at long ranges?

d) What is the "Doppler Dilemma" and what can be done about it?

See other questions.

a)

1) Reflectivity factor (Z) : $Z = \sum N_i D_i^6$, where N_i is the number of drops of diameter D_i per unit volume. Hence units are L^6/vol, and usually expressed as (mm^6/m^3). Often Z is expressed in log units call dB, where $Z(dB) = 10\log_{10}(Z(mm^6/m^3))$. So how is Z related to radar eq.?

$$P_R = \frac{C_1 |K|^2 Z}{r^2} = \frac{CZ}{r^2}$$ where $C_1 = C_1(P_t, \theta, \phi, G, \lambda, \text{pulse length})$, $|K|^2$ = complex index of refraction.

If we assume we a sampling only water ($|K|^2 \simeq 0.93$), then we get errors of about 7 dB if we are really sampling ice ($|K|^2 \approx 0.2$). Note that usually Z is evaluated assuming liquid drops.

2) Equivalent (or effective) reflectivity factor (Z_e)

Z_e is the conc. of uniformly distr. small ($D \ll \lambda$) drops that would return the power received.

Hence it has same units as Z

$$Z_e = \frac{\lambda^4 \eta}{\pi^5 |K|^2}$$

3) Reflectivity (η)

$\eta = \sum \sigma_i / Vol$, where σ_i is the backscattering cross sectional area of the particles, η is expressed as area/vol → length^{-1}, usually in units (cm^{-1}). η & Z are related by

$$\eta = \frac{\pi^5 |K|^2 Z}{\lambda^4}$$

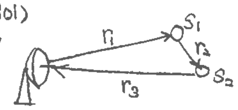
So reflectivity is what is really sensed by the radar – we have to make assumptions about $|K|^2$ to get at what we really want, particle distribution → rainfall, etc.

* (Power received at the receiver) multiple scatter return

MET5411 (Radar MET): Ray (?)

- A radar transmits a signal through an antenna to an isotropically radiating scatterer (s_1) located on at range r_1 from the radar. The scatterer radiates isotropically to a second scatterer (s_2) at a range r_2 from the first scatterer. The second scatterer then radiates isotropically back to the radar at range r_3. Derive an expression for the power received at the receiver. Be sure to define all variables. Express the result in dBZ.

Sol)



Assume S_1, S_2 scatter isotropically

Define A_1, A_2 as cross-sectional area of S_1, S_2.

Define G as the radar gain.

(1) Power received at $S_1 \equiv P_{S_1} = \dfrac{P_t G A_1}{4\pi r_1^2}$, Now assume S_1 scatters isotropically

peak transmitted power

(2) Power received at $S_2 \equiv P_{S_2} = \dfrac{P_{S_1} A_2}{4\pi r_2^2} = \dfrac{P_t G A_1 A_2}{(4\pi)^2 r_1^2 r_2^2}$

Now assuming S_2 scatters isotropically

(3) Power incident on antenna

$$P_{ant} = \frac{P_{S_2} A_{rad}}{4\pi r_3^2} = \frac{P_t G A_1 A_2 A_{rad}}{(4\pi)^3 r_1^2 r_2^2 r_3^2}$$

Where A_{rad}. = Effective radar cross sectional area. However we can define this as $A_{rad} = \dfrac{G\lambda^2}{4\pi}$, where λ = wavelength of radar.

(4) Power received $= P_r = \dfrac{P_t G^2 \lambda^2 A_1 A_2}{(4\pi)^4 r_1^2 r_2^2 r_3^2}$

$$= \left[\frac{P_t G^2 \lambda^2}{(4\pi)^4}\right] \frac{A_1 A_2}{(r_1 r_2 r_3)^2}$$

(5) To express answer in dB, consider two possibilities : dBm + dBZ

- dBm is dB calc. wrt mW

$\rightarrow P_r(\text{dBm}) = 10 \log_{10}[P_r(\text{mW})]$

- dBZ : This is a measure for reflectivity factor w.r.t units of mm^6/m^3

Recall above we found $P_r = \left[\dfrac{P_t G^2 \lambda^2}{(4\pi)^4}\right] \dfrac{A_1 A_2}{(r_1 r_2 r_3)^2}$, which may effectively be expressed as $C \dfrac{Z}{r^2}$

Solve for "Z" $= P_r \left[\dfrac{(4\pi)^4}{P_t G^2 \lambda^2}\right] (r_1 r_2 r_3)^2$

If we solve for this quantity in units of mm^6/m^3, then

$Z(\text{dBZ}) = 10 \log_{10}[Z(mm^6/m^3)]$

* clear air echoes

MET5411 (Radar MET): Ray (?)

- (a) What are the possible explanations for "clear air" echoes seen by a radar. (b) How might you design an experiment to distinguish between the different possibilities.

Sol)

By clear air echo we refer to measurable radar returns from target(s) which are not readily detectable by visual means. It is now largely accepted that clear air echoes are primarily caused by

(1) insects, birds, or other discrete single point targets.

and/or

(2) regions of the atmosphere where there are strong refractive index gradients.

Echoes falling into class (1) are called dot or point angels. They are absribed to the back-scattering by discrete single targets such as birds or insects. The second category of clear air echoes, (2), are often referred to as layer angels. They are characterized by having substantial lateral or vertical extents. They are usually caused by inhomogeneities of refractive index.

To determine if a clear air echo were of the dot or point angel category we need only look at the properties of the scatterers to devise an experiment to identify this category of echoes. Dot or point angels appear as small echoes whose dimensions are essentially determined by the pulse length + beam width of the radar. The dimensions of the target are small w.r.t. the resolution distances of the radar. Since the scatterers are small the Rayleigh approx is valid and so the backscatter cross-section of the dot angels scales as the radar wavelength to the -4 power,

$$\boxed{\sigma_{\substack{\text{dot}\\\text{angel}}} \propto \lambda^{-4}}$$

This suggests that we use a short wave length radar (1cm = K band, 3cm = X band) to see these targets. Larger angels have backscattering cross-sections which scale as λ. Thus a short wavelength radar will be most sensitive to dot angels. Conversely a S band (10cm) radar would be more sensitive to layer angels. To establish that insects can account for dot angels we could go to the laboratory and measure the scattering cross sections of various insects. This was done by Hajovsky et al. (1966). Their laboratory experiments agreed with results from Glover et al. (1966) who dropped live insects from an airplane and then tracked their descent to the ground.

Layer angels

To correlate the occurrence of layer angels with inhomogeneities in the refractive index we could conduct a series of experiments as has been done by several investigators in which vertically pointing radars + collocated soundings provided evidence of layer angels occurring near pronounced gradients in the refractive index. We've already noted that layer angels have a cross section which scales as the wavelength

$$\boxed{\sigma_{\substack{\text{layer}\\\text{angel}}} \propto \lambda}$$

Thus large wavelength radars (S = 10cm or higher) would be most useful in detecting layer angels. Kropfli et. al. (1968) tracked the flight of a helicopt[er] carrying a microwave refractometer (to measure spatial variations in the refractive index) with the 10.7 cm Wallops Island radar. Kropfli et al (1968) compared their observations to theoretical expressions assuming backscattering gradients in the refractive index. They found good agreeme[nt] between observations + theory.

Other types clear air echoes are observed. Their source can usually be traced to either a point type scatterer or layer scatterer. Examples of both include

- ring echoes associated with large flocks of birds — Eastwood et al. (1[...]) 23 cm radar
- sea breeze front (no precip.) — Atlas (1960); 1.25cm radar
  - → some say due to insects
  - some say " " discontinuities in temp. + moisture
- Clear air convection — refractive index gradients between edges of developing cumulus + dry environmental air — Hardy + Ottersten (1969)
- CAT — correlate aircraft measurement with radar obs.
  - Glover et. al. (1969) → CAT + clear air echoes associated with atmospheric layers having small positive Richardson #'s
- Kelvin Helmholtz waves — Browning (1971)
  - → low positive Richardson #'s + clear air echoes of breaking KH wa[ves]

$$R_i = \frac{\frac{g}{\theta}\frac{\partial \theta}{\partial z}}{\left|\frac{\partial V}{\partial z}\right|^2} \qquad R_i \le 0.25 \text{ for onset of KH waves}$$

(a)

Two general categories of clear-air echoes

(1) Particulates in the air (e.g., insects, dust, chaff, etc)

(2) Refractive index gradients

(1) Return from particulates is similar to return from precip.

(2) Refractive index gradients caused by sharp changes in temp + humidity (free electron conc. in the ionosphere). Caused by mainly by turbulence, primary cause. Mixing of warm/cold, dry/moist air on small scales (~ radar λ)

MET5411 (Radar): Ray (40 minutes)

- Question

a. Why is the NEXRAD radar a pulsed Doppler radar instead of a continuous wave (cw) radar?

b. Clear-air echoes are either from refractive index fluctuation or "insects". How would you devise an experiment to distinguish which is the source. → see the previous question

b. How might one distinguish between radar returns from insects and fluctuations in refractive index in radar returns from "clear air"?

Sol)

The backscattering cross-section for insects satisfies the Rayleigh approximation ($\alpha = \frac{\pi D}{\lambda} \ll 1$) → diameter of scatter

λ → wavelength of incident radiation

α → electrical size

Thus, (backscattering)

$\sigma_{insects} \propto \lambda^{-4}$ → leads to "dot" angels

In contrast, the backscattering cross-section for fluctuations in the refractive index are proportional to the wavelength, that is,

$\sigma_n \propto \lambda$ → leads to "layer" angels

The power returned to the radar is proportional to the sum of all scatters in the volume illuminated by the radar beam. To maximize the return from insects we should use a small wavelength ($\sigma_{insect} \propto \lambda^{-4}$ so small λ implies large σ_{insect}). A short wavelength radar (e.g. K-band $\lambda = 1$ cm) would more easily detects insects than signals due to fluctuations in the refractive index. To "see" signals due to fluctuations in the refractive index a long wavelength radar (S-band = 10 cm or L-Band = 20 cm) would be best due to the direct proportionality betweeen the backscattering coefficient for fluctuations in the refractive index and the wavelength.

MET5455 (Cloud Physics): Ray (45 minutes)

- Answer succinctly and completely:

A) What is an intensive and an extensive thermodynamic variable? Give three examples of each. Why is the Gibbs free energy used as a Thermodynamic potential in Cloud Physics?

B) Why do we so often see snow crystals at the ground in the winter when they form over such a narrow temperature range.

C) Discuss condensational growth on different types of CCN.

A)

Extensive variables are those which depends on the mass of the system
↳ density; specific volume, specific humidity.

Intensive variables are those which do not depend on the mass of the system
↳ pressure, temperature
↳ Intensive variables may be defined at a point anywhere in the system.

By system we mean a closed system, in which it is understood that the mass as well as chemical composition define the system itself. The rest of the properties define its state

The Gibbs free energy, g, of a body is defined by

$$\boxed{g \equiv u + p\alpha - Ts}$$

or in differential form

$$dg = du - Tds + pd\alpha + \alpha dp - sdT$$

↳ $Tds = du + pd\alpha \rightarrow$ 1st law of thermodynamics

$$dg = \alpha dp - sdT$$

we see that for isobaric, isothermal processes the Gibbs free energy is conserved (for a body in equilibrium)

For irreversible, spontaneous transformations the 2nd law of thermodynamics tells us

$$Tds > du + pd\alpha$$

↳ enstropy increases for spontaneous, irreversible processes

so

$$dg < \alpha dp - sdT$$

and for an isobaric, isothermal, irreversible process

$$dg < 0$$

Summarizing,

we define the Gibbs free energy by

$$g \equiv u - Ts + p\alpha$$

using the 1st + 2nd laws of thermodynamics we can show

$$dg \le -sdT + pd\alpha$$

where = holds for reversible, equilibrium transformations
< applies to irreversible, spontaneous transformations

For an isobaric, isothermal process

$$dg \le 0$$

The criterian for the thermodynamic equilibrium of a body at constant temp. + pressure is that the Gibbs free energy has a minimum value.

In cloud physics we are very much concerned with the phase changes of water. Consider the vapor ↔ liquid transition. The change in the Gibbs free energy represents the energy change associated with the vapor ↔ liquid transition. The change in the Gibbs free energy is a very useful + very important parameter when discussing nucleation of drops.

From Wallace + Hobbs, pp 100-101

In discussing the 1st law of thermodynamics we tacitly assumed that the only external work that a body can do is the work of expansion, $pd\alpha$. However, a body may also perform external work through other means (for example, electr or by creating new sfc area between two phases). In general, therefore, we should write the first law for a unit mass of a body as

$$dq = du + dW_{tot} \quad (1)$$

or for a reversible transformation

$$Tds = du + dW_{tot} \quad (2)$$

where dW_{tot} is the total work done by a unit mess of a body. Here, we are assuming that K.E. and the gravitational P.E. of the body are constant.

If da is the external work done by a unit mass of a body over and above any $pd\alpha$ work, that is, if

$$da = dW_{tot} - pd\alpha \quad (3)$$

then by combining

$$dg = du - Tds - sdT + pd\alpha + \alpha dp \quad (4)$$

↑ from $g = u - Ts + p\alpha \equiv$ definition of Gibbs free energy.

along with (2) + (3) we obtain, for a reversible process,

$$da = -dg - sdT - \alpha dp \quad (5)$$

Therefore, if both temp + pressure are constant

$$da = -dg$$

That is, the external work done by a body (exclusive of any $pd\alpha$ work) in a reversible, isothermal-isobaric process is equal to the decrease in the Gibbs free energy of the body. In the study of the phase changes of water we ofte assume the process is isothermal + isobaric. Thus the Gibbs free energy is a pertinent parameter to use in the study of water phase changes.

The following is from Haize "Cloud dynamics" Section 3.1.1

The particles in a cloud form by a process refered to as nucleation, in which water molecules change from a less ordered (vapor) to a more ordered (water) state. For example, vapor molecules in air may come together by chance collisions to form a liquid-phase drop. To see how this process takes place, consider the conditions required for the formation of a drop of pure water from vapor. This is a case of homogeneous nucleation.

If the embryonic drop of pure water has radius R, then the net energy required to accomplish nucleation is

$$\Delta E = (4\pi R^2)\sigma_{v\ell} - \frac{4}{3}\pi R^3 \eta_\ell (\mu_v - \mu_\ell)$$

The first term on the right is the work required to create a sfc of vapor-liquid interface around the drop. The factor $\sigma_{v\ell}$ is the work required to create a unit area of the interface. It is called the sfc energy or sfc tension. The second term on the right is the energy change associated with the vapor molecules going into the liquid phase. It is expressed in terms of the Gibbs free energy of the system. The Gibbs free energy of a single vapor molecule is μ_v while that of a liquid molecule is μ_ℓ. The factor η_ℓ is the number of water molecules per unit volume of the drop.

If the work required to create the sfc exceeds the change in Gibbs free energy ($\Delta E > 0$), the embryonic drop formed by chance aggregation of molecules has no change of surviving and immediately evaporates. If, on the other hand, the work required to create the sfc is less than the change in Gibbs free energy ($\Delta E < 0$), then the drop survives and is said to have nucleated.

→ a system approaches an equilibrium by reducing its energy.

B.

I'm not sure what answer Dr. Ray is seeking. Ice crystals can form in cold clouds (clouds with portions that extend above the 0°C level) regardless of season. The crystals form & can grow eventually reaching a mass such that they can not be supported by vertical air currents. Hence, they settle out. If on their earthward descent they encounter layers of above 0°C air they may partially or completely melt depending on factors such as the depth of the warm layer, the fallspeed of the crystals, the warmth of the layer, etc. If a warm layer extends to the sfc, liquid precip will fall. During the winter the entire temp. profile is below or near 0°C through the depth of the trajectories followed by falling crystals. Thus a variety of crystals reach the sfc

⋮

This response seems to simplistic.

Once ice crystals form they may grow by the deposition of water vapor. Ice crystals growing from the vapor phase can assume a wide variety of shapes (or habits). The basic habits are either platelike or prismlike. The simplest platelike crystals are plane hexagonal plates and the simplest prismlike crystals are solid columns which are hexagonal in cross section. Studies in the controlled laboratory environment and observations in natural clouds have shown that the basic habit of an ice crystal is controlled by the temp at which it grows. The availability of excess moisture for deposition plays a secondary role but on some temp. ranges may influence details regarding the type of growth upon the basic habit.

Temp. (°C)		excess vapor density over ice (g/m³)
0 to −4	platelike	0 to 0.1 ⟶ quasi-equilibrium
−4 to −10	prismlike	0.1 to 0.23 ⟶ edge growth
−10 to −22	platelike	0.23 to 0.45 ⟶ corner growth.
−22 to −50	prismlike	

Ice crystals are exposed to continually changing temps & supersaturations as they fall through clouds. Thus, even when growing solely by vapor deposition they can assume a wide range of quite complex shapes. Other growth mechanisms include

- riming — freezing of super-cooled droplets onto ice particles/crystals
- aggregation — collision and "sticking together" of ice particles
  - ↳ aggregation is somewhat more effective for intricate crystals such as dendrites as opposed to flat plates.
  - ↳ aggregation is more effective above −5°C since at these temps ice sfcs become "stickier".

The growth of ice crystals by deposition from the vapor phase, riming & aggregation can lead to a wide variety of solid precip. particles. Though a particular type of crystal may initiate under very specific temp. & moisture conditions, once formed the crystal may grow, building upon this structure and thus become heavy enough to settle out of the air (or be forced down by large scale vertical motions). Assuming the temperature profile remains subzero and the air is moist enough that the ice crystal/snowflake does not sublimate we can observe a large variety of snow crystals at the ground during the winter.

C)

The atmosphere contains many aerosols ranging in size from submicron to several tens of μm. Those aerosol that are wettable can serve as centers upon which water vapor can condense. (A sfc is said to be perfectly wettable if it allows water to spread out on it as a horizontal film. It is completely unwettable (or hydrophobic) if water forms spherical drops on its sfc.) We call atm. aerosol that can serve as the nuclei upon which water vapor condences Cloud condensation nuclei (CCN). As we shall discuss below, the larger the size of an aerosol and the larger its water solubility (or if it is unsoluble, the more readily it is wetted by water), the lower will be the supersaturation at which it can serve as a CCN.

It can be shown that the net increase in the energy of a system due to the formation of an embryonic drop of radius R is given by

$$\Delta E = (4\pi R^2)\sigma_{\ell v} - \left(\tfrac{4}{3}\pi R^3\right)\eta_\ell(\mu_v - \mu_\ell)$$

↑ sfc energy or sfc tension of water. ↑ change in Gibbs free energy due to condensation/evaporation (decrease if vapor → liquid)

The change in Gibbs free energy can be expressed, equivalently, as

$$\mu_v - \mu_\ell = kT\ln\left(\frac{e}{e_s}\right)$$

e — vapor pressure of ambient environment.
e_s — saturation vapor pressure over a plane sfc of water
k — Boltzmann's constant

Thus,

$$\Delta E = (4\pi R^2)\sigma_{\ell v} - \left(\tfrac{4}{3}\pi R^3\right)\eta_\ell kT\ln\left(\frac{e}{e_s}\right)$$

When air is supersaturated ($e > e_s$), $\ln\left(\frac{e}{e_s}\right) > 0$ so that ΔE can be positive or negative depending on the magnitude of R. For the embryonic droplet to survive we must have $\Delta E \le 0$. We can find the critical radius for which $\Delta E = 0$ (the drop just barely survives) by evaluating

$$\frac{\partial \Delta E}{\partial R} = 0$$

$$\downarrow$$

$$0 = 8\pi R_c\sigma_{\ell v} - 4\pi R_c^2 \eta_\ell kT\ln\left(\frac{e}{e_s}\right)$$

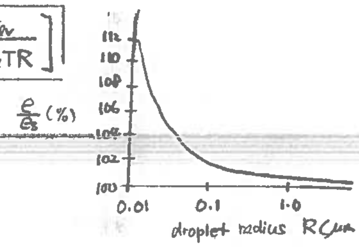
$$8\pi R_c\sigma_{\ell v} = 4\pi R_c^2\eta_\ell kT\ln\left(\frac{e}{e_s}\right)$$

$$2\sigma_{\ell v} = R_c\eta_\ell kT\ln\left(\frac{e}{e_s}\right)$$

$$R_c = 2\sigma_{\ell v}/\eta_\ell kT\ln\left(\frac{e}{e_s}\right)$$

This is Kelvin's formula. We may rewrite it as

$$\boxed{\frac{e}{e_s} = \exp\left[\frac{2\sigma_{\ell v}}{n_\ell kTR}\right]}$$



Since the supersaturations that develop in natural clouds due to the adiabatic ascent of air rarely exceed 1%, it follows that even embryonic droplets as large as 0.01μm, which might form by the chance collision of water molecules, will be well below the critical radius required for survival at 1% supersaturation. Consequently, droplets do not form in natural clouds by the homogeneous nucleation of pure water. Instead they form by what is known as heterogeneous nucleation on atmospheric aerosol, specifically CCN.

<u>Wettable</u>

Those aerosol which are wettable can serve as centers upon which water vapor can condense. Droplets can form + grow on these aerosols at much lower supersaturations than are required for homogeneous nucleation since the droplets form with a radius larger than that which they would have for homogeneous nucleation.

<u>Solute effect</u>

Some CCN are water soluble so that they dissolve when water condenses onto them. The equilibrium saturation vapor pressure over a solution drop is less than that over a pure water droplet of the same size. The saturation vapor pressure is proportional to the concentration of water molecules on the sfc of the droplet. In a solution droplet some of the sfc molecular sites are occupied by the molecules of salt (or ions if the salt dissociates) and thus the vapor pressure is reduced by the presence of the solute. It can be shown that the ratio of the vapor pressure over a solute drop to the saturation vapor pressure is given by a relation similar to Kelvin's formula but with an additional term to account for the presence of the solute. This formula is of the form

$$\frac{e}{e_s} = \exp\left[\frac{2\sigma_{\ell v}}{n_\ell kTR} - B\cdot\frac{1}{R^3}\right]$$

where B is a constant which depends on constant factors such the mass of the salt, density of water, etc. The more clearly see the effect of the solute we note that $e/e_s \sim 1$ so that a Taylor series expansion of

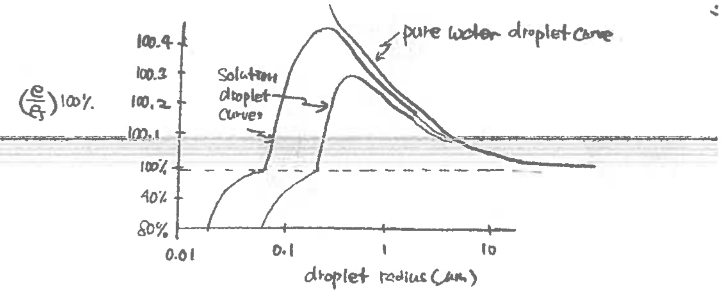
constants

$$\ln\left(\frac{e}{e_s}\right) = \frac{A}{R} - \frac{B}{R^3}$$

curvature effect

gives $$\frac{e}{e_s} - 1 \simeq \frac{A}{R} - \frac{B}{R^3}$$ ← solute effect

But $\frac{e}{e_s} - 1$ is nothing more than the supersaturation. Thus it is clear that the addition of a water soluble CCN reduces the supersaturation required for a droplet to survive. If we plot the variation of $\left(\frac{e}{e_s}\right)100\%$ of the air adjacent to a solution droplet as a function of droplet radius we obtain what is referred to as a <u>Köhler curve</u>.



Below a certain droplet size, the vapor pressure of the air adjacent to a solution droplet is less than that which is in equilibrium with a plane sfc of water at the same temp. As the droplets increase in size, the solutions become weaker and the kelvin curvature effect eventually dominates.

Consider a solution droplet which forms in a slightly saturated (possibly subsaturated) environment. If this ambient e/e_s ratio is less than the peak in Köhler curve, then the droplet will grow by condensation to that level of e/e_s. With additional growth the curvature effect would come into play as $\left.\frac{e}{e_s}\right|_{drop} > \left.\frac{e}{e_s}\right|_{ambient}$ and the droplet would evaporate back to the ambient e/e_s level. Conversely, if the droplet evaporated the vapor pressure would fall below that of the ambient environment. Thus the droplet finds itself in a stable equilibrium. We call such droplets <u>haze droplets</u>. All droplets to the left of the maxima in their respective Köhler curves are in the haze state.

Now consider a rising parcel of air. As the parcel rises, it expands, cools adiabatically, and eventually reaches saturation wrt. liquid water. Further uplift produces supersaturation which initially increases at a rate proportional to the updraft velocity. As the supersaturation rises CCN are activated, starting with the most effective ones. (Any droplet growing along a Köhler curve which has a peak supersaturation below that of the ambient air is said to be activated and will grow). When the rate at which moisture is being made available by the adiabatic cooling equals the rate at which it is condensing onto the CCN and droplets, the supersaturation reaches a maximum value. The concentration of cloud droplets is determined at this stage (which occurs with 100 m or so of the cloud base) and is equal to the concentration of activated CCN. Subsequently the growing droplets consume water at a rate greater than that at which it is made available by the cooling of the air. Thus the supersaturation decreases. The haze droplets evaporate slowly while the activated droplets continue to grow by condensation. Since the growth rate by condensation is inversely proportional to the droplet radius, the smaller activated droplets grow more quickly than the larger droplets. Consequently, the size of the droplets in the cloud tend to become increasingly uniform with time. We say that the distribution of cloud droplets is monodispersed.

MET5455 (Cloud Physics): Ray (40 minutes)

• Question

a. Why do we so often see snow crystals at the ground in the winter. Discuss.
b. Describe the difference between CCN and IN as many ways as you can.

MET5455 (Cloud Physics): Ray (45-60 minutes) *

• Using appropriate diagrams, discuss in detail homogeneous and heterogeneous nucleation. Be sure to include an adequate description of the "critical radius".

* hail formation & growth

MET5455 (Cloud Physics): Ray (45 minutes)

• Describe the life history of a hailstone (include both wet and dry growth processes) including "how, where, and why". Examine it both in the microphysical context and with respect to the cloud dynamical and microphysical context.

Sol)

Hailstones represent an extreme case of the growth of ice particles by riming. They form in vigorous convective clouds which have high liquid water contents. Observational evidence suggests that graupel most frequently serves as the hail "embryo". If a hailstone collects supercooled water droplets at too great a rate, its sfc temp. rises to 0°C and some of the water it collects will remain unfrozen. The sfc of the hailstone then becomes covered with a layer of liquid water and the hailstone is said to grow <u>wet</u>. Under these conditions some of the liquid water may be shed in the wake of the hailstone but some may also be incorporated into a water-ice mesh to form what is known as <u>spongy hail</u>. It has been deduced that the liquid fractions of large hailstones may amount to 20% or more. The entrapped liquid can later freeze if the stone enters colder or less dense cloud where the heat transfer will chill the stone below 0°C. During its lifetime the stone may undergo alternate wet & dry growth as it passes through a cloud of varying temp. and liquid water content, thus developing the layered structure that is often observed. If a thin section is cut from a hailstone & viewed in transmitted light, it is often seen to consist of alternate dark & light layers. The dark layers are opaque ice containing numerous small air bubbles and the light layers are clear, bubble free ice. Clear ice is more likely to form when the hailstone is growing wet. The sfc of a hailstone can contain fairly large lobes. Lobelike growth appears to be more pronounced when the accreted droplets are small and growth is near the wet limit. The development of lobes may be due to the fact that any small bumps on a hailstone will be areas of enhanced collection efficiencies for droplets.

An important aspect of hail growth is the latent heat of fusion released when the accreted water freezes. Owing to this heating, the temperature of a growing hailstone is several degrees warmer than its cloud environment. In the theory of hail development, the temp. is determined by assuming a balance condition for the hailstone heating rate.

The rate at which heat is gained as a result of the riming of a hailstone of mass m is

$$\frac{dQ_F}{dt} = \frac{dm_{col}}{dt}\left\{L_F - C_W(T(R)-T_W)\right\}$$

The factor $\frac{dm_{col}}{dt}$ is the rate of increase of the mass of the hailstone as a result of collecting liquid water. The hailstone is assumed to have a characteristic radius R. L_F is the latent heat of fusion released as the droplets freeze on contact with the hailstone. The second term, $(C_W(T(R)-T_W))$, is the heat per unit mass gained as the collected water drops of temp T_W come into temp equilibrium with the hailstone. The factor C_W is the specific heat of water. If the air surrounding the particle is assumed to be subsaturated, the temp T_W is approximated by the wet-bulb temp. of the air. This temp. may be several degrees less than the ambient air temp. when the humidity of the air is very low. If the air surrounding the particle is saturated $T_W = T_{ambient}$.

The rate at which the hailstone gains heat by deposition (or loses heat by sublimation) is

$$\frac{dQ_S}{dt} = (4\pi R)D_v\left[\rho_v - \rho_v(R)\right]V_{FS}L_S$$

where D_v is the diffusion coefficient for water vapor in air.
ρ_v is the vapor density in the ambient air.
V_{FS} is the ventilation factor
L_S is the latent heat of sublimation.

The rate at which heat is lost to the air by conduction is

$$\frac{dQ_C}{dt} = 4\pi R K_a\left[T(R)-T\right]V_{FC}$$

where K_a is the thermal conductivity
T is the ambient air temp.
V_{FC} is the ventilation factor for conduction.

In equilibrium, we have

$$\frac{dQ_F}{dt} + \frac{dQ_S}{dt} = \frac{dQ_C}{dt}$$

This may be solved for the hailstone equilibrium temp as a function of size. As long as this temp stays below 0°C, the sfc of the hailstone remains dry, and its development is called <u>dry growth</u>. The diffusion of heat away from the hailstone, however, is generally too slow to keep up with the release of heat associated with the riming (depositional growth ≪ riming growth). Therefore, if a hailstone remains in a supercooled cloud long enough, its equilibrium temp can rise to 0°C. At this temperature, the collected supercooled droplets no longer freeze spontaneously upon contact with the hailstone. However, a considerable portion of the collected water becomes incorporated into a water-ice mesh forming what is called spongy hail. This process is called <u>wet growth</u>

MET5455 (Cloud Physics): Ray (45 minutes, 1989) *

• If you were to write the code for a hail growth model, list all the realistic inputs you would need, and the assumptions that you would have to make. Outline the computations and decisions required. You may wish to make use of a "flow chart".

MET? (Mesoscale Meteorology): Ray (30 minutes, 1989) *

• Draw **and discuss** a diagram, relating the two most important factors governing storm type (give examples). What are typical values of Richardson number for supercell type storms? For multicell type storms?

MET? (Mesoscale Meteorology): Ray (30 minutes, 1989) *

• Give a succinct (but complete) summary of the life cycle of a squall-line. Include a discussion of the role of buoyancy, vorticity, updraft orientation, low-level shear, and any other important considerations.

MET5905 (Convection Dynamics): Ray (30 minutes)

- Modes of convection can be considered to arise in environments at different combinations of shear and buoyancy, or a kind of Richardson number space. Fully discuss this concept, and the range of *Ri* that might be associated with each mode. Be sure to include important details about the shear, its magnitude and directional change with height.

1/1

MET5905 (Convection Dynamics): Ray (30 minutes)

- Using diagrams (if necessary), and the relationship between wind shear and perturbation pressure and vorticity, explain the following processes.

a. Storm splitting (include effects of precipitation drag).

b. Vorticity generation at middle and low levels.

1/

MET5905 (Convection Dynamics): Ray (20 minutes)

- How is it believed that squall-lines form? What accounts for their evolution?

MET? (General): Ray (30 minutes)

- Assume the earth has a radius of 6000 km and that the moon's surface is 240,000 km away from the earth's surface. Assume you weigh 100 points on the surface of the earth. Also assume there is a tunnel from the surface of the earth, through the center of the earth to the other side of the earth. Briefly describe your weight as you move from the surface of the moon to the earth's surface and to the center of the earth. Write a formula which gives your weight as a function from the distance from the earth's surface. Plot the results.

Q: What is CISK?

Sol)

CISK stands for Conditional Instability of the Second Kind. The first applications were made in studies of the dynamics of hurricanes. The idea of cooperative interaction works for the development of a vortex as follows.

The clouds supply the heating to drive the vortex. The vortex, by providing the moisture convergence, maintains and organizes the cloud system. If there is a circular vortex in near gradient equilibrium over the ocean, its development into a hurricane by the CISK mechanism can be viewed as follows: Frictional convergence provides the moist supply in the BL. Mass and moisture are transported upward in the cumulus clouds. The heat released in the upper levels of the cloud causes the warm core to be established and a lowering of the sfc pressure. The increase of the pressure gradient at lower levels increases the vorticity of the near-gradient wind flow in the BL. This results in an enhancement of frictional convergence which increases the upward mass & moisture transport in the clouds and the entire development process keeps amplifying.

MET5455 Cloud Physics
Mid-Term Exam
Fall, 1997

1. What is the difference in the use of Gibbs free energy and Helmholtz free energy? Why would one be more appropriate than the other?

2. If the energy change involved in homogeneous nucleation is:

$$\Delta E = -n_L \frac{4}{3}\pi R^3 kT \ln\left(\frac{e}{e_s}\right) + 4\pi R^2 \sigma_{LV}$$

   What is the expression for the critical energy level ΔE^*? What can you say about the possibilities of homogeneous nucleation. Be as quantitative as possible.

3. What is the ventilation factor? What are the range of values? Can it be negative; if so, when?

4. What is the Van't Hoff factor? What would it be for an aqueous solution of NaCl?

5. Describe the essence of the warm rain process.

6. How is it possible for a completely insoluble aerosol particle to become a CCN? What equation is needed to theoretically explain your answer? Fully discuss nucleation on insoluble particles.

Mid-term (Ray) ; Ed's answer 44/50

3/5 1. Helmholtz free energy ($F = U - Ts$) is termed the "thermodynamic potential at constant volume" Isothermal
Gibbs free energy ($G = U - Ts + pV = F + pV$) is termed the "thermodynamic potential at constant pressure." = additional
The main difference is the additional work term in Gibbs free energy (pV). For meteorology, the Gibbs free energy is more appropriate since we typically work on constant temperature (T) and pressure (p) surfaces
Not for homogeneous grid

15/15 2. $\Delta E = -n_L \frac{4}{3}\pi R^3 kT \ln\left(\frac{e}{e_s}\right) + 4\pi R^2 \sigma_{LV}$ —— (1)

Critical energy ΔE^* occurs at the critical radius (r^*). To solve for r^*, take $\frac{\partial \Delta E}{\partial r}$ set equal to zero and solve for R (call this r^*). Let $S = e/e_s$.

$$\frac{\partial \Delta E}{\partial r} = -n_L 4\pi R^2 kT \ln S + 8\pi R \sigma_{LV} = 0$$

$$n_L 4\pi R^2 kT \ln S = 8\pi R \sigma_{LV}$$

$$R = r^* = \frac{2\sigma_{LV}}{n_L kT \ln(S)}$$

Putting this back into ΔE equation (1)

$$\Delta E^* = -n_L \frac{4}{3}\pi \left(\frac{2\sigma_{LV}}{n_L kT \ln S}\right)^3 kT \ln(S) + 4\pi \left(\frac{2\sigma_{LV}}{n_L kT \ln S}\right)^2 \sigma_{LV}$$

$$= -\frac{32\pi}{3} \frac{\sigma_{LV}^3}{(n_L kT \ln S)^2} + \frac{16\pi \sigma_{LV}^3}{[n_L kT \ln S]^2}$$

$$= \frac{48}{3} - \frac{32}{3} = \frac{16 \quad \pi \sigma_{LV}^3}{3(n_L kT \ln S)^2}$$

$$\therefore \boxed{\Delta E^* = \frac{16\pi \sigma_{LV}^3}{3[n_L kT \ln(S)]^2}} \quad \text{where } S = \left(\frac{e}{e_s}\right)$$

$$\Delta E^* = \frac{16\pi \sigma_{LV}^3}{3(n_L kT \ln(S))^2} f(m)$$

where $f(m) = \left[\frac{(2+m)(1-m)^2}{4}\right]$

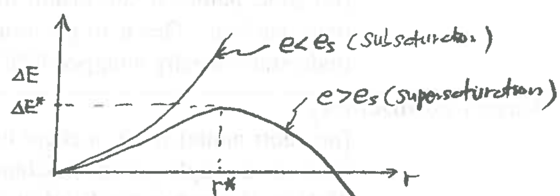
$m = \cos\phi$

ϕ = contact angle

$0 \le f(m) \le 1$

"wettable" $\phi = 0°$ — "not-wettable" $\phi = 180°$

Graphically this looks like:



Homogeneous nucleation occurs through chance collisions of tiny pure water droplets. If the droplet can reach or exceed the critical radius (r^*) it will grow spontaneously. The problem is that this ends up requiring high values of $S = \left(\frac{e}{e_s}\right)$. For a 0.01 µm radius drop $S = 1.125$ or 12.5% of supersaturation. This does not occur in nature, as S rarely exceeds 1%. Thus homogeneous nucleation is purely theoretical concept.

5/5 3. Ventilation factor is the ratio of growth rates for a falling drop vs stationary drop.

$$f = \frac{(dm/dt) \leftarrow \text{falling}}{(dm/dt) \leftarrow \text{stationary}}$$

Ranges of f (from class notes) : $1 \le f \le 15$

The ventilation factor was used in our growth rate by vapor diffusion equation:

$$r\frac{dr}{dt} = \frac{(S-1)f}{F_k + F_d}$$

We only use the ventilation factor in the case of evaporation (i.e. where $(S-1) < 0$). Thus, since r, F_k (thermodynamic term), F_d (vapor diffusion term) were all positive definite, we need $f > 0$ for $\frac{dr}{dt} < 0$ in the case of evaporation.

Also, through reading texts like Pruppacher + Klett + Rogers + You, the ventilation factor

$f = A + B^{1/3} N_{Re}^{1/2}$ where $A = 1$ (typically) + $B^{1/3}$ was a positive fraction like 1/2, 1/3, 1/4.

$$N_{Re}^{1/2} = \frac{2\rho r V}{\mu} = \frac{2rV}{\nu}$$

since $r > 0$, $V > 0$, $\nu > 0$, $N_{Re}^{1/2} > 0$ so $f > 0$

speed not velocity

Thus, by all of what I read $f > 0$ in all cases.

5/5

4. Van't Hoff factor is the number of free ions that come from dissociation of a molecule. These free ions are able to react with water vapor in the atmosphere to form droplets.

In the case a $NaCl \longrightarrow Na^+ + Cl^-$

there are 2 free ions, thus $\boxed{i = 2}$ in this case.

8/10

5. Essence of warm rain

water droplets in the atmosphere bond to form large droplets through two means of heterogeneous nucleation: (1) vapor diffusion and (2) Collisional Coalescence. In the case of vapor diffusion, water droplets condense on tiny aerosols called Cloud condensation nuclei (typically on Aitken nuclei ($r < 0.1 \mu m$)). hydrophilic or hydrophobic? Once these droplets form and begin to grow they naturally collide + coalesce onto other droplets. This process continues until the droplets grow large enough that they can no longer be suspended by updrafts and gravity effects take over and the droplet falls out as rain. has

do they grow forever —

ok. but too brief in details.

8/10

6. Completely insoluble aerosol become CCN?

Begin with the Köhler curve equation

$$S = \frac{e'(r)}{e_s(\infty)} = 1 + \frac{a}{r} - \frac{b}{r^3}$$

where $\frac{a}{r}$ = the curvature term, $\frac{b}{r^3}$ = the solution term

Since our problem involves insoluble aerosol

This reduces to $S = 1 + \frac{a}{r}$ where $a = \frac{2\sigma}{\rho_L R_v T}$

Soluble means "dissolves in water" an Insoluble aerosol would be one on which water condenses on the surface of the aerosol without any change in the aerosol. To do this water vapor would have to overcome the surface tension of the aerosol. This occurs if the aerosol is wettable ("hydrophylic")

Explain

MET5455 Cloud Physics
Final Exam
Fall, 1997

Make you answers short but complete.

1. A given population of aerosol particles may be approximated as a Junge distribution extending from 0.1 μm to 10 μm in diameter. That is:
$$n_i(D) = cD^{-3},\ 0.1\ \mu m \le D \le 10 \mu m$$
and 0, otherwise. If the total volume of these aerosols is $10^{-9}\ \mathrm{cm}^3/\mathrm{cm}^3$ of air, solve for the number density of aerosols in cm^{-3} and their total surface area in $\mathrm{cm}^2/\mathrm{cm}^3$.

2. In a developing cumulus cloud the cloud droplet spectrum has a Gaussian shape, centered at a mean radius of 4 μm with a standard deviation of 0.6 μm. Given that the cloud water content is 0.2 g/m^3, estimate the supersaturation in an updraft of 8 m/s. Assume a temperature of 0°C and a pressure of 80 kPa.

3. A hailstone with a diameter of 2 mm begins to fall from a height of 5 km above cloud base, where the ambient temperature is 250 K. It grows by accretion of cloud water under conditions such that its surface temperature is constant at 0 °C. The air has a lapse rate of 6 °C/km within the cloud. Assume zero updraft velocity; assume also that during growth a balance always exists between the rate of heat gained by the freezing of accreted water and the rate of heat loss by conduction to the air. Neglect sublimation effects and the heat capacity of the collected water. Show that the hailstone grows to a diameter of approximately 7 mm after falling 3 km. Assume that the fall speed depends upon the diameter as
$$v_f(D) = 1.4 \times 10^3 D^{1/2}$$
For the ventilation factor use $f = 0.3(\mathrm{Re})^{1/2}$, where Re is the Reynolds number of the flow about the stone.

Final exam (Rey) : Edi answer 100/100

1. Roger's & Yau (p91) State that "the size distribution of particles in an aerosol sample may be described by the distribution function $n_v(V)$, with the property that $n_v(V)dV$ is the number of particles per unit volume of air whose size are in the interval $V \rightarrow V+dV$. With sizes expressed as the diameter of equivolume spheres, the appropriate distribution function is $n_d(D)$, with the property that $n_d(D)dD$ is the number of particles per unit volume whose diameters fall in the interval $D \rightarrow D+dD$." The two size distributions are related by $n_d(D)dD = n_v(V)dV$

The relationship between volume & equivalent spherical diameter is

$$V = \pi D^3/6$$

Number density is given by $N(D) = \int_0^D n_d(D')dD'$ D' = dummy variable of integration

$N(D)$ is called the cumulative distribution of particle diameter. $n_d(D) = \frac{dN}{dD}$

The cumulative distribution function, then is given by:

$n_i(D)$ is our problem

$$n_e(D) = \frac{dN(D)}{d(\log D)} = \underbrace{\frac{dN}{dD}}_{n_d(D)} \frac{dD}{d(\log D)} = \boxed{n_d(D)\ln(10)\cdot D}$$

$$\frac{d(\log D)}{dD} = \frac{1}{\ln(10)}\frac{d\ln D}{dD} = \frac{1}{\ln(10)}\frac{1}{D}\frac{dD}{dD} = \frac{1}{\ln(10)D}$$

So the cumulative distribution is given by

$$\boxed{n_e(D) = n_d(D)\cdot\ln(10)\cdot D}$$

if we solve this equation for $n_d(D)$ we get

$$\boxed{n_d(D) = \frac{n_e(D)}{\ln(10)D}}$$

Next, we define the relationship for volume of particles with diameter in the interval $d(\log D)$ to be:

$$V(D) = \int_0^D \frac{\pi D'^3}{6} n_d(D')dD'$$

putting in the relationship for $n_d(D')$ above yields

$$V(D) = \int_0^D \frac{\pi D'^3}{6}\frac{n_e(D')}{\ln(10)D'}dD'$$

but the problem states that we assume a population for $n_e(D) = CD^{-3}$ for $0.1\,\mu m \le D \le 10\,\mu m$ and zero otherwise

So

$$V(D) = \int_{0.1\mu m}^{10\mu m} \frac{\pi D'^3}{6}\frac{CD'^{-3}}{\ln(10)D'}dD'$$

$$1\times10^{-9}\frac{cm^3}{cm^3} = \int_{0.1\times10^{-6}m}^{10\times10^{-6}m} \frac{\pi C}{6\ln 10}\frac{dD'}{D'} = \frac{\pi C}{6\ln(10)}\left[\ln(D')\right]_{0.1\times10^{-6}m}^{10\times10^{-6}m}$$

we solve this for the constant C

$$C = \frac{6\ln(10)\,10^{-9}\frac{cm^3}{cm^3}}{\pi\ln\left(\frac{10}{0.1}\right)} = \boxed{9.5493\times10^{-10}\frac{cm^3}{cm^3}}$$

Next we go back to $N(D) = \int_0^D n_d(D')dD'$ to get number density

$$N(D) = \int_{0.1\mu m}^{10\mu m}\frac{CD'^{-3}}{\ln(10)D'}dD' = \frac{9.5493\times10^{-10}}{\ln(10)}\int D'^{-4}dD' = \frac{9.5493\times10^{-10}}{-3\ln(10)}D^{-3}\Big|_{0.1\times10^{-4}cm}^{10\times10^{-4}cm}$$

$$= \frac{9.5493\times10^{-10}\frac{cm^3}{cm^3}}{-3\ln(10)}\left[1\times10^{9} - 1\times10^{15}\ cm^{-3}\right]$$

$\boxed{N(D) = 1.3824\times10^5\ cm^{-3}\text{ of air}}$ number density

Total surface area is then defined as

$$S(D) = \int_0^D \pi D'^2 n_d(D')dD' = \int_0^D \pi D'^2\frac{CD'^{-3}}{\ln(10)D}dD' = \frac{\pi C}{\ln(10)}\left[\frac{1}{-D}\right]_{0.1\times10^{-4}}^{10\times10^{-4}cm} = \boxed{1.29\times10^{-4}\ cm^2/cm^3\text{ of air}}$$

2. Roger's & Yau (p106) Define the rate of change of saturation ratio as

$$\frac{dS}{dt} = Q_1 \frac{dz}{dt} - Q_2 \frac{d\chi}{dt}$$

dz/dt = vertical air velocity (w)

$d\chi/dt$ = rate of condensation measured in units of mass of condensation per unit mass of air.

Q_1 & Q_2 are thermodynamic variables given by

$$Q_1 = \frac{1}{T}\left[\frac{\varepsilon L g}{R' C_p T} - \frac{g}{R'}\right] = \frac{1}{273\,°K}\left[\frac{(.622)(2.5\times10^6 \frac{J}{kg})(9.81\frac{m}{s^2})}{(287\, J/kg°k)(1005\frac{J}{kg°k})(273°k)} - \frac{9.81\, m/s^2}{287\, J/kg°k}\right] = 5.844\times10^{-4}\, m^{-1}$$

$$Q_2 = \rho\left[\frac{R'T}{\varepsilon e_s} + \frac{\varepsilon L^2}{PTC_p}\right] = 1\frac{kg}{m^3}\left[\frac{(287\frac{J}{kg°k})(273°k)}{(.622)\, 611\, Nm^{-2}} + \frac{(.622)(2.5\times10^6)^2 \frac{J^2}{kg^2}}{(8\times10^4 \frac{N}{m^2})(273°k)\, 1005\frac{J}{kg°k}}\right] = 383\, kg/kg$$

The value of these constants are given as

$\varepsilon = 0.622$, $L = 2.5\times10^6\, J/kg$, $g = 9.81\, m/s^2$, $R' = 287\, J/kg°k$, $C_p = 1004\, J/kg°k$,

$p = 800\, hPa = 8\times10^4\, Pa$, $e_s = 611\, N/m^2$, $\rho_{air} = 1\, kg/m^3$, $T = 273°k$

$$1\, J = 1\, Nm = 1\frac{kg\, m^2}{s^2} \quad \text{so} \quad J/kg = \frac{m^2}{s^2} \quad \text{so} \quad \left(\frac{m}{s^2}\right)/J/kg = \frac{m\, s^2}{s^2\, m^2} = \frac{1}{m}$$

If we assume all drops are of the same size then

$$\frac{ds}{dt} = \omega - \eta s \quad \text{where} \quad s = S - 1 \leftarrow \text{supersaturation}$$

$\omega = 100\, Q_1 u \qquad \eta = 4\pi \rho_L \nu_0 \bar{r} Q_2 / F_k + F_d \qquad$ p103 → table 7.1

$$\omega = 100 \cdot 5.844\times10^{-4}\, m^{-1}\; 8\, m/s = 0.46752\, s^{-1}$$

$$F_k = \frac{L\rho_L}{kT}\left(\frac{L}{R_v T} - 1\right) \quad \text{where} \quad k = 2.40\times10^{-2}\frac{J}{m s °k}$$

$R_v = 461\, J/kg°k$

$\rho_L = 10^3\, kg/m^3$

$$= \boxed{7.20\times10^9\, s\,m^{-2}}$$

$$F_d = \frac{\rho_L R_v T}{D e_s(T)} = \boxed{7.46\times10^9\, s\,m^{-2}}$$

But we are interested in the maximum value of supersaturation which occurs when $\frac{ds}{dt} = 0$ or at $\omega = \eta s$

$$\text{or} \quad S_\infty = \frac{\omega}{\eta} = \frac{100\, Q_1 u (F_k + F_d)}{4\pi \rho_L \nu_0 \bar{r} Q_2} \quad \text{where } \bar{r} = \text{mean radius} = 4\times10^{-6}\, m$$

If we assume that the initial concentration is based on the third moment of the Gaussian distribution (i.e. first moment = mean, second moment = standard deviation), then

$$\boxed{\nu_0 = 6.84\times10^8\, kg^{-1}}$$

$$\text{and} \quad S_\infty = \frac{100(5.844\times10^{-4}\, m^{-1})(8\, m/s)(7.20\times10^9\, s/m^2 + 7.46\times10^9\, \frac{s}{m^2})}{4\pi(10^3\, kg/m^3)(6.84\times10^8\, kg^{-1})(4\times10^{-6}\, m)(383\, kg/kg)}$$

$$= 0.51923 \quad \text{or} \quad 0.52\%$$

$$\therefore \boxed{S_\infty = 0.52\%}$$

3. Begin with the balance condition for hailstone heating rates given by the following equilibrium condition:

$$\frac{dQ_L}{dt} + \frac{dQ_v}{dt} = \frac{dQ_s}{dt}$$

where $\frac{dQ_L}{dt}$ = the heating rate due to accretion of supercooled liquid droplets.

$\frac{dQ_v}{dt}$ = heat gained by sublimation

$\frac{dQ_s}{dt}$ = heat lost to the air by conduction.

The parameters of the problem tell us to neglect sublimation effects, thus the balance eq. above becomes

$$\frac{dQ_L}{dt} = \frac{dQ_s}{dt}$$

Each of these rates are given as

$$\frac{dQ_L}{dt} = \pi R^2 E M V_f(D)\left[L_f - C(T_s - T)\right]$$

and

$$\frac{dQ_s}{dt} = 4\pi R K (T_s - T) f$$

we are further told to neglect the heat capacity of collected water, thus dQ_L/dt simplifies to the following form

$$\frac{dQ_L}{dt} = \pi R^2 E M V_f(D) L_f$$

Our balance eq thus becomes

$$\pi R^2 E M V_f(D) L_f = 4\pi R K (T_s - T) f$$

where R = radius of the hailstone, E = effective collision efficiency, M = cloud liquid water content,

T_s, T = temperatures of the hailstone, ambient cloud, respectively,

$V_f(D)$ = fall speed $= (1.4\times10^3\ cm^{1/2}/sec)\,D^{1/2} = 6.26\ m/s$

L_f = latent heat of fusion $= 3.34\times10^5\ J/kg$

K = heat conduction coefficient $= 2.24\times10^{-2}\ J/ms\cdot k$

$f = 0.3(Re)^{1/2}$ = ventilation factor where $Re = 2rV_f/\nu = 545.3$

ν = kinematic viscosity $= \mu/\rho = 2.296\times10^{-5}\ m^2/s$, μ = dynamic viscosity $= 1.60\times10^{-5}\ kg/ms$ based on interpolation of Table 7.1 (p103) with $T = 250°k$

$\rho = 0.697\ \frac{kg}{m^3}$ density of air at 5km, using $\rho = \frac{P}{R_d T}$ for $P \approx 500\ mb$ (5km)

$f = 0.3(545.3) = 7$, $T_s = 273°k$, $T = 250°k$

Next we need an equation for the rate of growth of a droplet (or in this case, hail).

$$\frac{dR}{dt} = \frac{EM}{4\rho_s} V_f(D) \longrightarrow \text{based on eq. (8.15) on p131}$$

but we know that $dR/dt = \frac{dR}{dh}\frac{dh}{dt}$, where $dh/dt = V_f(D)$, thus our rate of growth eq., as a function of height becomes

$$\frac{dR}{dh} V_f(D) = \frac{EM}{4\rho_s} V_f(D) \quad \text{or} \quad \boxed{\frac{dR}{dh} = EM/4\rho_s}$$

Next we return to the balance eq., and solve for EM to yield

Balance eq: $\pi R^2 E M V_f(D) L_f = 4\pi R K (T_s - T) f$

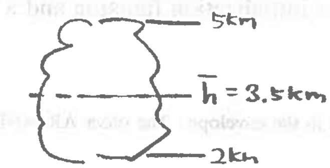
Solving for EM yields: $EM = \frac{4K(T_s - T) f}{R L_f V_f(D)}$

Inserting this into the rate eq. yields

$$\frac{dR}{dh} = \frac{4K(T_s - T) f}{4\rho_s R L_f V_f(D)}$$

Next we know that $T_s = 0°C$ (273°k), but we need a relationship to find T as a function of height (h) in the atmosphere. Here we assume that $T = T_0 + \gamma h$ where T_0 is an initial temperature for the profile and γ = lapse rate.

We know that the ambient temp (T) is 250°k for the cloud. We can assume that this condition occurs for mid-cloud such that



$T = 250°k$ at $h = 3.5$

$$250°k = T_0 + (-6°/km)(3.5\ km)$$

$$T_0 = 250 + 21 = \underline{271°k}$$

Thus $T_0 = 271°k$ and our rate eq becomes

$$\frac{dR}{dh} = \frac{K(T_s - T) f}{R L_f V_f(D) \rho_s} = \frac{K(T_s - (T_0 + \gamma h)) f}{R L_f V_f(D) \rho_s}$$

If we separate and integrate we get

$$\int_{r_i}^{r_f} R\,dR = \int \left[\frac{K(T_s - T_0 - \gamma h) f}{L_f V_f(D) \rho_s}\right] dh$$

$$\frac{R^2}{2}\Big|_{r_i = 2mm}^{r_f} = \frac{K(T_s - T_0) f}{L_f V_f(D) \rho_s}\int_{2km}^{5km} dh - \frac{K f \gamma}{L_f V_f(D) \rho_s}\int_{2km}^{5km} h\,dh$$

$$\frac{r_f^2 - r_i^2}{2} = \frac{K(T_s - T_0) f}{L_f V_f(D) \rho_s} h\Big|_{2km}^{5km} - \frac{K f \gamma}{L_f V_f(D) \rho_s}\frac{h^2}{2}\Big|_{2km}^{5km}$$

Inserting constants & solving for r_f yields

$$r_f^2 = r_i^2 + \frac{2K(T_s - T_0) f}{L_f V_f(D) \rho_s}(5km - 2km) - \frac{2 K f \gamma}{L_f V_f(D) \rho_s}\left[\frac{(5000)^2 - (2000)^2}{2}\right]$$

$$= (0.001 m)^2 + \frac{2(2.21\times10^{-2})(2)(7.0)(3000)}{(3.34\times10^5)(6.26)(910)} - \frac{(2.21\times10^{-2})(7.0)(-.006)}{(3.34\times10^5)(6.26)(910)}((5000)^2 - (2000)^2)$$

$$= 1\times10^{-6} m^2 + 9.756844\times10^{-7} m^2 + 1.024466\times10^{-5} m$$

$r_f^2 = 1.222\times10^{-5} m^2$, taking square root of both sides yields

$r_f = 3.49576\times10^{-3} m$ or $0.0035 m$

$D_f = 2\cdot r_f = 2(0.0035) = 0.007m$ or $\boxed{7mm}$

The problem is complete. In fact the final radius is 7mm, based on the parameters provided in the problem.