Eric

* Clausius - Clapeyron eq.

MET4420: E. Smith (45min to 1 hour, 1994, 1996) **

• Derive the equation of Clausius-Clapeyron, and then integrate it to express an algebraic solution for e (state all your assumptions). Draw the triple point diagram, identifying all of its key elements, and explain the discontinuity between the vaporization equilibrium line and the sublimation equilibrium line at the triple point itself.

Sol)

• First law of thermodynamics

$dq - dw = du$ where q = ext. heat, w = work, u = int. energy

$\rightarrow dq = dw + du = p d\alpha + du = p d\alpha + C_v dT$

Allow $C_v = \left(\frac{du}{dT}\right)_v$, $C_p = \left(\frac{du}{dT}\right)_p$, $C_p = C_v + R$ (use $p\alpha = RT$)

$\rightarrow \boxed{dq = C_v dT + p d\alpha = C_p dT - \alpha dp}$

Define enthalpy $h \equiv u + p\alpha \rightarrow dh = du + \alpha dp + p d\alpha$

$\rightarrow \boxed{dq = dh - \alpha dp}$

Define entropy (S) where $dS = \frac{dq_{rev}}{T} \rightarrow \boxed{TdS = du + p d\alpha}$

• Consider a closed system where liquid water is in equilibrium with saturated vapor in atm. Consider following reversible process; where α_1 (α_2) = spec. volume of liquid (vapor) at temp (T):

	Initial	Final	
1	(a) $T-dT$, $e_s - de_s$	(b) T, e_s	adiabatic compression
2	(b) T, e_s	(c) T, e_s	heat → isothermal expansion (evap)
3	(c) T, e_s	(d) $T-dT$, $e_s - de_s$	adiabatic expansion
4	(d) $T-dT$, $e_s - de_s$	(a) $T-dT$, $e_s - de_s$	cool → isothermal compression (cond.)

From theory of Carnot cycle, we know

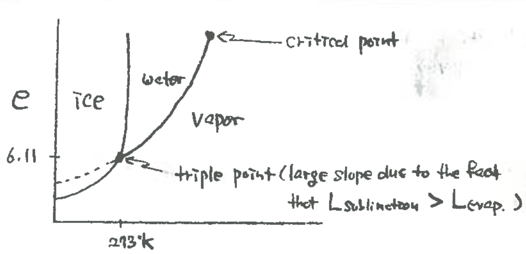
$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_1 - Q_2}{T_1 - T_2}$$

So, for isothermal processes (temp. = T), $Q = L$, latent heat of cond/evap.

For adiabatic processes, $\Delta Q = (\alpha_2 - \alpha_1) de_s$, $\Delta T = dT$

Subst. into Carnot expression

$$\rightarrow \frac{L}{T} = \frac{(\alpha_2 - \alpha_1) de_s}{dT} \rightarrow \boxed{\frac{de_s}{dT} = \frac{L}{T(\alpha_2 - \alpha_1)}} \rightarrow \text{C.-C. eq.}$$



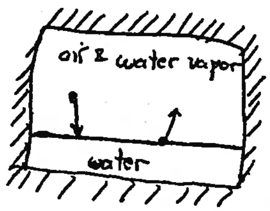
* See the attached notes.

Question: Clausius-Clapeyron Eqn

Response: As indicated

Rogers & Yau

Consider a closed and thermally insulated container partly filled w/ water



Molecules from the surface layer of water are in agitation and some break away as vapor molecules. On the other hand some of the vapor molecules collide w/ the sfc and stick. Condensation & evaporation take place simultaneously. For a given temperature an equilibrium condition will eventually be reached when the two processes have the same rate. Then the temperature of the air & vapor equals that of the liquid and there is no net transfer of molecules from one phase to ~~other~~ the other. The space above the water is then said to be saturated w/ water vapor. The partial pressure due to the water vapor in this condition is called the saturation vapor pressure, e_s.

Heat is required to change phase from liquid to vapor because the kinetic energy of the vapor molecules exceeds that of liquid molecules @ the same temperature. We denote by L_v the heat required to unit mass of liquid to vapor, with pressure & temperature held constant. For this transition from phase 1 (liquid) to phase 2 (vapor)

$$L_v = \underbrace{\int_{q_1}^{q_2} dq = \int_{u_1}^{u_2} du + \int_{\alpha_1}^{\alpha_2} p\, d\alpha}_{\text{integrate First law of thermodynamics } dq = du + p\,d\alpha}$$

pressure & temperature are constant in this process under the conditions of thermodynamic equilibrium

$$L_v = \underbrace{T}_{\text{since temperature is constant in this process}}\int_{q_1}^{q_2} \frac{dq}{T} = \int_{u_1}^{u_2} du + \int_{\alpha_1}^{\alpha_2} p\, d\alpha$$

But we have a quantity called entropy s defined by

$$ds = \frac{dq}{T}$$

Thus

$$\boxed{L_v = T\int_{s_1}^{s_2} ds = \int_{u_1}^{u_2} du + \int_{\alpha_1}^{\alpha_2} p\, d\alpha}$$

and

$$T(s_2 - s_1) = u_2 - u_1 + p(\alpha_2 - \alpha_1)$$

this pressure is the saturation vapor pressure (e_s) in this problem

$$Ts_2 - Ts_1 = u_2 - u_1 + e_s\alpha_2 - e_s\alpha_1$$

$$\boxed{u_1 + e_s\alpha_1 - Ts_1 = u_2 + e_s\alpha_2 - Ts_2}$$

This particular combination of thermodynamic variables remains constant in an isothermal, isobaric change of phase. We define the Gibbs free energy, g, as

$$\boxed{g = u + p\alpha - Ts}$$

From the above equality between phases 1 & 2 we also have

$$\boxed{g_1 = g_2}$$

Though the Gibbs free energy is constant during an isobaric, isothermal phase transition, it varies w/ temperature & pressure. Its dependence on these variables is shown in the differential eqn

$$dg = du + p\,d\alpha + \alpha\,dp - T\,ds - s\,dT$$

But from the 1st law of thermodynamics

$$du + p\,d\alpha = T\,ds$$

so

$$\boxed{dg = \alpha\,dp - s\,dT}$$

and in our particular application

$$dG_1 = dG_2$$

$$\alpha_1 de_s - s_1 dT = \alpha_2 de_s - s_2 dT$$

$$(\alpha_1 - \alpha_2)\, de_s = (s_1 - s_2)\, dT$$

$$\boxed{\frac{de_s}{dT} = \frac{s_1 - s_2}{\alpha_1 - \alpha_2}}$$

This is a general form of the Clausius-Clapeyron (C-C) equation. However, it is awkward to use in this form. We do not normally measure the entropy.

Recall that for the process under consideration

$$L_v = T\int_{s_1}^{s_2} ds = T(s_2 - s_1)$$

so

$$s_2 - s_1 = \frac{L_v}{T}$$

We use this to obtain a (slightly) more useful form of the C-C eqn.

$$\boxed{\frac{de_s}{dT} = \frac{L_v}{T(\alpha_2 - \alpha_1)}}$$

Recall that subscript 1 denotes the water phase
2 denotes the vapor phase

① We will <u>assume</u> $\alpha_2 \gg \alpha_1$. That is, the specific volume of water vapor is much larger than that of liquid water. For ordinary atmospheric conditions this is true ($\rho_{water} \gg \rho_{water\ vapor}$).

② We will assume that water vapor behaves like an ideal gas. That is,

$$e_s \alpha_2 = R_v T$$

temperature

gas constant for water vapor.

vapor pressure @ saturation

specific humidity of water vapor

Then By assumption ① we get

$$\frac{de_s}{dT} = \frac{L_v}{T\alpha_2}$$

By assumption ②

$$\frac{de_s}{dT} = \frac{L_v}{T} \cdot \frac{e_s}{R_v T} = \frac{L_v e_s}{R_v T^2}$$

or

$$\boxed{\frac{d \ln e_s}{dT} = \frac{L_v}{R_v T^2}}$$

This is the CC eqn following our two assumptions.

Note: The above form of the CC eqn may be used ~~to express~~ for the equilibrium conditions between water vapor & ice provided

(a) $T \leq 0°C$

(b) e_s is the saturation vapor pressure w/ respt to a plane ice surface

(c) L_v is the latent heat of sublimation, L_s

At temperatures above 0° only ~~water~~ liquid water can be in equilibrium w/ the vapor

Now we would like to integrate the CC eqn to get e_s as a function of temperature. This is complicated by the fact that the latent heat of vaporization is a function of temperature itself. (L_v varies by ~ 6% over the temperature range −30°C to +30°C). We can see this from the ~~starting eq~~ integral equation we started from

$$L_v = \int_{u_1}^{u_2} du + \int_{\alpha_1}^{\alpha_2} p\, d\alpha$$

1 ≡ liquid water
2 ≡ water vapor
$p = e_s$

$$L_v = u_2 - u_1 + e_s(\alpha_2 - \alpha_1)$$

Again assume $\alpha_2 \gg \alpha_1$ and assume $e_s \alpha_2 = R_v T$

$$L_v = u_2 - u_1 + \frac{R_v T}{\alpha_2}(\alpha_2)$$

$$L_v = u_2 - u_1 + R_v T$$

so

$$\frac{dL_v}{dT} = \underbrace{\frac{du_2}{dT}}_{c_{vv}} - \underbrace{\frac{du_1}{dT}}_{c} + R_v$$

c → specific heat capacity of liquid water

c_{vv} → specific heat capacity of water vapor @ constant volume

$$\frac{dL_v}{dT} = (c_{vv} + R_v) - c$$

→ by definition this is c_{pv} ≡ specific heat of water vapor @ constant pressure

$$\boxed{\frac{dL_v}{dT} = c_{pv} - c}$$

Now c_{pv} and c are functions of temperature but if we <u>assume</u> they are constants, then we can integrate over temperature to get

$$\boxed{L_v(T) = L_v(T_0) + (c_{pv} - c)(T - T_0)}$$

where $L_v(T_0)$ is a constant of integration

This eqn ~~states the~~ ~~estimates~~ approximates the temperature dependence of L_v. We say "approximate" since c_{pv} & c are themselves functions of T.

⋮

We could use this equation for $L_v(T)$ in our version of the CC eqn but as a first approx we will, rather, assume L_v is constant w/rspt to temperature. If we desire we could introduce the above T dependence of L_v and repeat the steps below.

We will <u>assume</u> L_v is independent of temperature. Now integrate the CC eqn

$$\int_{T_0}^{T} \frac{d\ln e_s}{dT}\, dT = \int_{T_0}^{T} \frac{L}{R_v} \frac{dT}{T^2}$$

$$\frac{L}{R_v}\left(\frac{T - T_0}{T T_0}\right)$$

$$\ln\left(\frac{e_s(T)}{e_s(T_0)}\right) = -\frac{L}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

or

$$\boxed{e_s(T) = e_s(T_0)\exp\left(\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right)}$$

Where $e_s(T_0)$ must be experimentally determined @ a temperature T_0

It is found

$e_s(T_0) \sim 611$ Pa for $T_0 = 0°C$.

$L_v(T_0) \sim 2.5 \times 10^6$ J/kg

$R_v = 461 \frac{J}{kg\,K}$

(B) Draw the triple point diagram, Identify all its key elements. Explain the discontinuity betw the vaporization & sublimation equilibrium lines @ the triple point.

(From Iribarne & Godson, Section 4.5, 4.6

For a closed system of constant composition the number of independent variables ν whose values have to be known (ie, fixed) in order to determine completely the equilibrium state of a heterogeneous system (its variance) is equal to the number of components c minus the number of phases φ plus two:

$$\nu = c - \varphi + 2$$

This <u>phase rule</u> was derived by JW Gibbs

Consider now a heterogeneous system of one component (water vapor) and three phases (solid, liquid, vapor (gas)). We need three independent variables in order to specify the state of this system. If the three phases are present simultaneously we have 3 pairs of variables but also three pairs of conditions for equilibrium, say,

$$p_s = p_\ell = p_v$$
$$T_s = T_\ell = T_v$$
$$G_s = G_\ell = G_v$$

c = 1 component
$\varphi = 3$ phase
$\nu = 1 - 3 + 2 = 0$ ← no degrees of freedom (a point)

Therefore in this triple equilibrium phase all three independent variables are fixed and define what is called the <u>triple point</u>

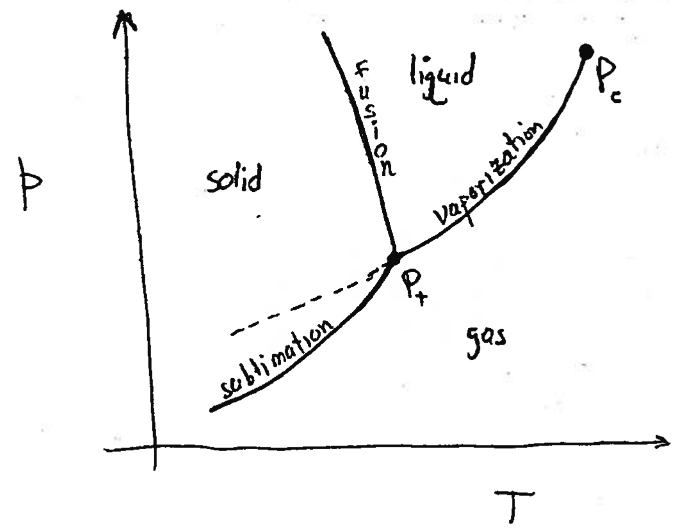
If we impose the condition of equilibrium between two phases we have four variables which must three conditions

$$p = p'$$
$$T = T'$$
$$G = G'$$

c = 1 component
$\varphi = 2$ phases
$\nu = 1 - 2 + 2 = 1$ degree of freedom ↳ a line

with the remaining state variable free.

As we have seen above, phase transition equilibria correspond to only one independent variable and may be represented by curves $p = f(T)$. They define in the p, T plane the regions where the water is in solid, liquid or gaseous state



The pressure of the vapor when it is in equilibrium w/ the condensed phase @ a certain temperature is called the vapor ~~phase~~ pressure of the condensed phase @ that temperature. The curves for sublimation and vaporization equilibria are thus the vapor pressure curves of ice and water. The curve for fusion corresponds to the ~~equilib~~ equilibrium ice-water. Its steep slope is negative unlike what happens w/ similar curves for most other substances. The three curves meet @ the triple point P_T, where the three phases coexist in equilibrium.

The extension of the vapor pressure curve for water @ temperatures lower than the triple point (represented by the --- line above) corresponds to supercooled water (metastable equilibrium).

The vapor pressure curve ends for water, for high temperatures, at the critical point P_c, beyond which there is no discontinuity betw the liquid & gaseous phases.

The pressure, temperature, and specific volumes for the three phases @ the triple point are

$$p_t = 6.107\ \text{mb}$$

$$T_t = 273.16\ \text{K}$$

$$\alpha_{it} = 1.091\times 10^{-3}\ m^3 kg^{-1}$$

$$\alpha_{wt} = 1.000\times 10^{-3}\ m^3 kg^{-1}$$

$$\alpha_{vt} = 206\ m^3 kg^{-1}$$

could the discontinuities in specific volume @ the triple point explain the discontinuities ~~in slope~~ in the phase diagram curves @ P_t?

The values for the critical point P_c are

$$P_c = 2.22\times 10^{7}\ Pa$$

$$T_c = 647\ K$$

$$\alpha_c = 3.07\times 10^{-3}\ m^3 kg^{-1}$$

Note that w/ 3 independent variables required to define the state of the water substance system ~~we can~~ the pT ~~diagram~~ phase diagram is merely a projection of a 3d thermodynamic surface onto this plane. ~~we can~~ This thermodynamic ~~surface~~ surface is one for which $f(p,\alpha,T)=0$.

⋮

We shall now consider more carefully the type of gaseous phase in which we shall be concerned: a mixture of dry air and water vapor plus one or more of the other phases of water vapor. Two effects of interest occur w/ the addition of dry air which modify the values of the saturation pressures over water & ice.

(A) displacement of the equilibrium caused by the increase in the total pressure

Ⓑ solubility of the gas in water (solubility in ice is negligable)

According to Raoult's law, ② produces a decrease in the vapor pressure proportional to the molar fraction of the dissolved gas. ~~Another change so that the~~

We have 3 types of departures from the ideal case of pure water or ice in the presence of vapor behaving as an ideal gas

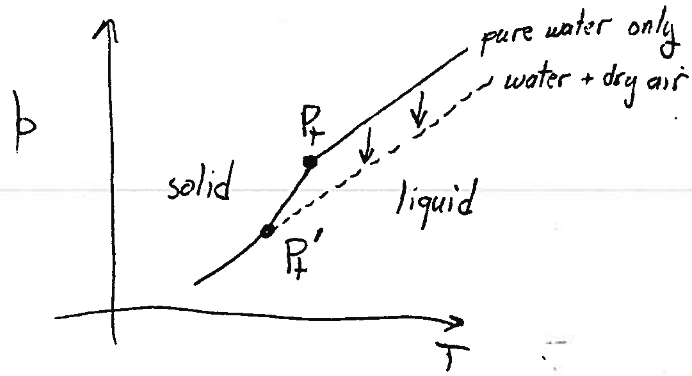
① total pressure is not the sum of the partial pressures of two ideal gases (as Dalton's law implies ~~as~~ for ideal gases) since neither water vapor nor dry air are ideal gases

② the condensed phase is under a total pressure augmented by the presence of dry air

③ the condensed phase is not pure water substance but contains dissolved gas.

effect B

The effect of dissolved air is to lower the whole curve of water-vapor equilibrium as shown below



The solubility of air gases in ice is truly negligable. Therefore, if solubility was the only effect, the equilibrium ice-water vapor would now occur @ the new triple point P_t'.

effect A

However, we must consider the effect of pressure which will be different for the two condensed phases.

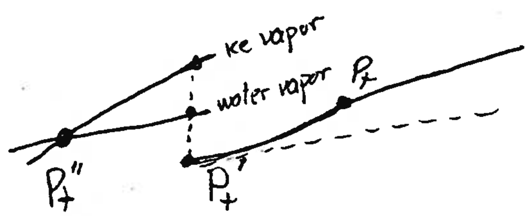
- This leads to two increments of pressure, (one for ice, one for liquid.) which must be added to P_t'. Through each of these two new points we use the CC eqn to extend the corresponding ice-vapor, water-vapor equilibrium curves until they intersect at the final, mixed ~~system~~ component triple point, P_t''.

~~ice, liquid water~~

@ this point ice, water saturated w/air, and water vapor are in equilibrium @ a total pressure of $p_d + e_t''$

p_d — partial pressure of dry air.

e_t'' — vapor pressure @ P_t''



$P_t \equiv$ water system triple point

$P_t' \equiv$ triple point for air-water mixture + effect of solubility

$P_t'' \equiv$ " " " " " " " " " + effect of partial pressures

For an total pressure of 1 atmosphere (101.325 kPa) solubility decreases the triple point by ~ 0.0028 K, the pressure effect decreases it by an additional ~ 0.0075 K, with the net effect being a decrease of about 0.01 K

↓

the triple point temperature of water only system of 273.16 K is reduced to 273.15 K for the dry air + water system.

* saturation

MET4420: E. Smith (45min to 1 hour, 1995, 1996) *

• Explain in mathematical and physical terms, why two unsaturated parcels of air at different temperatures can mix and achieve saturation.

* formation of steam fog

MET4450 (Atmospheric Physics II): ? (?, 1998)

• Explain the formation of steam fog in the context of an adiabatic-isobaric mixing process involving two sources of unsaturated air. Be quantitative and use schematic diagrams to help clarify your explanation. Conclude your by answering whether clouds could form under a similar mixing process, including an explanation of either a yes of no answer?

MET? (?): E. Smith (?)

- Show that the radiance from a steady emitting body is independent of distance from the body but the irradiance is dependent on the square of the square?

Sol)

Assume the power output from the body is P. At a radius R from the source, the power is related to the irradiance (F) through the concentric sphere by

$$P = A_s \cdot F$$

where A_s = the sfc area of the sphere.

Since $A_s = 4\pi R^2$ then

$$F = P/4\pi R^2$$

and therefore

$$F \propto 1/R^2$$

Furthermore, since the radiance (N) is given by

$$N = F/d\Omega$$

where $d\Omega$ is the solid angle.

subtended by the source whose cross-sectional area is A_c, and since

$$d\Omega = A_c/R^2$$

then $N = [P/4\pi R^2] / [A_c/R^2]$

$$= P/4\pi A_c$$

and N is thus independent of R.

MET? (Atmospheric radiation): E. Smith (?) * — M.S. question

• Irradiance and radiance

a) Derive a relationship between solar irradiance (flux) (H) and the distance (D) between the sun and the point of observation. Assume $R_0 << D$ where R_0 is the radius of the sun itself.

b) Does the radiance (intensity) (N) obey the same relationship? No.

a) Assume power output from the sun = P_0

So irradiance/flux at distance D ($D \gg R_0$)

$$= H = \frac{P}{4\pi D^2} \Rightarrow H \propto D^{-2}$$

b) Radiance defined as flux per solid angle. The solid angle subtended by the sun at distance D is

$$\Delta\omega = \frac{dA}{r^2} = \frac{\pi R_0^2}{D^2}$$

So $N = \frac{H}{\Delta\omega} = \frac{P}{4\pi D^2} \cdot \frac{D^2}{\pi R_0^2} = \frac{P}{4\pi^2 R_0^2}$ → N independent of D!

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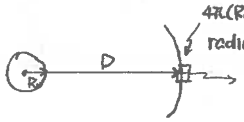
a)

The flux of outgoing radiation passing through a sphere of radius (R_0+D) concentric with the sun is given by

$$P = 4\pi(R_0+D)^2 * H \qquad (1)$$

where H is the solar irradiance.

→ By definition the radiant flux per unit area through which it passes is the irradiance (W/m²). The radiant flux is the rate of energy transfer by electromagnetic radiation (W).



$4\pi(R_0+D)^2$ represents the sfc area of a sphere of radius R_0+D.

→ The radiant flux P at a distance R_0+D from the center sun is $4\pi(R_0+D)^2 H$ where H is the solar irradiance.

In obtaining (1) we assumed that virtually no solar energy was absorbed in space and any planets, meteors, etc. along the radial path from the sun to the point in question absorb only an infinitesimal fraction of the emitted radiation. From (1) we see

$$H = \frac{P}{4\pi(R_0+D)^2}$$

or since $R_0 \ll D$

$$\boxed{H = \frac{P}{4\pi D^2}} \qquad * \text{ Irradiance is proportional to } \left(\frac{1}{\text{distance to point}}\right)^2$$

which demonstrates that

$$\boxed{H \propto D^{-2}}$$

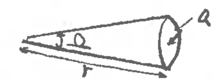
That is, solar irradiance, H, is inversely proportional to the square of the distance, D, from the sfc of the sun.

b)

At large distances from the sun the radiance, N, is approx. equal to $H/\delta\omega$ where $\delta\omega$ is the arc of solid angle subtended by the sun when viewed from a point in space.

The solid angle Ω is defined as the ratio of the area σ of a spherical sfc intercepted by the cone to the square of the radius,

$$\Omega = \frac{\sigma}{r^2}$$



In our case with $D \gg R_0$ the solid angle $\delta\omega$ is given by

$$\delta\omega = \frac{A_c}{D^2}$$

where A_c is the cross sectional area of the sun ($A_c = \pi R_0^2$)

with $\delta\omega$ given, the radiance N is

$$N = \frac{H}{\delta\omega} = H \cdot \left(\frac{A_c}{D^2}\right)^{-1}$$

But from part (a) the irradiance $H = \frac{P}{4\pi D^2}$

So

$$N = \frac{P}{4\pi D^2} \cdot \frac{D^2}{A_c} \qquad * \text{ Radiance is independent of distance to point.}$$

$$\boxed{N = \frac{P}{4\pi A_c}} \longleftarrow \text{independent of D (compare with irradiance } H = \frac{P}{4\pi D^2})$$

Which demonstrates that unlike irradiance, the radiance is independent of the distance D from the source to the point of observation.

MET? (Atmospheric radiation): E. Smith (?) —M.S. question

• If an incident radiation field is described by $N(\theta) = N_0 \tan\theta$, where θ is the zenith angle, briefly describe the visual appearance of such a field and derive an expression between the irradiance H and N_0.

Sol)

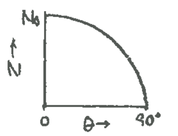
There is obviously a mistake in the formulation of this problem, as $N(\theta) = N_0 \tan\theta \longrightarrow N \to \infty$ at $\theta = 90°$, completely unrealistic. I believe the problem should consider $N = N_0 \cos\theta$. This makes perfect sense as $N = N_0$ (max) when $\theta = 0$ (sun directly overhead) and $N = 0$ when $\theta = 90°$ (sun at horizon).

Irradiance $(H) = \int N d\Omega = \int N d\mu d\phi$

$$\rightarrow H = \int_0^{2\pi}\int_0^{\pi/2} \underbrace{(N_0 \cos\theta)}_{N} \underbrace{(\sin\theta\, d\theta)}_{d\mu} d\phi$$

$$= 2\pi \int_0^{\pi/2} N_0 \cos\theta \sin\theta\, d\theta$$

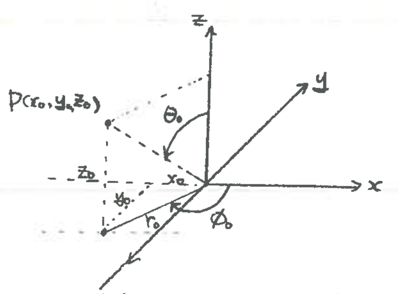
$$= 2\pi N_0 \underbrace{\left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2}}_{=1/2} = \pi N_0$$



$$\therefore \boxed{H = \pi N_0}$$

Sol)

First, lets orient ourselves. Consider an x, y, z Cartesian coord. system as shown below. (To help fix this in our mind let z be along the axis of rotation of the earth increasing toward the north pole; let x increase due east; y increase due north)



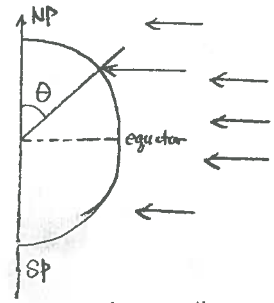
Consider point P located at x_0, y_0, z_0. We can also locate point P using a polar coord. system $\Rightarrow P(r_0, \phi_0, \theta_0)$

↳ θ is the zenith angle measured CW from the positive z axis.

↳ azimuth angle usually measured CW from positive x axis.

↳ radial distance in x, y plane to projection of point onto the $r\phi$ plane.

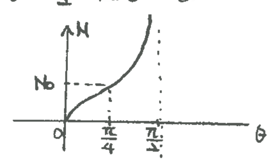
We describe these polar coord. quantities since they will help in the discussion below. Generally the zenith angle $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ but in the current problem a negative radiance does not make sense so we restrict θ to the range 0 to $\pi/2$. In a earth based reference frame we might consider the N.H. where the zenith angle plays the role of co-latitude.



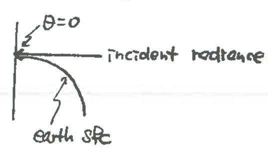
Since the distance between the sun and the earth is so great compared to the relative size of both the sun + the earth we can consider the solar energy to approach the earth as parallel beam radiation. This will help in the physical interpretation of the given eq. for the incident radiation,

$$N = N_0 \tan\theta$$

Between 0 + $\frac{\pi}{2}$ $\tan\theta$ ranges from 0 to ∞



At $\theta = \pi/4$, the incident radiance is the constant N_0. That is the constant N_0 is the incident radiance at a zenith angle of $\pi/4$. Note that as $\theta \to 0$ the incident radiance $N \to 0$. In our earth based perspective this is reasonable since at a zenith angle of 0 the incident radiation is parallel to the earths sfc (we are assuming the earth is a perfect sphere and neglecting any atmospheric effe on the incident radiance). This is shown below



Increasing the zenith angle (moving equatorward) N increases. This makes sense since the relative area of the earths sfc which the incident radiance sees is increasing. (The projection of N onto the earth's sfc is increasing in absolute value).

However as we near the equator ($\theta \to \frac{\pi}{2}$) the incident radiance tends to infinity. This is clearly not physically realistic. Based on our greatly simplified perspective we expect the incident radiance to be a max at $\theta = \frac{\pi}{2}$. However, we also expect this max to be some finite number.

To derive a relation between N_0 + H (solar irradiance) we recall from the previous question (b) that we found

$$N = \frac{P}{4\pi A_c}$$

$$P = 4\pi (R_0 + D)^2 H \sim 4\pi D^2 H \quad \text{since } R_0 \ll D$$

so

$$N = \frac{4\pi D^2 H}{4\pi A_c} = \frac{D^2 H}{A_c}$$

A_c is the cross sectional area of the sun $\Rightarrow A_c = \pi R_o^2$

so $N = \dfrac{D^2 H}{\pi R_o^2} = \dfrac{D^2 H}{A_c}$

~~We use the given expression for N.~~

$$N = N_o \tan\theta = \frac{D^2 H}{A_c} = \frac{D^2 H}{\pi R_o^2}$$

Solving for N_o

$$\boxed{N_o = (\cot\theta)\left(\frac{D^2}{A_c}\right)H \quad \text{or} \quad N_o = (\cot\theta)\left(\frac{D^2}{\pi R_o^2}\right)H}$$

$\cot\theta \to \infty$ as $\theta \to 0$

$\cot\theta \to 0$ as $\theta \to \frac{\pi}{2}$

at $\theta = \frac{\pi}{4}$,

$$\boxed{N_o = \frac{D^2}{A_c}H}$$

* other approach.

Since radiance is irradiance per unit solid angle,

$$\text{irradiance} = \int_{\text{solid angle}} (\text{radiance})\, d\Omega$$

We defined the solid angle Ω as $\Omega = \dfrac{\sigma}{r^2}$

To obtain a differential solid angle we construct a sphere whose central point is denoted as O. Assuming a line through point O moving in space and intersecting an arbitrary sfc located at distance r from point O, then the differential area in polar coords is given by

$$d\sigma = (r\,d\theta)(r\sin\theta\, d\phi)$$

→ azimuth angle

→ zenith angle

→ radial distance

Hence as

$$d\Omega = \frac{d\sigma}{r^2}$$

then

$$d\Omega = \frac{1}{r^2}(r\,d\theta)(r\sin\theta\, d\phi)$$

$$\boxed{d\Omega = \sin\theta\, d\theta\, d\phi}$$

We use this in our problem to write

$$H = \int_\Omega N\, d\Omega$$

as

$$H = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} (N_o \tan\theta) \sin\theta\, d\theta\, d\phi$$

↑ integral over zenith angle.

↑ integral over azimuth

Evaluate this integral

$$H = \int_0^{2\pi}\int_0^{\pi/2} N_o \sin^2\theta \cos^{-1}\theta\, d\theta\, d\phi$$

Assume N_o is constant over θ & ϕ

$$H = 2\pi N_o \underbrace{\int_0^{\frac{\pi}{2}} \sin^2\theta \cos^{-1}\theta\, d\theta}$$

→ from CRC math tables

$$\int \frac{\sin^2\theta}{\cos\theta}\, d\theta = -\sin\theta + \log\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$$

$$H = 2\pi N_o \left[\left(-\sin\frac{\pi}{2} + \log\left[\tan\left(\frac{\pi}{4}+\frac{\pi}{4}\right)\right]\right) - \left(-\sin 0 + \log\left[\tan\frac{\pi}{4}\right]\right)\right]$$

$$H = 2\pi N_o \left[-1 + \log\left[\tan\left(\frac{\pi}{2}\right)\right] - \log(1)\right]$$

$$H = 2\pi N_o \left[\log\left(\tan\left(\frac{\pi}{2}\right)\right) - 1\right]$$

∞ so $\log \to \infty$

This is not a well behaved function as $\theta \to \frac{\pi}{2}$.

Lee says that Dr. Smith made a mistake in this problem and that

$$N = N_o \cos\theta$$

then

$$H = \int_\Omega N\, d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi/2} N_o \cos\theta \sin\theta\, d\theta\, d\phi$$

$$= 2\pi N_o \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta$$

$$= 2\pi N_o \left.\frac{\sin^2\theta}{2}\right|_0^{\frac{\pi}{2}}$$

$$\therefore \boxed{H = \pi N_o}$$

→ For isotropic radiation (radiant intensity or radiance independent of direction) the flux density or irradiance is simply π times the radiance.

* Stefan-Boltzmann relationship.

MET4450: E. Smith (1 hour, 1995)

- Given the Planck Function:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5[\exp(hc/\lambda kT) - 1]}$$

derive the Stefan-Boltzmann relationship and explain why the Stefan-Boltmann constant σ contains residual uncertainty.

Sol)

Recall Plank function

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5[\exp(hc/\lambda kT)-1]}$$

To calc. total intensity, we integrate over all wavelengths

$$\rightarrow B(T) = \int_0^\infty B_\lambda(T)\,d\lambda = \int_0^\infty 2hc^2\lambda^{-5}[\exp(hc/\lambda kT)-1]^{-1}\,d\lambda$$

Now we will employ a change of variable for ease of integration,

$$X = \frac{hc}{\lambda kT}$$

$$\rightarrow dX = -\frac{hc}{\lambda^2 kT}\,d\lambda \longrightarrow d\lambda = -\frac{\lambda^2 kT}{hc}\,dX = \frac{-hc}{X^2 kT}\,dX\ , \text{ as } \lambda = \frac{hc}{XkT}$$

Also, $\begin{cases}\lambda \to 0 : X \to \infty \\ \lambda \to \infty : X \to 0\end{cases}$

So substitute for λ + $d\lambda$

$$\rightarrow B(T) = \int_\infty^0 2hc^2\left(\frac{hc}{XkT}\right)^{-5}[e^X-1]^{-1}\left(-\frac{hc}{X^2kT}\right)dX$$

$$= -\int_\infty^0 (2hc^2)\left(\frac{XkT}{hc}\right)^5\left(\frac{hc}{X^2kT}\right)[e^X-1]^{-1}\,dX$$

$$= \frac{2k^4T^4}{h^3c^3}\int_0^\infty \frac{X^3}{e^X-1}\,dX$$

Now the exponential integral can be expanded in terms of a gamma function, which when evaluated $\rightarrow$ integral $= \pi^4/15$.

Substitute into above

$$\rightarrow B(T) = \left(\frac{2k^4}{h^3c^3}\,\frac{\pi^4}{15}\right)T^4 = bT^4$$

If the body radiates isotropically, then in flux form

$$\boxed{B^F(T) = \pi b T^4 = \sigma T^4}$$

where $\sigma = \pi b \equiv$ Stefan-Boltzmann constant $\approx 5.67\times10^{-8}\ \mathrm{W\,m^{-2}\,deg^{-4}}$

Recall Plack's law contains many assumptions and simplifications :

1) Assume Boltzmann dist. $(N = N_0 e^{-\Delta E/kT})$
2) Expanded series for total energy considering all energy states.
3) Used average energy for all oscillators.
4) Used Rayleigh - Jeans formula (small f, large T) to define energy
5) Again assumed small f, large T simplifying energy from Boltzmann formulation.

$\Longrightarrow$ All of this leads to uncertainty in our Stefan-Boltzmann constant σ.

* upward infrared irradiance

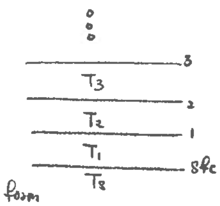
MET? (Atmospheric radiation): E. Smith (?) → Ph.D question

• Show that for an isothermal, surface-atmosphere system the upward infrared irradiance is invariant with height.

Sol)

For an isothermal sfc-atm system, temp. of sfc (T_s) is the same as temp. of each atm level ($T_1, T_2, \cdots$)

$\rightarrow T_s = T_1 = T_2 = T_3 = \cdots$



For an azimuthally indep. flux field,

$$\frac{dI_\nu^\uparrow}{d\tau}(\tau,\mu) = I_\nu^\uparrow(\tau,\mu) - B_\nu(T(\tau))$$

→ Solution for IR can be written in following form

$$I_\nu^\uparrow(\tau,\mu) = I_\nu^\uparrow(\tau_s,\mu)\, e^{-(\tau_s-\tau)/\mu} + \int_\tau^{\tau_s} B_\nu[T(\tau')]\, e^{-(\tau'-\tau)/\mu}\frac{d\tau'}{\mu}$$

For an isothermal sfc-atm, $T(\tau') = T_s$ → indep. from τ.

Also $I_\nu^\uparrow(\tau_s,\mu) = B_\nu(T_s)$

$$\rightarrow I_\nu^\uparrow(\tau,\mu) = B_\nu(T_s)\, e^{-(\tau_s-\tau)/\mu} + B_\nu(T_s)\underbrace{\int_\tau^{\tau_s} e^{-(\tau'-\tau)}\, d\tau'/\mu}_{e^{-(\tau'-\tau)}\big|_{\tau_s}^{\tau} = 1 - e^{-(\tau_s-\tau)/\mu}}$$

$$\rightarrow I_\nu^\uparrow(\tau,\mu) = B_\nu(T_s)\left[e^{-(\tau_s-\tau)/\mu} + 1 - e^{-(\tau_s-\tau)/\mu}\right]$$

$\rightarrow I_\nu^\uparrow(\tau,\mu) = B_\nu(T_s)$! indep. of τ and hence height!

MET?: E. Smith (45 minutes - 1 hour)

- Write down the complete form of the one dimensional Radiative Transfer Equation for unpolarized radiation in differential equation form. Describe what simplifications are necessary to obtain the Beer's Law solution, the Schwarzchild solution, and the solution for a multiple scattering but non-emitting atmosphere. Write down or describe the form of these three solutions assuming a 2-stream framework for the multiple scattering case.

Sol)

Complete form of RTE : $dI_\lambda = -I_\lambda + J_\lambda$ (J_λ = source function)

$I_\lambda = I_\lambda(\theta,\phi)$

$dI_\lambda = -I_\lambda k_{a\lambda} \rho ds$ → loss : absorption from path

$\quad - I_\lambda k_{s\lambda} \rho ds$ → loss : scattering from path

$\quad + B_\lambda(T) k_{a\lambda} \rho ds$ → gain : emission into path

$\quad + \mathbb{P}[I_\lambda] k_{s\lambda} \rho ds$ → gain : scattering into path

where $\mathbb{P}$ = function of radiance which scatters into path

Definitions

1) extinction : $k_{e\lambda} = k_{a\lambda} + k_{s\lambda}$

2) vol. coeff. : $\beta_{e\lambda} = \rho k_{e\lambda}$

3) optical depth (wrt vert. path) : $d\tau/\mu = -\rho k_{e\lambda} ds = -\beta_{e\lambda} ds$, $\mu = \cos\theta$

4) single scatter albedo : $\tilde{\omega} = \frac{\beta_s}{\beta_e} \longrightarrow (1-\tilde{\omega}) = \frac{\beta_a}{\beta_e}$

$\rightarrow dI_\lambda = -I_\lambda \beta_{e\lambda} ds + B_\lambda(T)\beta_{a\lambda} ds + \mathbb{P}[I_\lambda]\beta_{s\lambda} ds$

divide by $d\tau/\mu = -\beta_{e\lambda} ds$

$$\rightarrow \mu \frac{dI_\lambda}{d\tau} = +I_\lambda - \frac{\beta_{a\lambda}}{\beta_{e\lambda}} B_\lambda(T) - \frac{\beta_{s\lambda}}{\beta_{e\lambda}} \mathbb{P}[I_\lambda]$$

recall definition of $\tilde{\omega}$

$$\rightarrow \boxed{\mu \frac{dI_\lambda}{d\tau} = \underbrace{I_\lambda}_{\text{extinction}} - \underbrace{(1-\tilde{\omega})B_\lambda(T)}_{\text{emission}} - \underbrace{\tilde{\omega}\mathbb{P}[I_\lambda]}_{\text{scattering}}}$$

Now let's examine our scattering function.

$$\mathbb{P} = \frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I_\lambda(\tau,\mu',\phi') P(\mu,\phi;\mu',\phi') \overbrace{d\mu' d\phi'}^{d\Omega}$$

$\because$ (normalized P: $\int_{4\pi} P = 4\pi$)

We can separate the direct and diffuse contributions of $\mathbb{P}$

$\rightarrow \mathbb{P} = \mathbb{P}_{direct} + \mathbb{P}_{diffuse}$

For direct beam, incident at $\mu = -\mu_0$; $\phi = \phi_0$, where beam pointed down ($-\mu_0$)

$$\rightarrow \mathbb{P}_{direct} = \frac{1}{4\pi}[\pi F_0] P(\mu,\phi;-\mu_0,\phi_0)[e^{-\tau/\mu_0}]$$

$$\mathbb{P}_{diffuse} = \frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I_\lambda(\tau,\mu',\phi') P(\mu,\phi;\mu',\phi') d\mu' d\phi'$$

(where $\mu', \phi' \neq -\mu_0, \phi_0$)

- Beer-Bouger-Lambert form, or Beer's law

Consider extinction only (No emission, no scattering)

$$\rightarrow \boxed{dI_\lambda = -I_\lambda k_{a\lambda}\rho ds = -I_\lambda \beta_{a\lambda} ds = -I_\lambda\, d\tau/\mu}$$

$$\rightarrow \int_0^\tau \frac{dI_\lambda}{I_\lambda} = \int_0^\tau -\frac{d\tau}{\mu} \rightarrow \ln I_\lambda(\tau) - \ln I_\lambda(0) = \ln\left[\frac{I_\lambda(\tau)}{I_\lambda(0)}\right] = \frac{-\tau}{\mu}$$

$$\rightarrow \boxed{I_\lambda(\tau) = I_\lambda(0)e^{-\tau/\mu} = I_\lambda(0)\mathcal{T}_\lambda}$$

Note : This solution holds for extinction by abs + scattering as well

- Schwarzchild-Milne form or Schwarzchild's eq.

Consider extinction + emission (no scattering) ⟶ $\tilde{\omega} = 0$, or $(1-\tilde{\omega}) = 1$

$\rightarrow dI_\lambda = -I_\lambda k_{a\lambda}\rho ds + B_\lambda(T) k_{a\lambda}\rho ds$ divide by $k_\lambda \rho ds = -d\tau$

(Note : μ included in τ here)

$\rightarrow -\frac{dI_\lambda}{d\tau} + I_\lambda = B_\lambda(T)$ multiply by integrating factor $e^{-\tau}$

$$\rightarrow \int_{\tau_0}^{\tau}(-e^{-\tau}dI_\lambda + e^{-\tau}I_\lambda d\tau) = -\int_{\tau_0}^{\tau} d[I_\lambda e^{-\tau}] = \int_{\tau_0}^{\tau} B_\lambda(T)e^{-\tau}d\tau$$

For $\tau_0 = 0$

$$\Rightarrow \boxed{I_\lambda(\tau) = \underbrace{I_\lambda(0)e^{-\tau}}_{\text{atten. boundary}} + \underbrace{\int_0^\tau B_\lambda(T(\tau'))e^{-\tau'}d\tau'}_{\text{atten. atm. emission}}}$$

→ LW (IR) radiation not scattered by clear atm ($\lambda \gg D$)

So these eqs. valid for that case.

- Scattering form of RTE

Consider extinction and scattering source (no emission)

Same as conservative scattering, $\tilde{\omega} = 1 \longrightarrow (1-\tilde{\omega}) = 0$

$$\mu\frac{dI_\lambda}{d\tau} = I_\lambda - \frac{\tilde{\omega}}{4\pi}\int_{4\pi} I(\tau,\Omega')P(\Omega,\Omega')d\Omega' - \frac{\tilde{\omega}}{4\pi}\pi F_0 P(\Omega,-\Omega_0)e^{-\tau/\mu_0}$$

- Single scattering — scatter only once (neglect diffuse)

$$\rightarrow I_\lambda - \mu\frac{dI}{d\tau} = \frac{\tilde{\omega}}{4\pi}\pi F_0 P(\Omega,-\Omega_0)e^{-\tau/\mu_0}$$

mult. integrating factor $e^{-\tau/\mu}$, integrate over τ

$$\rightarrow I_\lambda(\tau,\mu,\phi) = I(\tau_0,\mu,\phi)e^{-(\tau-\tau_0)/\mu} + \int_{\tau_0}^{\tau}\frac{\tilde{\omega}}{4}F_0 P(\Omega,-\Omega_0)e^{-\left(\frac{\tau-\tau_0}{\mu}\right)\left(\frac{\tau'}{\mu_0}\right)}\frac{d\tau'}{\mu}$$

If $I_\lambda(\tau_0,\mu,\phi) = 0$ at boundary $\rightarrow I_\lambda(\tau,\Omega) \propto P(\Omega,-\Omega_0)$!!

- Two-stream – consider multiple scattering (both diffuse + direct beam)

Expand $P(\tau,\Omega)$ as series of Legendre Polynomials

Consider case where LP index = −1 + 1 (→ "2 streams")

This also corresponds to $\mu = +\mu_1$ (up) + $-\mu_1$ (down)

$$\rightarrow \text{up}: \mu_1\frac{dI\uparrow}{d\tau} = I\uparrow - \tilde{\omega}(1-b)I\uparrow - \tilde{\omega}bI\downarrow - S^- e^{-\tau/\mu_0}$$

$$\text{down}: -\mu_1\frac{dI\downarrow}{d\tau} = I\downarrow - \tilde{\omega}(1-b)I\downarrow - \tilde{\omega}bI\uparrow - S^+ e^{-\tau/\mu_0}$$

where $b = (1-g)/2$, g = asymm. factor $\begin{cases} g=1 & : \text{pure forward scatt.} \\ g=-1 & : \text{" backward "} \\ g=0 & : \text{Rayleigh scatt.}\end{cases}$

and $S^\pm = \frac{1}{4}F_0\tilde{\omega}(1 \pm 3g\mu_1\mu_0)$

so b = integrated fraction of back-scatt. rad.

$1-b$ = " " " forward-scatt. rad.

So problem now reduced to solving a coupled pair of differential eqs.

2-stream solution rep. the phase function as long as λ does not get too long w.r.t. scattering → exag. forward peak (2 stream not rep.)

- δ-Eddington

This adjusts for the 2-stream problem when the forward peak is exag. Essentially it rescales the problem by including the forward peak in the direct beam term.

*Schwarzchild-Milne form of RTE

MET? (Atmospheric radiation): E. Smith (?) *

- Provide a mathematical expression for the Schwarzschild-Milne form of the Radiative Transfer Equation (RTE).

a) Which terms of the complete RTE have been ignored?

b) Provide a finite difference formulation of infrared flux at the top of a clear atmosphere based on a transformation of the above equation to a simplified broad-band flux emissivity expression.

c) Explain how the method of partitioning infrared transfer into cooling exchange between internal layers (CIL), cooling-to-space (CTS) and cooling to ground (GTG) can be exploited in a GCM. Which of these terms is most important with respect to tropospheric cooling? ⟶ See Newtonian Cooling Question.
↳ CTS?

- Schwarzchild eq :

$$I_\lambda(\tau) = I_\lambda(0)e^{-\tau/\mu} + \int_\tau B_\lambda(T(\tau))e^{-\tau'/\mu}\,d\tau'$$

(a) Scattering terms ignored!

(b)
If we assume the earth's sfc emits as a blackbody, express sfc emission of Planck function in above expression. To yield flux, integrate over solid angle (θ & ϕ). Assuming azimuthal indep., above reduces to integral over μ (or θ). So

$$F_\nu\uparrow(\tau) = 2\pi B_\nu(T_s)\int_0^1 e^{-(\tau_s-\tau)}\mu\,d\mu + 2\pi\int_0^1\int_\tau^{\tau_s} B_\nu(T(\tau'))e^{-(\tau'-\tau)/\mu}d\tau' d\mu$$

$(\because d\tau' = d\tau/\mu)$

(Note: Here is where we sometimes define exp. integral functions – not necessary here)

We note the above expression is for (1) monochromatic, and (2) τ form

We can integrate over ν (or λ – whatever) to transform to band form

$$\rightarrow F_{\bar\nu}\uparrow(\tau) = \int_{\Delta\tau} F_\nu\uparrow(\tau)\frac{d\nu}{\Delta\nu}$$

$$= 2\pi B_\nu(T_s)\int_{\Delta\nu}\int_0^1 e^{-(\tau_s-\tau)}\mu\,d\mu\frac{d\nu}{\Delta\nu}$$

$$+ 2\pi\int_{\Delta\nu}\int_0^1\int_\tau^{\tau_s} B_{\bar\nu}(T(\tau'))e^{-(\tau'-\tau)/\mu}d\tau' d\mu\frac{d\nu}{\Delta\nu}$$

define flux transmission

$$\mathcal{T}_{\bar\nu}^F(\tau) = 2\int_0^1\int_{\Delta\nu} e^{-\tau/\mu}\frac{d\nu}{\Delta\nu}\mu\,d\mu \rightarrow \frac{d\mathcal{T}_{\bar\nu}^F}{d\tau} = -2\int_0^1 e^{-\tau/\mu}\frac{d\nu}{\Delta\nu}d\mu$$

$$\rightarrow \boxed{F_{\bar\nu}\uparrow(\tau) = \pi B_\nu(T_s)\mathcal{T}_{\bar\nu}^F(\tau_s-\tau) + \int_{\tau_s}^{\tau} B_\nu(T(\tau'))\frac{d\mathcal{T}_{\bar\nu}^F(\tau'-\tau)}{d\tau'}d\tau'}$$

For broad band flux, we'll integrate over all ν (or λ...)

$$\rightarrow F_{BB}\uparrow(\tau) = \int_0^\infty \pi B_\nu(T_s)\mathcal{T}_\nu^F(\tau_s-\tau)d\nu + \int_0^\infty\int_{\tau_s}^{\tau} B_\nu(T(\tau))\frac{d\mathcal{T}_\nu^F(\tau'-\tau)}{d\tau'}d\tau'$$

From Stefan–Boltzmann law, $\int_0^\infty \pi B_\nu(T)d\nu = \sigma T^4$

$$\rightarrow \boxed{F_{BB}\uparrow(\tau) = \sigma T_s^4\mathcal{T}_{BB}^F(\tau, T_s) + \int_{\tau_s}^{\tau}\sigma T^4(\tau')\frac{d\mathcal{T}_{BB}^F(\tau-\tau', T)}{d\tau'}d\tau'}$$

where $\mathcal{T}_{BB}^F(\tau, T) = 1 - \mathcal{E}_{BB}^F(\tau, T) = \int_0^\infty \pi\frac{B_\nu(T)}{\sigma T^4(\tau)}d\nu$

$\mathcal{T}_{BB}^F(\mathcal{E}_{BB}^F) \equiv$ Broad band flux transmission (emissivity)

So, how do we calc. $\mathcal{T}_{BB}^F$ or $\mathcal{E}_{BB}^F$? Normally $\mathcal{T}_{\bar\nu}^F$ is derived from either theory or experiment for small spectral bands. Then, the broadband value is given by summing over the finite number of bands (e.g. – to encompass a specifical IR regime).

$$\rightarrow \mathcal{E}_{BB}^F(\tau, T) = 1 - \mathcal{T}_{BB}^F(\tau, T) = \sum_{i=1}^{N}\pi\frac{B_{\bar\nu_i}(T)}{\sigma T^4}[1 - \mathcal{T}_{\bar\nu_i}^F(\tau)]\Delta\nu_i$$

where N = number of spectral bands. So our system is

$$\boxed{\begin{aligned} F_{BB}\uparrow(\tau) &= \sigma T_s^4\mathcal{T}_{BB}^F(\tau, T_s) + \int_{\tau_s}^{\tau}\sigma T^4(\tau')\frac{d\mathcal{T}_{BB}^F(\tau-\tau', T)}{d\tau'}d\tau' \\ F_{BB}\downarrow(\tau) &= \int_{\tau_0=0}^{\tau}\sigma T^4(\tau')\frac{d\mathcal{T}_{BB}^F(\tau'-\tau, T)}{d\tau'}d\tau' \end{aligned}}$$

So, B.C.s : 1) TOA : $F_{BB}\downarrow(\tau=0) = 0$
2) sfc : $F_{BB}\uparrow(\tau=\tau_s) = \sigma T_s^4$

MET?: E. Smith (45 minutes, 1989) *

• Question

a) Write out an expression in integro-differential form for the complete radiative transfer equation; be sure and define your notation.
b) Show (or derive) the Beer-Bouguer-Lambert form of the solution and the Schwarzschild-Milne form of the solution (the latter is the non-scattering case).
c) From these two types of solutions, what approximations were made and what types of atmospheric radiation problems would be appropriate for applying these two forms of solutions?
d) If the full R.T.E. is to be solved analytically, what essential mathematical step must be taken in dealing with the scattering phase function? I am not asking for a derivation here.

* radiative heating + cloud top cooling

MET4450 (Atmospheric Physics II): E. Smith (45 min to 1 hour, 1995, 1998) **

• Derive the expression for radiative heating rate from first principles and then explain the concept of infinite cloud top cooling in the framework of the heating rate equation. Why don't we see infinite cloud top cooling? If it were to occur, what would happen to the atmosphere?

* Flux profiles – cooling rate

MET?: E. Smith (45 minutes) **

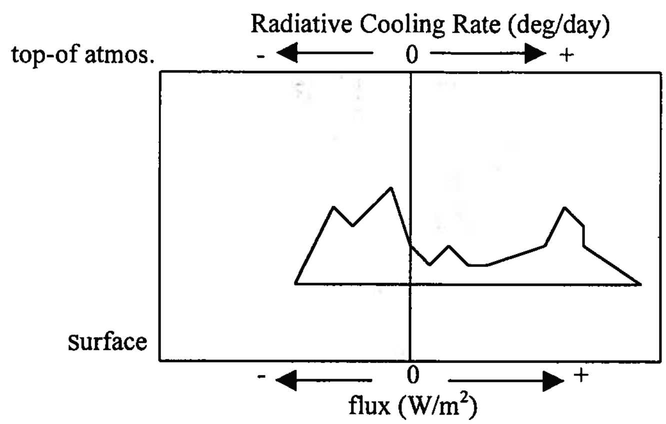
- Consider the diagram given below of a cloud layer within the atmosphere:

a) Draw schematically the upward ($F\uparrow$) and downward ($F\downarrow$) infrared flux profiles between the surface and top-of-atmosphere.

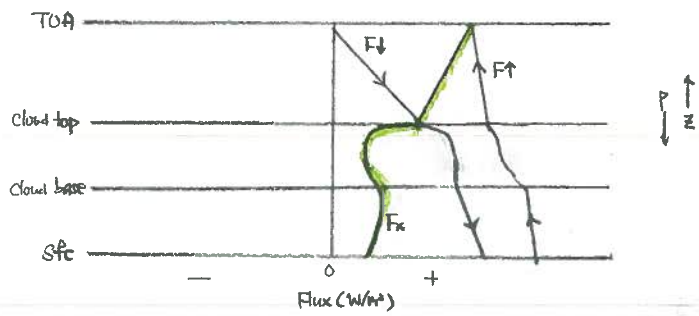
b) Draw schematically the associated flux divergence profile $\left(F\uparrow - F\downarrow\right)$.

c) Draw schematically the representative radiative cooling profile associated with the situation.

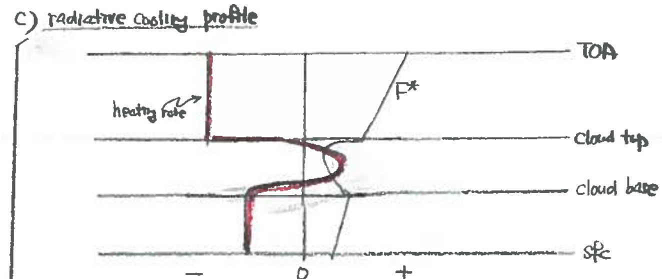
d) How would the variation of the altitude of a cloud layer modulate the bulk troposphere cooling rate and what are the climate implications of varying the mean altitude of cloud base.



Note: $\frac{dT}{dt} = \overbrace{\left(\frac{g}{C_p}\right)}^{-\Gamma_d}\frac{dF^*}{dp}$, $F^* = F\uparrow - F\downarrow$



c) radiative cooling profile



a) See above diagram for sketch of profiles. Note the following:

① $F\downarrow = 0$ at TOA

② Slope of $F\uparrow$ less than slope of $F\downarrow$ for clear air.

③ Note ~ discontinuities of $F\uparrow$ + $F\downarrow$ at cloud boundaries

④ Discont. of $F\downarrow$ greater than discont. of $F\uparrow$ at cloud top. The cloud is much more active for IR than the air above → large discont.

b) $F^* = F\uparrow - F\downarrow$: net flux out

flux divergence

See above diagram for sketch. Note the following:

① Slope of F^* greater above cloud than below cloud.

② Positive slope of $\frac{\partial F^*}{\partial z}$ → net flux out of layers (above + below cloud)

③ Discontinuity at cloud top as F^* changes slope discontinuously. Also ~ discont. at cloud base (although realistic profiles of fluxes should lead to a more smooth curve for F^*)

See above diagram for sketch. Note the following

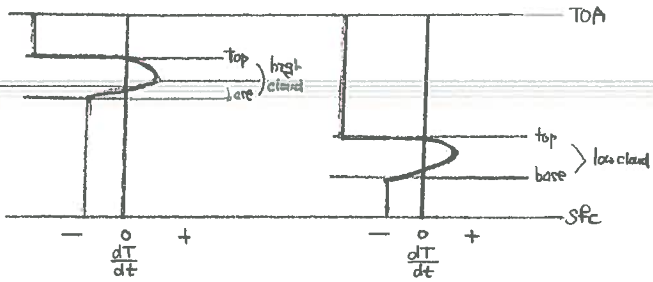
① $\frac{dT}{dt} = +\frac{g}{C_p}\frac{dF^*}{dp} = -\frac{1}{\rho C_p}\frac{dF^*}{dz}$ → positive slope with height = cooling.

② Greater slope of F^* above cloud → greater cooling above cloud (than below). This is consistent with the way clouds are described as a "blanket", which acts to retain heat below the clouds. Hence one would expect inhibited cooling below cloud.

③ Heating throughout most of cloud layer.

④ Discontinuity in cloud top cooling. Also, note we have smoothed the profile for cloud base with our smoothed F^* profile. If we had drawn discont. in F at both boundaries, we obviously would have gotten discont. in $\frac{dT}{dt}$ at those boundaries as well.

d)

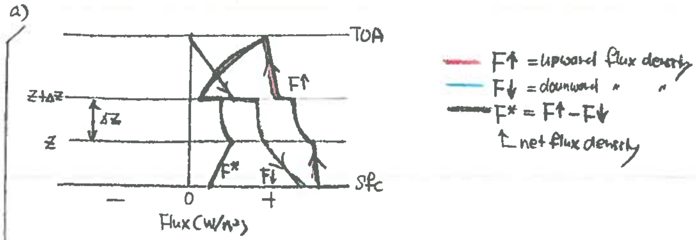


① High cloud case : Smaller F↑, F↓ from cloud as cloud temp. is cold. F↑ at cloud base is higher due to sfc, atm, and clouds below. We get warming at cloud layer, also cooling is greater above cloud than below.

② Low cloud case : Larger F↑, F↓ from cloud as cloud temp is warm. Again we get warming in cloud layer, also cooling is greater above cloud than below.

The big difference is that for high cloud, a larger depth of the troposphere (below the cloud) has suppressed IR cooling. Hence a higher cloud deck (assuming it's optically thick enough) will reduce the bulk IR cooling rate of the troposphere. This would lead to a climatologically warmer troposphere. However, this neglects an important feedback mechanism. As the troposphere gets warmer, there is more convection and more cloud. However, the increased cloud could increase the albedo, which increases the amount of solar radiation reflected to space. With less solar radiation reaching the atm below (as well as the sfc), heating would decrease (as well as the sfc temp.), which would reduce the flux from the sfc and hence from the atm./ Bottom line ; this important feedback is crucial to mean atm. temp. and unfortunately has not been incorporated well into climate models.

a)



• points to note

① The slope of the net flux F* above the cloud top is larger than that below the cloud base. As we shall later see this means that the IR cooling rate is larger above the cloud top than it is below the cloud base. (Intuitively makes sense as cloud deck serves as a "blanket" which keeps sfc to cloud base temps higher than they would be in the absence of the cloud deck)

② The discontinuity of downwelling flux (F↓) is larger than that of upwelling flux (F↑) on the cloud top. → The (low) cloud is a much more active for IR than the air above, thus the discontinuities at cloud top.

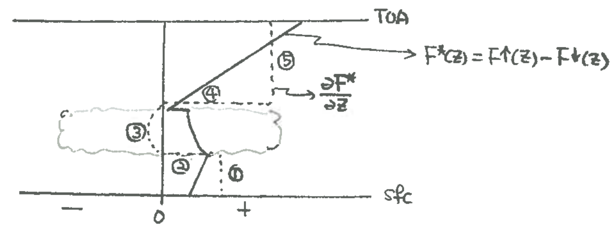
③ Zero incoming (downward) IR at TOA.
↳ zero or practically zero.

b)

At each level z the net IR flux density is

$$F^*(z) = F\uparrow(z) - F\downarrow(z).$$

Intuitively, if in a layer Δz thick there is a net positive IR flux density (more IR coming in than going out), the layer will warm. Conversely, a net flux divergence across a layer Δz thick will lead to IR cooling of the layer. Thus, w.r.t IR heating/cooling of a layer the parameter of interest is the vertical gradient of the net flux density $\Rightarrow \frac{\partial F^*}{\partial z}$. This is the slope of the F* curve.



① From cloud base to the sfc F* has an approx. constant, positive slope ⇒ F* increases from the sfc up

② A discontinuity arises at the cloud base because F* goes from increasing with height to decreasing with height in the cloud layer. This reflects the fact that there is a net warming due to IR in the cloud layer. ③ reflects the negative slope of F* in the cloud layer.

④ A larger discontinuity in $\frac{\partial F^*}{\partial z}$ occurs at cloud top. ⑤ Above the cloud top the net IR increases with height. Furthermore, this rate of increase in F* is greater than that below the cloud base. Hence a layer discontinuity

c)

Consider a cloud of thickness Δz. The change in the net IR flux density across the layer Δz is

$$\Delta F^* = F^*(z+\Delta z) - F^*(z)$$

The idea of conservation of energy may be used to examine how ΔF* heats or cools the layer Δz. We will express the effects of heating/cooling in terms of the rate of change of layer temperature. It is conventionally given by

$$\Delta F^*(z) = -\rho C_p \Delta z \frac{\partial T}{\partial t} \quad \left(\text{consider } 1^{st} \text{ law } C_p\frac{dT}{dt} - \alpha\frac{dp}{dt} = J\right)$$

where ρ is the density of air in the layer, C_p is the specific heat at constant pressure. The heating rate/cooling rate for the layer Δz is therefore

$$\boxed{\frac{\partial T}{\partial t} = -\frac{1}{\rho C_p}\frac{\Delta F^*(z)}{\Delta z}}$$

Using the hydrostatic relation $\Rightarrow \frac{\Delta p}{\Delta z} = -\rho g$

we can rewrite this as

$$\boxed{\frac{\partial T}{\partial t} = +\underbrace{\frac{g}{C_p}}_{\text{the adiabatic lapse rate.}}\frac{\Delta F^*(p)}{\Delta p}}$$

The point to note here is that the radiative heating/cooling profile is given by $-\frac{\partial F^*}{\partial z}$

That is, $\frac{\partial T}{\partial t} = \left(-\frac{1}{\rho C_p}\right)\frac{\partial F^*}{\partial z}$, $F^* = F\uparrow - F\downarrow \equiv$ net IR flux density in the layer

Qualitatively we see

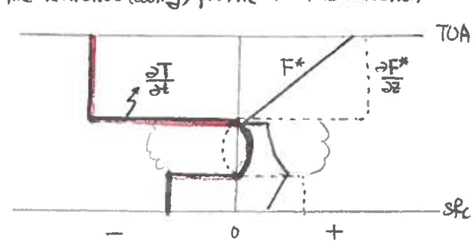
heating $\rightarrow \frac{\partial T}{\partial t} > 0$ when $\frac{\partial F^*}{\partial z} < 0 \rightarrow$ vertical convergence of net upward flux density

cooling $\rightarrow \frac{\partial T}{\partial t} < 0$ when $\frac{\partial F^*}{\partial z} > 0 \rightarrow$ " divergence of " " " "

heating ≡ convergence of radiative flux

cooling ≡ divergence of " "

Below is the radiative (cooling) profile for this situation



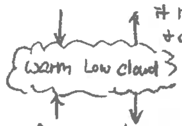
Note: The cloud top cooling is greater than the cooling below the cloud base
⇒ we see the insulating effect of clouds. The cloud top exchange of radiation is greater with the space above than the sfc below.

d)

For a low cloud we have

downward IR at cloud top is less than upward IR. downward IR due to atm above + clouds above

Since low cloud temp. is relatively high it radiates actively in both upward + downward directions.



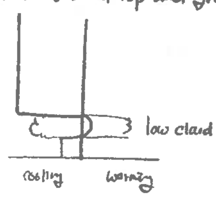
large F↑ since sfc is warm

large F↓ since cloud temp is relatively high for a low cloud

F↑ is generally greater than F↓ below cloud base

⇒ The net effect is a convergence of net upward IR and so the temp. of the low cloud layer increases. Above + below the cloud layer the atm cools with the cooling rate greater above cloud top and greater below cloud base.

Because the cloud layer is low, a greater depth of the atm experiences the larger above cloud cooling rate.

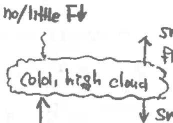


For a high cloud

The net effect of ↑↓ IR fluxes is to warm the high cloud layer. In fact, the high cloud layer may experience more warming than the low cloud layer.

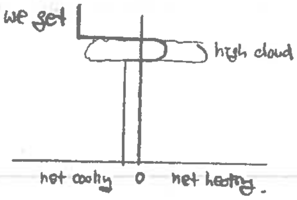
no/little F↓

small upward + downward IR flux density from cloud because cloud temp. is low



large upward IR flux density due to sfc, clouds, + atm. below. The F↑ at the cloud base of a high cloud is generally greater than that at the base of a low cloud.

If we take the $\frac{\partial T}{\partial t}$ IR heating profile from the low cloud case and apply it here we get



As before the net IR cooling is greater above the cloud top than below the cloud base. However, because the cloud base is high the bulk of the troposphere experiences the smaller (in absolute magnitude) IR cooling rate. Hence the net IR cooling of the entire troposphere is reduced when we have a high cloud base.

* infrared radiative cooling (cloud)

MET? (Radiation Problem): E. Smith (1 hour) ← Treadon's question

• A) Explain why infrared radiative cooling at the top of a stratus cloud deck would be expected to be large, B) the influence of such cooling on cloud development, and C) how the development would be modulated by the diurnal cycle of solar radiation absorption by above-cloud, in-cloud, and below-cloud layers.

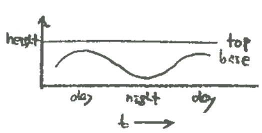
A) see flux profiles → cooling rates (the previous or other questions)

B) First, the cloud deck (if thick enough) is nearly opaque to IR radiation and strongly absorbs + scatters solar radiation. For the case of the sfc warmer than the cloud base, the cloud bottom absorbs IR from the sfc and is heated. The top of the cloud cools by emitting IR, but is heated by sunlight. So total cloud top cooling is modulated by diurnal cycle (see part C). The cloud layer may be destabilized by this cloud top cooling, and may produce small scale turbulence. This turbulent mixing in the boundary layer is mainly confined below the inversion, which above generally large scale subsidence is present. This large scale subsidence establishes a "capping" effect, which confines the moisture + heat fluxes from the sfc to a shallow layer, which helps to maintain the stratus. It has been found that a large amount of the IR cloud top cooling is balanced by adiabatic warming from subsidence, which acts to reduce turbulence produces.

C) Obviously at night, there is ~ no solar radiation present in the atm. So how will that affect the stratus? Model experiments have produced the following results for two cases

① Case #1 – No significant subsidence above cloud layer.

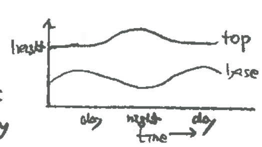
During day, absorption of SW reached a max deeper in the cloud than did the max of LW cooling. In the case of no large subsidence of the layer,



net cloud top cooling remained after sunrise. The cloud top height remained ~ constant, while the base raised during the day due to solar heating.

② Case #2 – Large subsidence above cloud layer

In this case, the top descends and base ascends during day time. During daytime, combined effect of adiabatic warming of subsidence and warming by



solar radiation overcomes cloud top cooling → lowering of cloud top. Another way of looking at it is that the max of radiative cooling shifts lower into the cloud, which causes the collapse of the cloud above the max. The cloud base rises as described before.

Sol)

* see earlier response for up/down IR profiles + radiative heating/cooling.

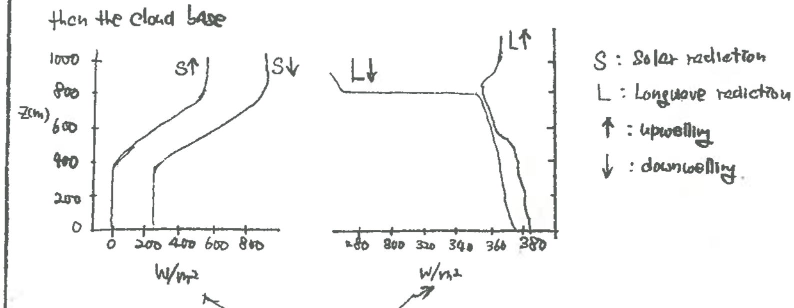
From Emanuel's text pp425-426

Strato cumulus – topped mixing layers are common over cold ocean water such as the eastern North pacific and North Atlantic. These are regions of large scale subsidence in the atmosphere and upwelling of cold water in the ocean. Air near the sfc generally flows equatorward, and as it passes over successively warmer water, the boundary layer gradually deepens and the strato cumulus layer breaks up and reforms as a trade-cumulus boundary layer. The stratocumulus may cover very large areas of the eastern ocean basins and, because their albedos are high, they reflect much of the incoming solar radiation. For this reason, they are important factors in the earth's climate and thus it is of some importance to understand the physical processes that lead to their development and breakup.

Stratocumulus and trade-cumulus boundary layers form in regions of large scale subsidence, often constituting the descent branches of tropical circulations such as the Hadley and Walker cells. In the case of the Hadley cell, the meridional temp gradient through much of the troposphere is held very close to a critical curve corresponding, through thermal wind balance, to constant angular momentum at the tropopause. At the same time the zonal temperature gradient is close to zero in the Walker cell. In both cases, the vertical temperature gradient is close to moist adiabatic everywhere above the boundary layer, corresponding to the vertical temp. profile in the ascent region.

In the descent region, above the boundary layer, the thermodynamic balance is simply between adiabatic warming and radiative cooling. However, this relationship (balance) does not hold all the way to the surface. Naturally, the atm adjacent to the sfc becomes convectively unstable, and a convective BL will form. Suppose that this BL eventually comes into equilibrium with its tendency to expand upward balanced by the large scale subsidence. What role do short + longwave radiation play in determining the depth of the layer and whether it is filled with cloud?

The existence of cloud at the top of the BL has several profound effects on the properties of the layer. In the first place, cloud of modest thickness (~100m) is nearly opaque to IR and also strongly absorbs SW radiation. Below are vertical profiles of radiative fluxes in a typical cloud deck over land or water that is warmer than the cloud base



note difference in magnitude of radiation

The base of the cloud absorbs upwelling IR from the sfc and is heated unless the sfc is colder than cloud base. The top of the cloud cools by emitting IR but may be strongly heated by sunlight. At night, stratocumulus decks are strongly destabilized by IR effects but may be stabilized during the day. If the air above the cloud is sufficiently dry, the cloud top may be unstable to cloud-top entrainment instability. This serves as an additional source of turbulence in the cloud. Sfc heating and/or sfc wind also generate turbulence in the stratocumulus-topped mixed layer, and strong wind shear in and across the top of the layer may also lead to turbulence.

From Cotton & Anthes pp 352-362

Role of vertical shear of the horizontal wind

The boundary layer large eddies are driven in part by the destabilization of the cloud layer by cloud top radiative cooling. The undulation of the cloud top occurs when the BL eddies are fully developed. The penetration of the convective elements into the capping inversion locally enhances the shear and thus aids in the breakdown of Kelvin-Helmholtz-like billows. The entrained air subsequently reduces the liquid water content and lowers the rate of radiative cooling, which then stabilizes the cloud layer. As a result, the large eddy circulation is quenched. The cycle begins to repeat itself due to the recovery of radiative cooling and increase of liquid water content. Drizzle enhances the reduction in liquid water content near cloud top by drop settling, and hence reduces the rate of radiative cooling, thus speeding up the process. This process is called the Sporadic Cloud Radiative Mechanism (SCRIM) by Chen & Cotton (1987)

→ radiative cooling, cloud water production, drizzle formation, and wind shear cooperatively interact to generate sporadic episodes of entrainment.

A different relationship is found between cloud top radiative cooling and entrainment in the stable or weakly convective stratocumulus (Sc) cloud. It is generally believed that the local instability created by cloud top radiative cooling is transmitted through the depth of the cloud layer and subcloud layer by strong vertical mixing caused by penetrative plumes. Thus, radiative cooling causes enhanced mixed layer turbulence in the form of penetrative plumes, which, in turn, causes greater rates of cloud-top entrainment.

In the case of more stable Sc, it appears that the local instability caused by cloud top radiative cooling generates small scale turbulence. The small scale turbulence interacts with the local wind shear at the top of the radiatively cooled layer. This causes sporadic turbulent breakdown or shear-driven entrainment. In some cases, these does not appear to be any direct communication between radiation cooling at cloud top and the energetics of the entire depth of the cloud layer. Almost all the cloud-top radiative cooling is balanced locally by entrainment. As a result there is no net generation of positive buoyancy and associated convective transport. Such clouds may be more properly called stratus than Sc and exhibit properties more similar to As and Ci than boundary layer stratus.

Role of drizzle

With regards to the coupling among drizzle, radiation, and turbulence, Chen and Cotton (1987) experimented with the sensitivity of the simulated Sc properties to the drizzle process. Removal of water by drizzle lowered the max liquid water content near cloud top by about 40%. Since the cloud-top longwave cooling rate is determined by the liquid water path, reduction in cloud-top liquid water reduced the radiative cooling rate, which further reduced the liquid-water production. As a consequence of the reduced cloud-top radiati cooling, the cloud layer is stabilized somewhat and the rate of entrainment at cloud top is reduced when the drizzle process is present.

It is important to recognize that drizzle may be more important to the dynamics of nocturnal stratocumulus. This is because it is observed frequently that there is a wel defined nighttime maximum in marine precip. Kraus (1983) found the largest diurnal amplitude to occur in midlat. regions where the rain is mostly non-convective. He speculated that absorption of SW radiation during the daytime resulted in less liquid-water production than at night. Thus, drizzle effects could be more important in the somewhat deeper and wetter, nocturnal Sc clouds.

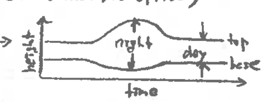
Role of large scale subsidence

It is generally recognized that large-scale subsidence plays an important role in estab the environmental conditions favorable for formation of marine Sc. That is, large sc subsidence establishes the pronounced capping inversion which serves as an upper limit to the atmospheric BL and confines the moisture and heat fluxes from the ocean sfc to a shallow layer. The overlying air mass is also dried out by the sinking motion. Whi subsidence may establish environmental conditions favorable for maintaining a solid stratus deck, too much subsidence may be responsible for the breakup of Sc clouds.

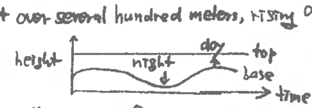
In a modelling study Chen & Cotton found that a significant fraction of the LW radiative cooling at cloud top is balanced by subsidence warming. As a consequence, the upward heat flux in the cloud layer and buoyancy production of turbulence are reduced.

Diurnal variations in marine Sc

Using a second-order turbulent transport model, Oliver et al (1978) simulated a diurnal varying Sc cloud structure. The stratus top rises and the cloud base lowers during the nig and thickens until sunrise, when the stratus begins to dissipate due to solar insolation. T location of maximum solar heating well into the interior of the cloud layer may be only be possible in clouds that are optically thin (i.e, contain little cloud liquid water).



Bougeault's (1985) simulation of the diurnal cycle of a Sc layer with a higher order closur model did not exhibit any significant variation in cloud-top height. The cloud base varied in height over several hundred meters, rising during the daytime and decending during night time.



Chen & Cotton (1987) found that absorption of SW radiation occurred deeper in the cloud layer than LW radiation cooling. SW heating reached a max about 100 m in th cloud interior, while LW cooling was max only 25 to 30 m in the cloud interior. (Not that this difference in radiative depths is much less than predicted by Oliver et al.). Chen & Cotton found that in the absence of large-scale subsidence and a moist layer above capping inversion, a net cloud-top radiative cooling remained after sunrise. They found, lik Bougeault, that the cloud top did not descend. Solar heating did, however, contribute to a ri in cloud base and a reduction in the maximum liquid-water content in the cloud layer. O when rather large subsidence was imposed did both the cloud top descend and the cloud b rise after sunrise. The lowering process took over 4 hours to commence as the combined warming of large scale subsidence & SW heating were initially insufficient to offset LW radiative cooling. The lowering process commenced by a reduction in cloud water content and cloud fractional coverage in the upper 100 m of the cloud layer. As a result of the reduced liquid-water path, the max radiative cooling shifted to lower levels causing an abrupt colla of the cloud top into a thinner cloud layer.

Twomey (1983) computed SW & LW radiation heating profile based on aircraft measurement of cloud microphysical parameters, temp, and moisture through the depth of the marine BL. He found the radiative warming prevailed near cloud top, with SW & LW radiative influenc being about equal and opposite. He concluded that the presence of a warm, moist layer abov the capping inversion reduced the rate of LW radiative cooling at cloud top.

Cotton & Chen also examined the influence of a moist layer above the capping inversion on th evolution of a simulated Sc layer. The moist layer weakened the radiative cooling at cloud top Instead of the max radiation cooling being localized near cloud top, it was distributed between c top and the top of the overlying moist layer. Because the moist layer is transparent to SW radi solar heating was unaffected. As a result, the cloud layer became far more responsive to sola

* response of the atmosphere to cloudiness

MET?: E. Smith (1 hour)

• What would the response of the atmosphere be to changes in cloudiness according to the following scenarios (respond to each part separately).

a) Significant increase of low level cloudiness in tropics (stratus and/or cumulus)
b) Significant increase of high level cloudiness in tropics (layered cirrus)
c) Significant increase of low level cloudiness in mid-latitudes (stratus and/or cumulus)
d) Significant increase of high level cloudiness in mid-latitudes (layered cirrus).

*radiative equilibrium temp.

MET? : E. Smith (?)

• Compute the radiative equilibrium temperature of the small black disk suspended high above the planar snow field shown in Fig. 1. Assume heat transfer to the disk occurs only through radiative transport. Assume also that the snow field radiates as a blackbody at ~~237~~ 273 °K in the thermal infrared and reflects solar radiation as a perfect diffuse (Lambertian) reflector. The solar zenith angle is given as 15°. Neglect scattering and absorption in the atmosphere. The Stephen-Boltzmann constant is $5.67\times10^{-8}\ \mathrm{Wm^{-2}\,deg^{-4}}$. The solar constant is $1353\ \mathrm{Wm^{-2}}$.
What is the radiative equilibrium temperature after sunset?

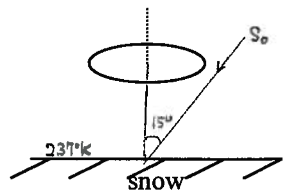


Fig. 1 Disk over snow field

Sol)

We assume the disk radiates as a blackbody at temp T_D. The radiative equilibrium temp. is that value of T_D for which

$$F_{OUT} = F_{IN}$$

at the disk where $F_{OUT} \Rightarrow$ blackbody emission by disk. From Stephen-Boltzmann law, $F_{OUT} = \sigma T_D^4 = F_D$, and $F_{IN} \Rightarrow$ net radiation incident upon the disk.

F_{IN} has several components depending on the time of day.

Lets start by assuming that the sun is up in the day. The SW radiation from the sun warms the earth sfc. The earth's surface radiates to maintain an equilibrium with the incident energy. We are told to assume that the snow covered sfc emits IR as though it were a blackbody at $T_s = 273k$. The longwave (IR) from the snow covered sfc is

$$F_{LW}^{\uparrow} = \sigma T_s^4$$

All $F_{LW}^{\uparrow}$ will reach the disk since we are told to neglect scattering + absorption by the atm.

At a particular time of day the zenith angle of the sun is 15°. Some SW radiation is intercepted by the disk. The amount (per unit area) is the projection of the incomming SW onto the normal to the disk. This amount is

$$F_{SW}^{\downarrow} = S_0 \cos\theta$$

where $S_0 = 1353\ \mathrm{W/m^2}$ is the solar constant.

$\theta = 15°$ is the solar zenith angle.

Again we neglect scattering + absorption by the atm. So is the incident solar SW at the TOA. Of this amount about 50% reaches the earth's sfc. The other 50% is absorbed, scattered, reflected by the atm. We neglect this scattering.

Finally we have the solar radiation reflected off the snow covered sfc.

A perfect diffuse Lambertian reflector is one which isotropically reflects incident radiation without any loss. This means that of the amount S_0 incident on the snow covered sfc at zenith angle 15°, all is reflected in all directions $\Rightarrow$ A portion will be reflected at a zenith angle of 0° $\Rightarrow$ normal to the sfc.

Thus we have this contribution to the incident radiation at the disk

$$F_{SW}(\text{reflected}) = S_0$$

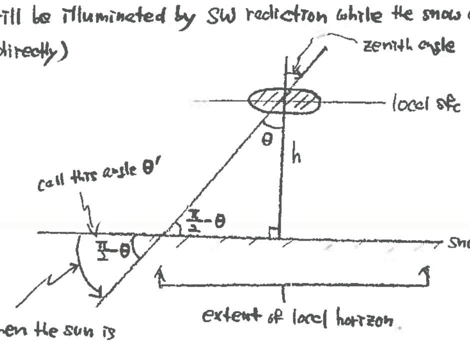
Radiative equilibrium at the disk requires

$$\boxed{\text{Daytime}\quad F_D = F_{LW}^{\uparrow} + F_{SW}^{\downarrow} + F_{SW}(\text{reflected})}$$

At night time the sun is far below the local horizon and we have no input from the solar SW radiation. Thus at night the radiative balance is

$$\boxed{\text{Night}\quad F_D = F_{LW}^{\uparrow}}$$

If the disk is very high above the snow covered sfc it is possible for it to still be illuminated by SW radiation while the snow covered sfc is not illuminated (directly)



when the sun is below the horizon but within this wedge the disk will still receive SW radiation

Let θ' be the "below the horizon" angle as indicated in the figure. The zenith angle in terms of θ' is $\theta = \frac{\pi}{2} - \theta'$

Clearly θ' ranges from 0 to some upper limit dependent upon the height, h, of the disk above the sfc. The larger h is (the higher the disk is above the sfc) the greater this upper limit. For calculation purposes below we will arbitrary let $\theta' = 15°$.

The radiative balance at the disk for just before sunrise/just after sunset is

$$\boxed{F_D = F_{LW}^{\uparrow} + F_{SW}'^{\downarrow}}$$

where $F_{SW}'^{\downarrow} = S_0 \cos\theta$ but where $\theta = \frac{\pi}{2} - \theta'$

Now lets compute values for these 3 cases.

Case 1 : Daytime with $\theta = 15°$

$$F_D = F_{LW}^{\uparrow} + F_{SW}^{\downarrow} + F_{SW}(\text{reflected})$$
$$\sigma T_D^4 = \sigma T_s^4 + S_0\cos\theta + S_0$$
$$T_D = \left(T_s^4 + \frac{S_0(1+\cos\theta)}{\sigma}\right)^{1/4}$$
$$= \left((273k)^4 + \frac{1353(1+\cos 15)}{5.67\times10^{-8}}\right)^{1/4}$$
$$\simeq 478.6K$$

Case 2 : late at night

$$F_D = F_{LW}^{\uparrow}$$
$$\sigma T_D^4 = \sigma T_s^4$$
$$T_D = T_s = 273k$$

Case 3 : Just before sunrise / Just after sunset with $\theta' = 15°$

$$F_D = F_{LW}^{\uparrow} + F_{SW}'^{\downarrow}$$
$$\sigma T_D^4 = \sigma T_s^4 + S_0\cos\theta$$

where $\theta = 90° - 15° = 75$

$$T_D = \left(T_s^4 + \frac{S_0\cos\theta}{\sigma}\right)^{1/4}$$
$$\simeq 329.1K$$

* Greenhouse / Global warming

→ or MET5421

MET4450: E. Smith (45 minutes to 1 hour, 1989, 1994) **

- The popular conception of the root cause of global warming is the greenhouse influence of ever growing concentrations of trace gases. Considering the radiative issues involved in this problem, provide succinct answers to the following questions. Try and use straightforward, quantitative arguments.
  a. Describe the nature of the greenhouse effect using basic radiative principles.
  b. What are the similarities and differences between the atmospheric greenhouse and a man-made greenhouse? Provide an explanation in terms of directional solar and infrared fluxes and other possible heat transfer mechanisms that have bearing on this problem.
  c. Why would Methane, whose concentration is increasing but whose current background concentration is ≈ 200 times below that of CO_2, be nearly as important as CO_2 in contributing to global warming?
  d. How does polar ozone depletion impact the global warming process? Consider this question from the point of view of a simple two layer atmosphere containing a troposphere and a stratosphere.
  e. What role do clouds and water vapor play in the overall process of greenhouse warming?

a.
"Greenhouse effect" – In clear atmosphere, ~50% of incoming solar radiation is transmitted through atmosphere to be absorbed by sfc. IR radiation emitted by earth is largely absorbed by CO_2, H_2O, & CH_4, which then reradiates → trapping of thermal IR radiation → warming, and is called the "atmospheric effect." It has also been called the "greenhouse effect, but is this analogy accurate?

b. Greenhouses do allow solar radiation (VIS) in, and trap heat → warming. However, are the mechanisms the same? No, actually the heating is attributable to the glass inhibiting convection, that is stopping the warmed air from rising out of the greenhouse and removing heat. The trapping of heat is then mechanical in nature (a physical barrier trapping warmer air which would otherwise rise out), and not due to the absorption of IR radiation.

c.
Methane is a more complex molecule and has a much larger absorption cross section, making it a more efficient absorber. Hence even at much lower concentration, it can effect greenhouse warming comparing with CO_2. Also the level of CH_4 is rising faster than CO_2 (proportionally), so its relative importance may increase even further.

d.
The primary role of concern for O_3 is that O_3 absorbs nearly all UV radiation, allowing only a small amount to reach the sfc. O_3 depletion would result in more solar energy (in the form of UV) to reach the sfc. The first effect would be for more absorption at sfc → warmer sfc temp → more sfc IR emission → warmer lower atmosphere. However the stratosphere is also affected. Absorption of O_3 → heating in the stratosphere, so O_3 depletion → less heating in stratosphere, so stratosphere would cool. When accounting for the meridional distribution of O_3, the meridional temp gradient in the stratosphere would change → general circulation of the atmosphere would be altered. Also, UV is harmful to all life, so depletion → death of vegetation → change in albedo, effecting radiation balence.

e. Role of clouds & H_2O vapor

H_2O vapor plays a similar role as CO_2, that is, it is very active absorber/emitter in IR. However, unlike CO_2, H_2O is not well mixed but highly variable. One general statement can be made, though; the more active weather (e.g. rain & clouds), the more H_2O transport back & forth between ocean sfc & atmosphere → higher concentration of H_2O in the air → more greenhouse warming. However, the main impact is its distribution (that is, where on earth is these higher concentration of H_2O than somewhere else → effects on energy budgets, general circulation, etc.)

Clouds: The effect of clouds is complicated, and as such has not been very well dealt with in climate models w.r.t. global warming. Much depends on cloud type, amount, latitude, and timing (seasonal & diurnal). The net effect of thin cirrus will be to let much of the VIS through, but still absorb/re-emit much of IR → warming. This problem largely ignored by climate modelers. Thicker clouds are even more active in IR, but are less transparent to VIS → opposing contribution (blanket effect → warming below; increased albedo → cooling above). So the level of the cloud also plays a role, effecting the depth of the atmosphere effected. Since more radiation is incident on tropics, cloud effects are different there than higher lats. The same holds true for mid-lats on a seasonal basis (due to earth's inclination). Diurnal cycles are also important. At night there is no incident solar radiation, so clouds prevent cooling to space → atmosphere warming. During day, however, the albedo effect can lower amount of solar radiation absorbed by earth/atmospheric system.

*** broadband flux expression**

MET4450/5421: E. Smith (1 hour, 1993) *****

• The intensity expression for upwelling infrared (IR) radiation in a non-scattering atmosphere is expressed (in monochromatic form) at an arbitrary level τ as:

$$*I_\nu^\uparrow(\tau) = F\uparrow_\nu(\tau) = \underbrace{B_\nu(T_s)e^{-(\tau_s-\tau)/\mu}}_{\text{attenuated sfc emission}} + \overbrace{\int_\tau^{\tau_s} B_\nu(T(\tau'))e^{-(\tau'-\tau)/\mu}d\tau'/\mu}^{\text{attenuated atm emission}} \qquad (1)$$

where T_s and τ_s are the surface temperature and optical depth at the surface respectively. Optical depth extends from 0 at z = TOP-OF-ATMOSPHERE to τ_s at z = 0 and is given with respect to the normal path to the surface; μ is the cosine of the zenith angle. Based on a definition of the exponential integral:

$$\varepsilon_n(\tau) = \int_1^\infty \frac{e^{-\tau\bullet\chi}}{\chi^n}d\chi \qquad (2)$$

derive a broadband flux expression from (1) and (2) involving $\mathcal{E}_2(\tau)$ and $\mathcal{E}_3(\tau)$. [Hint: a change of variable for χ involving μ will help].

Sol)

To obtain flux form intensity, we must integrate over solid angle

$\rightarrow F_\nu(\tau) = \int_0^{2\pi}\int_0^1 I_\nu(\tau,\mu,\phi)\mu d\mu d\phi = 2\pi\int_0^1 I_\nu(\tau,\mu,\phi)\mu d\mu$

We will assume I_ν independent of azimuth(ϕ)

$F_\nu(\tau) = 2\pi B_\nu(T_s)\int_0^1 e^{-(\tau_s-\tau)/\mu}\mu d\mu + 2\pi\int_\tau^{\tau_s}\int_0^1 B_\nu(T(\tau))e^{-(\tau'-\tau)/\mu}\mu d\mu\frac{d\tau'}{\mu}$

Now we note $B_\nu(T(\tau'))$ independent of μ → bring outside inner integral

$\rightarrow F_\nu(\tau) = 2\pi B_\nu(T_s)\int_0^1 e^{-(\tau_s-\tau)/\mu}\mu d\mu + 2\pi\int_\tau^{\tau_s} B_\nu(T(\tau'))\left[\int_0^1 e^{-(\tau'-\tau)/\mu}d\mu\right]d\tau'$

Now we let $\chi = \frac{1}{\mu} \rightarrow d\chi = -\frac{1}{\mu^2}d\mu \rightarrow \chi=1 \rightarrow \mu=1,\ \chi=\infty \rightarrow \mu=0.$

So $\mathcal{E}_n(\tau) = \int_1^\infty \frac{e^{-\tau\chi}}{\chi^n}d\chi = \int_1^0 e^{-\tau/\mu}\mu^n\left(-\frac{1}{\mu^2}\right)d\mu = \int_0^1 e^{-\tau/\mu}\mu^{n-2}d\mu.$

So for $n=2$,

$\mathcal{E}_2(\tau) = \int_0^1 e^{-\tau/\mu}d\mu$

for $n=3$,

$\mathcal{E}_3(\tau) = \int_0^1 e^{-\tau/\mu}\mu d\mu$

$\rightarrow F_\nu(\tau) = 2\pi B_\nu(T_s)\mathcal{E}_3(\tau_s-\tau) + 2\pi\int_\tau^{\tau_s} B_\nu(T(\tau'))\mathcal{E}_2(\tau'-\tau)d\tau'$

For a broad band flux, we must integrate over wavenumber (or wavelength, etc.)

$$\rightarrow \boxed{F_{\Delta\nu} = \int_{\nu_1}^{\nu_2} F_\nu d\nu = 2\pi\int_{\nu_1}^{\nu_2} B_\nu(T_s)\mathcal{E}_3(\tau_s-\tau)d\nu + 2\pi\int_{\nu_1}^{\nu_2}\int_\tau^{\tau_s} B_\nu(T(\tau'))\mathcal{E}_2(\tau'-\tau)d\tau'd\nu}$$

As this is for IR radiation, we may choose to integrate over the IR regime, e.g. $\lambda = 4\text{-}100\,\mu m \rightarrow \nu = .01 \rightarrow .25\,\mu m^{-1}$. Also, we know $\nu = \frac{1}{\lambda} \rightarrow d\nu = -\lambda^{-2}d\lambda$. Hence the above expression for broad band flux (in wavenumber form) could be expressed in wavelength form.

Sol) → see Liou, section 4.3 (pp93-95)

Based on the definition of the exponential integral

$\mathcal{E}_n(\tau) = \int_1^\infty \frac{e^{-\tau x}}{x^n}dx$

$\frac{d\mathcal{E}_n(\tau)}{d\tau} = -\int_1^\infty \frac{-xe^{-\tau x}}{x^n}dx = -\int_1^\infty \frac{e^{-\tau x}}{x^{n-1}}dx = -\mathcal{E}_{n-1}(\tau)$

Now introduce a change of variables

Let $x = \frac{1}{\mu}$ as $x\rightarrow 1, \mu\rightarrow 1$; $x\rightarrow\infty, \mu\rightarrow 0$

$dx = -\frac{d\mu}{\mu^2}$

Then

$\mathcal{E}_n(\tau) = \int_1^\infty \frac{e^{-\tau x}}{x^n}dx = \int_1^0 \frac{e^{-\tau/\mu}}{(\frac{1}{\mu})^n}\left(-\frac{d\mu}{\mu^2}\right) = \int_0^1 \frac{e^{-\tau/\mu}}{1}\cdot\mu^{n-2}d\mu$

$\rightarrow \boxed{\mathcal{E}_n(\tau) = \int_0^1 e^{-\tau/\mu}\mu^{n-2}d\mu}$

We can replace τ with any variable as long as we are consistent. We will find it helpful to consider

$\boxed{\mathcal{E}_n(\tau'-\tau) = \int_0^1 e^{-(\tau'-\tau)/\mu}\mu^{n-2}d\mu}$

In particular for $n=2$ and $n=3$ we have

$$\boxed{\begin{aligned}\mathcal{E}_2(\tau'-\tau) &= \int_0^1 e^{-(\tau'-\tau)/\mu}d\mu \\ \mathcal{E}_3(\tau'-\tau) &= \int_0^1 e^{-(\tau'-\tau)/\mu}\mu d\mu\end{aligned}}$$

Now for a given upward monochromatic intensity (or radiance), the upward, monochromatic flux density (irradiance) is defined by the normal component of I_λ integrated over the entire hemispherical solid angle. It may be written as

$F_\nu^\uparrow(\tau) = \int_\Omega I_\nu\cos\theta\sin\theta\,\overset{\text{zenith angle}}{d\theta}\,\overset{\text{azimuthal angle}}{d\phi}$

We have defined

$\mu = \cos\theta$ → as θ ranges over 0 to $\frac{\pi}{2}$,

$d\mu = -\sin\theta d\theta$ $\quad\mu$ " " 1 to 0.

So

$\int_\Omega I_\nu\cos\theta\sin\theta d\theta d\phi = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_\nu\cos\theta(\sin\theta d\theta)d\phi$

$= \int_0^{2\pi}\int_0^1 I_\nu\mu d\mu d\phi$

So $\longrightarrow$ monochromatic irradiance (flux density)

$$F_\nu^\uparrow(\tau) = \int_0^{2\pi}\int_0^1 I_\nu^\uparrow(\tau)\,\mu\, d\mu\, d\phi$$

$\hookrightarrow$ monochromatic radiance (intensity)

Since IR (terrestrial) radiation is independent of azimuth angle we may evaluate the $\int_0^{2\pi}(\quad)d\phi$ integral $\Rightarrow 2\pi$.

Thus,

$$F_\nu^\uparrow(\tau) = 2\pi\underbrace{\int_0^1 I_\nu^\uparrow(\tau)\,\mu\, d\mu}\qquad (3)$$

$\hookrightarrow$ This represents the radiance (intensity) integrated over zenith angle θ up to level τ in the atmosphere.

If we substitute (1) for the RHS of (3),

$$F_\nu^\uparrow(\tau) = 2\pi B_\nu(T_s)\int_0^1 e^{-(\tau_s-\tau)/\mu}\,\mu\, d\mu + 2\pi\int_\tau^{\tau_s} B_\nu[T(\tau')]\int_0^1 e^{-(\tau'-\tau)/\mu}\,\mu\, d\mu\cdot\frac{d\tau'}{\mu}$$

But we recognize

$$\int_0^1 e^{-(\tau'-\tau)/\mu}\, d\mu = \mathcal{E}_2(\tau'-\tau)$$

and $$\int_0^1 e^{-(\tau_s-\tau)/\mu}\,\mu\, d\mu = \mathcal{E}_3(\tau_s-\tau)$$

Thus

$$\boxed{F_\nu^\uparrow(\tau) = 2\pi B_\nu(T_s)\mathcal{E}_3(\tau_s-\tau) + 2\pi\int_\tau^{\tau_s} B_\nu[T(\tau')]\,\mathcal{E}_2(\tau'-\tau)\,d\tau'}$$

This is the upward monochromatic flux density at depth τ in the atm for wavenumber ν (a single band).

To evaluate the total upward fluxes at level τ for the entire IR spectrum integration over the wavenumber is required

$$F^\uparrow(\tau) = \int_0^\infty F_\nu^\uparrow(\tau)\,d\nu = 2\pi\int_0^\infty B_\nu(T_s)\mathcal{E}_3(\tau_s-\tau)\,d\nu + 2\pi\int_\tau^{\tau_s}\int_0^\infty B_\nu[T(\tau')]\,\mathcal{E}_2(\tau'-\tau)\,d\nu\, d\tau'$$

$\uparrow$ all-band upward flux density (irradiance)

If we limit the integration to a certain range of wavenumbers covering the wavenumbers of peak emission we get a <u>broad band</u> flux density.

$$\boxed{F_{\nu_1,\nu_2}^\uparrow(\tau) = \int_{\nu_1}^{\nu_2} F_\nu^\uparrow(\tau)\,d\nu = 2\pi\int_{\nu_1}^{\nu_2} B_\nu(T_s)\mathcal{E}_3(\tau_s-\tau)\,d\nu + 2\pi\int_\tau^{\tau_s}\int_{\nu_1}^{\nu_2} B_\nu[T(\tau')]\,\mathcal{E}_2(\tau'-\tau)\,d\nu\, d\tau'}$$

We may take $\nu_1 = 100\,\text{cm}^{-1} = \frac{1}{100}\,\text{cm}$ wavelength

$\nu_2 = 1800\,\text{cm}^{-1} = \frac{1}{1800\,\text{cm}}$ wavelength

Evaluation of the double integral above is computationally very expensive as depending on the limits (ν_1, ν_2) we must carry out many line by line calculations. The solution to this has been to consider not monochromatic radiance but rather finite spectral intervals of bands for which the effective transmission functions can be derived by experiment or by theory.

* broadband flux expression

MET? : E. Smith (?)

- Derive a broadband flux expression from the solution of upward intensity:

$$I_\nu \uparrow(\tau) = B_\nu(T_s)e^{-(\tau_s-\tau)/\mu} + \int_\tau^{\tau_s} B_\nu(T(\tau'))e^{-(\tau'-\tau)/\mu}d\tau'/\mu$$

and $$\varepsilon_n(\tau) = \int_1^\infty \frac{e^{-\tau\bullet\chi}}{\chi^n}d\chi$$

involving $\varepsilon_2(\tau)$ and $\varepsilon_3(\tau)$.

Here T_s and τ_s are the surface temperature and optical depth at the surface, respectively. Optical depth extends from 0 at z = TOA to τ_s at $z = 0$ and is given with respect to the normal path to the surface; μ is the cosine of the zenith angle. [Hint: a change of variables for χ involving μ will help].

See the previous question

Sol)

Based upon a definition of exponential integral

$$\mathcal{E}_n(\tau) = \int_1^\infty \frac{e^{-\tau x}}{x^n}dx$$

It is clear that

$$\frac{d\mathcal{E}_n(\tau)}{d\tau} = -\int_1^\infty \frac{e^{-\tau x}}{x^{n-1}}dx = -\mathcal{E}_{n-1}(\tau)$$

i.e. $$\int_1^\infty \frac{e^{-\tau x}}{x^{n-1}}dx = \mathcal{E}_{n-1}(\tau)$$

Noting that

$$x = \frac{1}{\mu}, \quad d\mu = -\frac{dx}{x^2} \quad \left(\begin{array}{l} \mu \to 0, \ x \to \infty \\ \mu \to 1, \ x \to 1 \end{array} \right.$$

then

$$\int_0^1 e^{-(\tau'-\tau)/\mu}\mu\, d\mu = \int_\infty^1 e^{-(\tau'-\tau)x}\left(-\frac{dx}{x^3}\right)$$
$$= \int_1^\infty \frac{e^{-(\tau'-\tau)x}}{x^3}dx$$
$$= \mathcal{E}_3(\tau'-\tau)$$

and similarly,

$$\int_1^\infty \frac{e^{-(\tau'-\tau)x}}{x^2}dx = \mathcal{E}_2(\tau'-\tau)$$

Now from a given upward intensity, we obtain the monochromatic upward flux density as

$$F_\nu\uparrow(\tau) = \int_0^{2\pi}\int_0^1 I_\nu\uparrow(\tau,\mu)\,\mu\, d\mu\, d\phi$$

since IR radiation is not dependent on azimuth angle ϕ

$$= 2\pi B_\nu(T_s)\int_0^1 e^{-(\tau'-\tau)/\mu}\mu\, d\mu$$
$$+ 2\pi \int_\tau^{\tau_s} B_\nu[T(\tau')] \int_0^1 e^{-(\tau'-\tau)/\mu}\cancel{\mu}\, d\mu \frac{d\tau'}{\cancel{\mu}}$$

Consequently

$$F_\nu\uparrow(\tau) = 2\pi B_\nu(T_s)\mathcal{E}_3(\tau'-\tau) + 2\pi\int_\tau^{\tau_s} B_\nu[T(\tau')]\mathcal{E}_2(\tau'-\tau)d\tau'$$

⋮

See the previous question!

* blackbody cloud

MET5421: E. Smith (45 minutes, 1993)

• Consider a thin circular cloud with a radius of 1 km which behaves like a blackbody emitter and with a cloud base temperature of 10°C. How much energy is detected on a square cm aperture directly below the cloud? ↳ 283K

Sol)

As the cloud behaves like a blackbody emitter, we may assume it emits isotropically, and that the flux may be calculated from the Stefan-Boltzmann Law. Hence, flux density at cloud base $\equiv F_c = \sigma T_c^4$. Hence for a detector directly below the cloud, the flux incident upon the detector will also equal F_c. Hence the power detected by the detector = $F_c \cdot$ (cross-sectional area of detector). Let's plug in the numbers:

Power at detector $= P_d = F_c \cdot \text{area} = (\sigma T_c^4)(\text{area})$

$= (5.67\times10^{-8}\ \text{W m}^{-2}\text{K}^{-4})(284\ \text{K})^4(.01\text{m})^2$

$= 3.64\times10^{-2}$ W

As Power = Energy/unit time, to calculate the total energy detected one would have to account for sample time. Hence

→ energy at detector (J) = $(3.64\times10^{-2}$ J/s) (sample time (s))

Note: You may notice I did not utilize the cloud radius (1 km) in the calculation. That's because we are given the detector is directly below the cloud. As 1 km >> 1 cm (size of detector), it would not matter if the cloud were 10 km or 1000 km → the flux would be the same, and the field of view would still constitute that of an entire hemisphere w.r.t. the 1 cm² aperture. The total energy emitted by the cloud of course does depend on the cloud size in this problem. However, the amount of energy intercepted by the aperture directly below the cloud is independent of the cloud size, as long as it is >> detector.

* 1st thermodynamic eq + the role of radiation and cloud processes on global circulation

(Cloud/radiation roles in GCMs)

MET5421: E. Smith (1 hour, 1993)

• Express the first law of thermodynamics as local time derivative of temperature equated to all other relevant terms. First define all terms and then in the context of this law, explain the role of radiation and cloud processes in general circulation methods. Provide reasoned arguments why these processes must be parameterized and what large scale prognostic or diagnostic variables best link the cloud-radiation interactions.

* See Liou, sec. 7.4

Sol)

Recall the first law of thermodynamics :

$$C_p\frac{dT}{dt}-\alpha\frac{dp}{dt}=C_v\frac{dT}{dt}+p\frac{d\alpha}{dt}=\dot{q}$$

where $\dot{q}$ = rate of heating/unit mass due to radiation, latent heat.

In pressure coords, the vertical velocity $\omega=\frac{dp}{dt}$. We may also expand the total derivative of

$$\frac{dT}{dt}=\frac{\partial T}{\partial t}+V_H\cdot\nabla T+\frac{\partial T}{\partial p}\omega$$

Substitute into the first law

$$\rightarrow \frac{\partial T}{\partial t}+V_H\cdot\nabla T+\omega\frac{\partial T}{\partial p}-\frac{\alpha\omega}{C_p}=\frac{\dot{q}}{C_p}$$

$$\rightarrow \frac{\partial T}{\partial t}=\underbrace{-V_H\cdot\nabla T}_{\text{horiz. adv.}}+\omega\Big(\underbrace{\frac{\alpha}{C_p}}_{\text{adiabatic heating}}\underbrace{-\frac{\partial T}{\partial p}}_{\text{vertical adv.}}\Big)+\underbrace{\frac{\dot{q}}{C_p}}_{\text{diabatic heating}}$$

Finally, we may now expand $\dot{q}$ for known sources of diabatic heating.

So we have

$$\frac{\dot{q}}{C_p}=Q_{COND}+Q_{RAD}+Q_{DIFF}$$

→ heating/cooling from eddy thermal diffusion (horizontal + vertical)

→ radiative heating/cooling rate

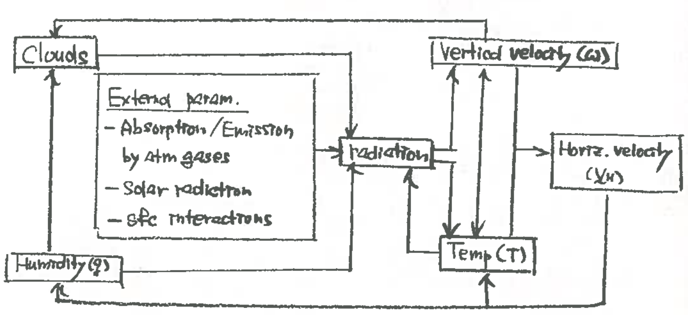
→ condensational heating/cooling rate.

So we see through the temp eq. that clouds + radiation interact with dynamic processes. There are complex feedbacks, best illustrated with a diagram (see below diagram).

(1) Radiation is driven by incoming solar radiation and its interaction with the atmosphere (absorption/emission by atmospheric gases, interaction with aerosols, clouds) and the sfc (reflection/absorption → albedo).

(2) Temperature affects radiation processes by thermal emission and temp. dependence of absorption coeffs.

(3) Radiative flux exchanges produce atm heating/cooling that directly affects local change of temp → indirectly affects vertical velocity.



(4) Temp + vertical velocity fields are then linked to large-scale horiz. velocity (wind) fields, which in turn affect temp, humidity and clouds

(5) Clouds in GCM's are usually determined from humidity and vertical velocity fie[lds]. Through these complex interactions + feedback, we see radiation is the powe[r] of the atmospheric engine.

Parameterization : We note that GCM's are global models, and have a minimum resolutio[n] corresponding to grid size (or wavenumber for spectral models). These models are inten[ded] to represent the general circulation and hence do not depict micro- or some meso-scale phenomena. Also, these small scale events are not sufficiently sampled to provide accurate initial conditions even if we attempted to have a high enough resolution in our model. However we have a problem: the complex interactions represented above do not occur only at the larger scale! In fact, much of the interac[tions] required to drive the general circulation occur due to convection and turbulence, which occurs at scales smaller than that of the model. So what do we do? We attempt to represent the effects of these phenomena based on quantities determined from large scale parameters → this is called parameterization.

So what phenomena must be parameterized?

GCM's first predict large scale variables for a time step (Δt). The two most significa[nt] variables for cloud + radiation param. are T + q. Vertical profiles of T + q ar[e] examined at each grid point for superadiabatic lapse rates ($\gamma>\gamma_d$) and/or supersaturation ($q>q_s$). If either or both conditions exist, the T + q fields are "adjusted" by amounts δT, δq. This simple param. scheme is termed "convective adjustment." $\omega, q\rightarrow$ clouds → (absorption, reflection, transmission).

Phenomena parameterized within GCM's

1) Convection : The physical processes and fields of vertical motion associated with convection occur at sub-grid scale processes, and hence must be param. Large scale properties used to param. convection include atm stability (functions of T + q profil[es]), convergence/divergence fields (function of horiz. wind fields), surface energy flux[es] (themselves param!) etc. These convection param. schemes must include affe[cts] of both deep penetrating convection + shallow convection.

2) radiative fluxes : $F\uparrow$ + $F\downarrow$ must be calculated at each grid-point for the model domain. As absorption/emission is occurring at the molecular scale, obviousl[y] the effects on the general circulation must be param. We note that boundary condition[s] (fluxes at TOA + sfc) are also required. We note that the effects of SW and LW radiation may be handled separately. Although we may assume gases such as N_2, O_2, CO_2, etc. are well mixed, important radiative gases such as H_2O are highly variable in space + time. Hence the large scale q field is important in our radiative param.

3) Cloud amount : Usually param. by condensation, a function of large scal[e] variables of vertical velocity + humidity. Note that most models can account for both stable and convective type clouds.

4) Particle distribution : We note that the effect of clouds + radiation depends on more than volume/area and total moisture. The effects of scattering are heav[ily] dependent of size distribution, as well as shape and phase of cloud particles (liquid, spherical vs frozen, crystalline).

Sol) See Liou Sec 7.4

The 1st law of thermodynamics in terms of temperature, may be expressed by (in $(\lambda, \theta, \sigma, t)$ coord. $\rightarrow \theta$ is co-lat.)

$$\frac{\partial T}{\partial t} = \boxed{-\frac{u}{a\sin\theta}\frac{\partial T}{\partial \lambda} - \frac{v}{a}\frac{\partial T}{\partial \theta} + \left(\frac{RT}{c_p\sigma} - \frac{\partial T}{\partial \sigma}\right)\dot{\sigma} + \frac{RT}{c_p}\frac{d\ln p_s}{dt}}$$

← horizontal + vertical adv. + other related terms which arise in σ coords.

$$\boxed{+\frac{Q}{c_p} + \frac{Q_R}{c_p} + F^T + \frac{g}{p_s}\frac{\partial \tau^T}{\partial \sigma}}$$

→ heat sources/sinks
(These are the diabatic terms

The temperature at a point may be changed due to the horizontal and/or vertical adv. of temperature. These terms are on the first line of the RHS of the above eq. The second line on the RHS contains the diabatic sources and sinks of temperature (heat) at the point. The advection terms can be explicitly resolved in terms of the prognostic variables in the model. The diabatic terms must be parameterized. We have the following diabatic terms

$Q \equiv$ condensational heating/cooling rate

$Q_R \equiv$ radiative heating/cooling rate

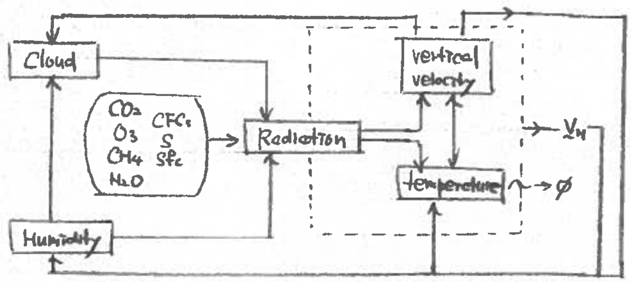
$F^T \equiv$ horizontal eddy thermal diffusion

$\tau^T \equiv$ vertical eddy thermal diffusion.

Radiative processes (Q_R) directly influence the dynamics and thermodynamics of the atmosphere through the generation of radiative heating/cooling rates, as well as net radiative fluxes available at the sfc. For example, the Hadley circulations in the tropics + arctic regions are maintained by differential radiative heating. Radiative flux exchanges are largely controlled by cloud fields. Through phase changes and the subsequent latent heat release, clouds also directly affect temperatures and wind fields.

In the context of a GCM, cloud cover + precip. are generally computed from specific humidity based on empirical threshold methods. Empirical eqs for the evaluation of cloud cover are tuned to available climatological data. The global cloud liquid water content (LWC) data base has not been available, and, therefore, is not a diagnostic variable in GCMs. Interactive radiation calculations require information concerning cloud position, cloud cover, and cloud LWC. Some measure of cloud particle size distribution is also important. Cloud position + cover are given from the model but cloud LWC and cloud type (eg. ice, water, or mixed clouds) must be assumed.

The figure in the next column illustrates the interactions among radiative processes, vertical velocity, temp, humidity, external perturbation parameters (CO_2, O_3, solar constant, aerosols, surfaces, etc), clouds, and large-scale motion. These interactions can be understood from the physical eqs that define dynamic + thermodynamic processes. First, radiative processes are driven by the energy emitted from the sun in terms of the solar constant and are governed by the compositions of the earth and the atmosphere, including the sfc albedo, absorbing gaseous profiles, and clouds (aerosols). Second, temperature affects the radiative processes through thermal emission and the temperature dependence of absorption coefficients. Radiative flux exchanges, in turn, produce atmospheric heating/cooling that directly affects the local time rate of change of temp. and indirectly affects the vertical velocity. Finally, temp. fields and vertical velocity are linked to the large scale horizontal winds that, in turn, affect temp., humidity, and cloud distribution. Basically, clouds and radiation interact with dynamic processes through the temp. eq. The feedbacks involving radiative transfer, cloud fields, and temperature distributions are extremely complex in the real atm. and remain so in their somewhat simplified, parameterized form in GCMs. Many of these feedbacks occur on subgrid scales and so they must be parameterized in terms of large scale variables or derived quantities. The formation of cloud leads to the release of latent heat which also has a significant effect on dynamic processes.



Although GCMs do not require the parameterization of the dynamical motions of the atmosphere, they can not explicitly resolve all the physical processes that occur on spatial scales that are smaller than the resolution employed to solve the model eqs. In GCMs a finite resolved horizontal scale must be specified as well as a vertical scale. Any physical process, such as radiation or cumulus convection, that occurs on scales smaller than this resolution must be represented via parameterization. The only model variables available for the parameterization are the large-scale fields predicted by the model. Relating the subgrid processes to these large scale values usually depends on knowledge of the fundamental physics involved in the process.

GCMs require the following radiative quantities: the net upward + downward radiative fluxes at the top of the atmosphere and sfc and internal atmospheric heating rates. The sfc fluxes are components of the sfc energy balance and contribute to the determination of the sfc temperature. The top of the atmospheric fluxes are required for determining the overall energy budget of the sfc-atm system. The net radiative heating profile is needed for the thermodynamic tendency calculations. This profile is the sum of radiative heating by visible and near infrared wavelength radiation and cooling by longwave radiation.

$$Q_R = Q_{sw} + Q_{lw}$$

The heating terms are proportional to the divergence of the net radiative flux and in terms of $K s^{-1}$

$$Q_{sw} = \frac{g}{c_p}\frac{\partial F_{sw}^{net}}{\partial p} \quad \text{and} \quad Q_{lw} = \frac{g}{c_p}\frac{\partial F_{lw}^{net}}{\partial p}$$

where $F_{sw}^{net} = F_{sw}^{\downarrow} - F_{sw}^{\uparrow}$ + $F_{lw}^{net} = F_{lw}^{\uparrow} - F_{lw}^{\downarrow}$

$F^{\uparrow}$ and $F^{\downarrow}$ are the upward and downward fluxes, respectively.

The object of a radiative transfer parameterization is to calculate F_{sw}^{net} and F_{lw}^{net} for clear and/or cloudy conditions at each gridpoint of the model domain. The processes that need to be parameterized are essentially of molecular scale for gaseous absorption and submicron scale for particular scattering. Since the source of the radiation is quite different for SW + LW radiation, these two processes are considered separately.

As moist parcels rise, cloud formation processes take place that considerably complicate the process of convective overturning. The physical processes and scale of vertical motions involved in the moist convective processes range from mirons to a few kilometers. The range of scales is much smaller than the spatial scales resolved by GCMs. Hence the effects of convective motions on the evolution of the thermal, moisture, and even dynamical properties in the model atmosphere must be parameterized. (Cumulus parameterization). These parameterizations are, in general, for deep clouds that occur in regions of moisture convergence. Over the years, several deep convective parameterizations have been developed for GCMs. Like any parameterization, the unresolved properties must be linked to large scale model variables. This linking process is known in the convective parameterization field as the "closure assumption."

Shallow, non precipitating convection must also be parameterized. Tiedke et al. (1988) describe one such scheme. The vertical flux of eddy moist static energy is assumed to depend on the vertical gradient of large scale variables. This amounts to a diffusion assumption being made about the transport by shallow clouds.

Prediction of cloud amount is of great importance to climate modeling. Clouds play a fundamental role in controlling the amount of solar and IR radiation available to the climate system. It is important to remember that the effect of cloud amount enters into a GCM only via the radiation. Clouds range in size from hundreds of meters to hundreds of kilometers in radius. Thus clouds can span both resolved and unresolved scales in a GCM. Unlike other model variables there is no fundamental prognostic eq. for cloud fraction. The cloud amount in a grid cell must be related to other prognostic variables or quantities derived from them. Cloud fraction must be prescribed in associated with condensation. When stable or convective condensation occurs in a grid box a cloud amount must be assigned. However, clouds can persist after the condensation and so advection of cloud amount should also be considered. Many cloud parameterizations distinguish between convective + stable clouds but few account for advection of cloud amount.

Historically, the most popular prognostic variable used in cloud amount parameterizations is relative humidity (RH). The parameterization may be as simple as defining cloud amount categories based on threshold RH values or may involve more complex linear or quadratic relations. All these schemes, however, are extremely simplistic. Current efforts in cloud parameterization are focused on predicting liquid/ice cloud water (LCW, ICW) in a GCM grid box. These methods are based on prognostic eqs for liquid/ice water formulated in terms of fundamental physical processes. There is still a need to relate LCW to cloud amount or some other radiative quantity.

W.r.t. the coupling of the cloud formation + radiation calculations, not only LWC/IWC but also cloud particle size distribution must be properly taken into account in the GCM. As an example of the importance of particle size we note that for a given LWC small cloud droplets reflect more SW than large droplets. In the case of ice crystal fundamental data on the scattering + absorption of nonspherical ice particles are needed in order to implement a reliable radiation parameterization involvi[ng] cirrus clouds. In the context of radiative parameterization, the asymmetry factor that determines the amount of forward scattering for irregular ice crystals is generally smaller than that for spherical droplets and regular hexagonal ice crystals. The cloud particle size distribution in terms of mea[n] effective size could be inferred from prognostic variables, including LWC/IWC humidity, and temperature. The necessary closure assumptions should be based on sound physical principles as well as on analyses of microphysical cloud data determined from reliable aircraft observations.

* 2-stream solution to the RTE

MET5421: E. Smith (1 hour, 1995)

• Mathematically <u>outline</u> the derivation of a 2-stream solution to the RTE. Be sure to state all of the main assumptions and simplifications. Discuss the conditions under which the δ–transform method is applicable and its advantages over the non-transformed solution. Discuss the respective strengths and weaknesses of the 2-stream technique in atmospheric prediction modeling and terrestrial remote sensing applications.

* 2-Stream flux type solution to the RTE

MET5421: E. Smith (1 hour, 1995)

- Outline the derivation of a 2-stream flux-type solution to the radiative transfer equation in a conservative scattering medium.

* Sum of exponentials technique

MET5421: E. Smith (1 hour)

- Develop the mathematical framework for the "Sum of Exponentials" technique for calculation of absorption by multiple gaseous constituents in a layer of the atmosphere. Explain the strengths and weaknesses of this technique.

Sol)

How do you calculate the total transmission through a layer of the atmosphere, when there may be several gases in that layer that effect the transmission?

Consider a spectral band $\Delta\nu$. We know

$$T_{\Delta\nu}(u) = \frac{1}{\Delta\nu}\int_{\Delta\nu} e^{-k_{a\nu}u}\, d\nu \qquad \left(\begin{array}{l} k_{a\nu} = \text{spectral abs. coeff} \\ u = \text{optical depth } (=\text{path}) \end{array} \right.$$

We now consider $f(k_a)$, a PDF (prob. density function), where $f(k_a)dk_a$ = the fraction of the band for which the abs. coeff. lies between k_a and $k_a + dk_a$ (effectively a weight).

So we have $T_{\Delta\nu} = \int_0^\infty f(k_a)\, e^{-k_a u}\, dk_a$

Now we define a cumulative PDF $g(k_a) = \int_0^\infty f(k)dk \rightarrow$ $k=0$; $g=0$, $k\to\infty$; $g=1$

$\rightarrow T_{\Delta\nu}(u) = \int_0^1 e^{-k(g(k))u}\, dg(k)$

Assuming a finite number of intervals, this integral may be expressed as

$$T_{\Delta\nu}(u) = \sum_{i=1}^{N} e^{-k_i u}(\Delta g_i), \quad (\Delta g_i) = \text{weights}.$$

Hence we can define the transmission of a layer from a single gas as a weighted "Sum of exponentials" of the abs. coeff. (for finite intervals) x path(u). To calculate the transmission for multiple gases, you use the transmission multiplication rule:

$$T_{\Delta\nu}(gas1, gas2, gas3) = T_{\Delta\nu}(gas1)\, T_{\Delta\nu}(gas2)\, T_{\Delta\nu}(gas3).$$

Note: If absorption lines of different gases do not overlap, then we could simply add the abs. for each gas $\rightarrow$ total abs., then calc. T, which would be equiv. to above. However this is invalid for overlap.

Advantages

1) Gain computational speed $\rightarrow$ replace large wavenumber integration with summation over small number of bands (using smooth PDF)

Speed vs. resolution trade off

Disadv.

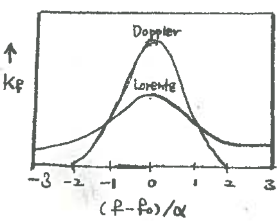
1) Lose spectral resolution – line structure replaced with smooth PDF
2) Lose ability to consider overlap between various lines.
3) Lose ability to correct for non-homogeneous paths (forcing you to employ scaling)

MET?: E. Smith (45 minutes)

• Explain the basic assumptions and procedures used to derive statistical band methods for absorption from the basic Lorentz line shape profile. Consider the regular band model of Elsasser, the random band model of Goody, and the S^{-1} random band of Malkmus in your discussion (provide a discussion of the principles, not formal mathematical derivations). Be sure to explain the meaning and application of the "transmission multiplication rule" to the problem of band modeling and the basic properties of all statistical band models in conjunction with this rule. Finally discuss whether the 3 types of models under consideration must restrict themselves to the Lorentz line shape.

Sol)

First, lets discuss the Lorentz line shape profile. It is due to "pressure broadening", or broadening of the absorption line due to collisions.



$$k_f = \frac{S}{\pi}\frac{\alpha}{[(f-f_0)^2+\alpha^2]} = S\cdot \text{func}(f-f_0)$$

S = line strength, α = halfwidth

f_0 = frequency of line

S normalized such that $\int_{-\infty}^{\infty} k_f\, df = S$

Half width depends on $P, T \rightarrow \alpha = \alpha_0 \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right)^{1/2} \rightarrow$ pressure broadening. $\alpha_0 = \alpha(STP)$

Recall absorption $A_\nu = 1 - \tau_\nu = 1 - e^{k_\nu \rho ds}$, $k = k(S, f-f_0)$ from above.

However for a band, we must the contributions of multiple lines + gases?

So is the transmission of a band ($\Delta\nu$) simply the product of the transm. of the gases? No! To explain, we consider the following example:

$A_{12} = A_1 + A_2$ $\quad\neq\quad$ $\tau_{12} = \tau_1\tau_2 = (1-A_1)(1-A_2)$

$\rightarrow \tau_{12} = 1 - A_1 - A_2$ $\qquad\qquad = 1 - A_1 - A_2 + A_1A_2$

So for our 3 important IR gases (H_2O, CO_2, O_3), in general,

$\tau_{\Delta\nu 123} \neq \tau_1\tau_2\tau_3$ due to partial overlapping and interdependence of τ's in some cases. However monochromatically, $\tau_{\nu 123} = \tau_1\tau_2\tau_3$.

So we must consider the process in a near-monochromatic fashion.

Typically a broadened line width $\sim 0.5\ cm^{-1}$. To consider $\sim$ monochromatic, we break up the broadened line into ~ 100 subdivisions. So for a typical band used ($\Delta\nu = 5\times10^4\ cm^{-1}$) $\rightarrow 100 \times 5\times10^4\ cm^{-1}/0.5\ cm^{-1} \rightarrow 10^7$ calcs! When considering the entire spectrum, this become unworkable. Hence the need for band models.

So when does the transmission multiplication rule hold? That is, under what conditions does $\tau_{123} = \tau_1\tau_2\tau_3$?

Only two statistical distribution situations exist in which this holds:

(1) Random line spacing

(2) Regular line spacing (line spacing of seperate gases are not correlated).

We'll now briefly discuss 3 different band methods

(1) Elsasser Regular band model: Assumes regular spacing of lines, and that successive lines of the same gas do not significantly overlap each other. So we can calc. $\bar{A} \propto \Sigma A_i$, summed over lines. Due to pressure broadening, there is some overlap $\rightarrow$ must assume lines of equal intensity $\rightarrow$ $A_S = A\Delta\nu/S$, S = line spacing. Assume Lorentz line shape for $K_{\Delta\nu}$. Use in calculation of transmission function, which must be numerically equal.

(2) Goody random band model: Assumes random line spacing. Consider $\Delta\nu = n\bar{S}$, n = number of lines, $\bar{S}$ = average line spacing. Then calculate mean transmission function (using trans. mult. rule), and assume each of the n intervals are equivalent for a sufficiently small $\Delta\nu$! Goody assumes the Lorentz line shape, and a Poisson distribution for the PDF.

(3) Malkmus random band model: Assumes random line spacing. Similar to Goody (Lorentz shape, uses PDF for line intensity). This model uses a different PDF. The Poisson dist. (i.e.–Goody) under estimates low intensity lines, like those for far IR. Malkmus chooses a distribution function where $d\nu c\ S^{-}$ This unfortunately leads to discont. problems at the end points, but this can be handled by quantifying the PDF differently at diff. regimes of the S(strength) dist.

As we have already mentioned, these band models are needed for computational requirements, as well as to account for the random nature and finite widths of the line behavior. We must be aware of the drawbacks and limitations, though:

1) Models cannot be better than lab data they use

2) Models are limited as they have only used Lorentz line shape

3) As they are statistical in nature, accuracy dependent on PDF used

4) Realize that results have a statistical uncertainty.

5) In general, models do not handle line wing absorption outside of the band under consideration.

* Newtonian cooling (mean meridional circulation)

MET5421: E. Smith (1 hour, 1995)

• Discuss the concept of Newtonian cooling, how it is formulated, and why it is used as a radiative dissipation scheme in such modeling problems as describing the general circulation of the stratosphere, or describing the mean meridional circulation (zonally symmetric flow). Also, what aspects of perturbed flows cannot be investigated or understood within the constraints of the Newtonian cooling approach? You may use specific examples.

* See Liou → sec 2.7.4 → p90.

Sol)

Method: Consider RTE for IR (neglect scattering) → Schwarzchild eq when considering $F\uparrow, F\downarrow$ (allow $F\downarrow(TOA)=0$ for IR), in transmission form, you get

$$F_{\bar{\nu}}\uparrow = \pi B_{\bar{\nu}}(T, z=0)\,{}^{o}T_{\bar{\nu}}^{F}(z) + \int_0^z \pi B_{\bar{\nu}}(z')\frac{d}{dz'}\,{}^{o}T_{\bar{\nu}}^{F}(z-z')dz'$$

$$F_{\bar{\nu}}\downarrow = \int_{z(TOA)\to\infty}^{z} \pi B_{\bar{\nu}}(z')\frac{d}{dz'}\,{}^{o}T_{\bar{\nu}}^{F}(z'-z)dz'$$

→ Net flux $= F_{\bar{\nu}}^{*} = F_{\bar{\nu}}\uparrow(z) - F_{\nu}\downarrow(z)$

$$= \pi B_{\bar{\nu}}(z_0)\,{}^{o}T_{\bar{\nu}}^{F}(z) + \int_0^{z(TOA)\to\infty} \pi B_{\bar{\nu}}(z')\frac{d}{dz'}\,{}^{o}T_{\bar{\nu}}^{F}(|z-z'|)dz'$$

Recall IR cooling rate

$$\equiv \left(\frac{\partial T}{\partial t}\right)_{IR} = \int_0^\infty \left(\frac{\partial T}{\partial t}\right)_\nu d\nu = -\frac{1}{\rho C_p}\frac{dF(z)}{dz}$$

Consider now an atm. with an isothermal temp. profile → local cooling rates are produced solely from the emission of a local layer →

$$\left(\frac{\partial T}{\partial t}\right)_{space}(z) = \frac{1}{\rho C_p}\pi B_{\bar{\nu}}(z)\frac{d}{dz}\,{}^{o}T_{\bar{\nu}}^{F}(z_\infty - z)$$

(Cooling to space (CTS) approx: most suitable for upper atm)

$$\to Q = \left(\frac{\partial T}{\partial t}\right)_{IR} = \underbrace{\left.\frac{\partial T}{\partial t}\right)_{space}}_{\text{func. of level temp}} + \underbrace{\left.\frac{\partial T}{\partial t}\right)_{\text{layer exchange}}}_{\text{func. of temp profile}}$$

Now we'll outline a param. scheme based on the CTS approx in which perturbations in radiative cooling may be expressed in terms of temp. perturbations. Let T_0 = std temp, ΔT = temp. pert.
Q_0 = " Q, ΔQ = pert.

$$\frac{dQ}{dT} \simeq \frac{Q(T_0+\Delta T) - Q(T_0-\Delta T)}{2\Delta T} = a_0(z),\quad a_0 = \text{Newtonian cooling coeff.}$$

$$Q = Q_0 + \Delta Q = Q_0 + a_0(z)\Delta T$$

The Newtonian coefficients are calculated by doing band calculations over relevant absorbers (CO_2, O_3, etc) These can be calc. apriori, and hence the simple param. (as a func. of temp.) can be exploited in GCMs and other modelling applications.

Applications to stratosphere:

The general circulation of the strat. (~15–90 km) is driven by differential vert. + horiz. radiative heating. Abs. of solar insolation is primarily by O_2 + O_3. This abs. largely balanced by radiative cooling throughout strat. by emission/abs. of IR rad. by CO_2 + O_3. Calc. of IR cooling by CO_2 + O_3 can be very calc. (from line-by-line, to band modelling). The CTS approx. is a more simple method, whose computational req. are more suitable to atm. modeling applications.

Limitations: This param. scheme uses pre-calc. Newtonian coeffs as a function of temp. Hence, the gases are assumed to be well mixed. Hence this approach could not be used in the troposphere, where the effects of clouds + H_2O are significant, while their distrib. + conc. are hardly variable

* Sun-synchronous satellite

MET5471 (Planetary Atmospheres): ? (?, 1998)

• Explain the nature of a sun-synchronous satellite orbit in quantitative terms, and explain how and why such an orbit can be achieved by an artificial satellite in Earth orbit. Conclude by describing the advantages and disadvantages of a sun-synchronous low earth orbiting satellite versus a geosynchronous orbiting satellite in conjunction with making measurements of the Earth's atmosphere.

* Venus, Earth, Mars — radiative, thermodynamic, dynamic behavior

MET? 5471 (Satellite MET and Planetary atmospheres): E. Smith (?) *

- Compare and contrast the radiative, thermodynamic, and dynamic behavior of the atmospheres of the three principle inner planets-Venus, Earth, Mars. In your discussion be concise and quantitative. You may use diagrams, however, be sure to label the axes and indicate the units of any variables. Carefully focus on the important aspects of the atmospheres which makes it possible to equate or to distinguish their behaviors.

Sol)

Planet	sfc pressure (x1000 mb)	T_{sfc} (K)	Major gases (%)
Mercury	10^{-15}	440	He(98), H(2) → Solar wind (no atm)
Venus	90	730	CO_2(96), N_2(3.5)
Earth	1	288	N_2(77), O_2(21), H_2O + Ar(1)
Mars	.007	218	CO_2(95), N_2(~3)

Radiation properties/Greenhouse:

planet	Planetary Albedo	T_{sfc}(K)	Equiv. Temp(K)	Diff(K)
Venus	~0.7	~750	~250	~500
Earth	~0.3	~300	255	~45
Mars	~0.15	~230	225	~5

→ varies quite a bit both (1) diurnally and (2) clear vs dust storms.

* Satellite vertical temp profile retrieval

MET5471: E. Smith (1 hour)

• Explain the principles of retrieving the vertical temperature profile of the atmosphere using a filtered radiometer at infrared wavelengths flown on a satellite looking down at the atmosphere. Discuss all aspects of the problem (essential wavelengths, fundamental principles of technique, nature of measurements, influence of surface, behavior of weighting functions, etc.). The mathematical development of the Radiative Transfer Equation needed for a physical relaxation solution.

Consider this same problem, but now with the radiometer placed on the earth's surface looking up. What is the nature of the weighting functions.

*Satellite retrieval eq. (temp. profiles)

5471
MET?: E. Smith (?) *

- If we write the satellite retrieval equation in the following form:

$$N_{\Delta\nu}(p_o) = B_{\Delta\nu}(T_s)\tau_{\Delta\nu}(p_s) + \int_{\ln(p_s)}^{\ln(p_o)} B_{\Delta\nu}(T_P)\frac{\partial \tau_{\Delta\nu}(p)}{\partial \ln p}\, d\ln p$$

Where $B \equiv$ Planck function
$\tau \equiv$ Transmittance
$p_o \equiv$ Pressure at satellite altitude
$p_s \equiv$ Pressure at surface
$T_s \equiv$ Temperature at surface

a) How is a profiling radiometer designed to obtain temperatures at different levels in the atmosphere?

b) What is $\partial\tau_{\Delta\nu}(p)/\partial\ln p$ and how does it behave as a function of height (a properly labeled diagram will suffice)?

c) How does the vertical structure of the $\partial\tau_{\Delta\nu}(p)/\partial\ln p$ term change as $\Delta\nu$ is shifted toward the center of a strong absorption band; how does it change as $\Delta\nu$ is shifted away from the center of the absorption band? What is the physical explanation for these changes? Are the selection of $\Delta\nu$, for temperature retrieval purposes, arbitrary?

d) By application of the mean value theorem, derive a basic relaxation equation which could be used solve for the temperature profile iteratively (physical retrieval). What assumptions and simplifications must be made?

— LIOU 7.3.4 p 263-264.

Sol)

Recall Schwartzchild eq (no scattering into path) — consider upwelling case where $\tau_s = \tau$ at sfc, $\tau_0 = \tau$ at TOA ≈ 0

$$\rightarrow I_\nu(\tau) = I_\nu(\tau_s)e^{-(\tau_s-\tau)} + \int_\tau^{\tau_s} B_\nu(T(\tau'))e^{-(\tau'-\tau)}d\tau'$$

$$\rightarrow I_\nu(\mathrm{TOA}: \tau=0) = I_\nu(\tau_s)e^{-\tau_s} + \int_0^{\tau_s} B_\nu(T(\tau'))e^{-(\tau'-\tau)}d\tau'$$

In transmission ($\mathcal{T}$) form

$$\rightarrow I_\nu(\mathrm{TOA}) = I_\nu(\tau_s)\mathcal{T}_\nu(\tau_s) + \int_{\mathcal{T}_s}^{1} B_\nu(T(\mathcal{T}'))d\mathcal{T}'$$

So in press. coords

$$\rightarrow I_\nu(\underset{p=0}{\mathrm{TOA}}) = I_\nu(P_s)\mathcal{T}(P_s) + \int_{P_s}^{0} B_\nu(T(p))\frac{\partial\mathcal{T}(p)}{\partial p}dp$$

Finally in lnP coords

$$\rightarrow I_\nu(\underset{\ln p=0}{\mathrm{TOA}}) = I_\nu(P_s)\mathcal{T}(P_s) + \int_{\ln P_s}^{0} B_\nu(T(p))\frac{\partial\mathcal{T}(p)}{\partial \ln p}d\ln p$$

(assume $P = 1$mb at TOA $\rightarrow \ln p = 0$)

Assumptions made in temp. sounding problem

1) Emitting constituent is well mixed. $\rightarrow$ radiance a function of T, λ (or ν), and $\mathcal{T}$ (transmission) $\rightarrow$ $\mathcal{T}$ can be pre-calculated for diff. $\Delta\nu$ bands.
2) Selection of $\Delta\nu_i$ should not overlap strong abs. lines of other gases.
3) Conditions of L.T.E. are preferred (not good assumption above ~ 45-50 km)
4) Use a climo. profile $\rightarrow$ solve for deviation $\delta T_{prof} = T_{prof} - \overline{T}_{prof}$
5) Assume good estimate of sfc temp. (use indep.-retrived T_{sfc})
6) Expand Plank func. using Taylor series expension (dropping high order terms)
$\rightarrow$ Choose band pass $\Delta\nu_i$ narrow enough that this is good approx.

$$\Rightarrow B_{\Delta\nu}(T_{prof}) \simeq B_{\Delta\nu}(\overline{T}_{prof}) + \delta T_p\frac{dB_{\Delta\nu}(\overline{T}_{prof})}{dT}$$

Substitute into above Schwartzchild eq in lnp coord., group known/meas. quantities on left,

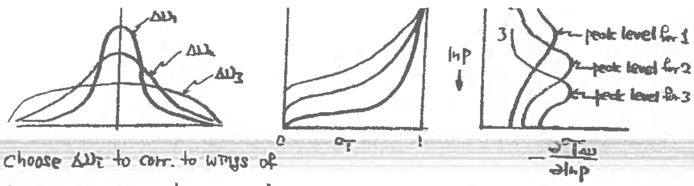
$$\rightarrow I_\nu(P_o) - I_\nu(P_s)\mathcal{T}(P_s) - \int_{\ln P_s}^{0} B_\nu(\overline{T})\frac{\partial\mathcal{T}}{\partial\ln p}d\ln p = \int_{\ln P_s}^{0}(\delta T)\frac{\partial B_{\Delta\nu}(\overline{T})}{\partial T}\frac{\partial\mathcal{T}}{\partial\ln p}d\ln p$$

So problem reduces to finding δT_i over N spectral bands ($i = 1, N$). This is a non-linear inversion problem, with severals options to reach solution.

Recall Schwartzchild eq in lnP coords.

$$\rightarrow I_\nu(P_o) = I_\nu(P_s)\mathcal{T}(P_s) + \int_{\ln P_s}^{0} B_\nu(T(p))\left[\frac{\partial\mathcal{T}(p)}{\partial\ln p}\right]d\ln p$$

The term $\left[\frac{\partial\mathcal{T}(p)}{\partial\ln p}\right]$ is normally referred to as the weighting function, and when multiplied by the Planck function represents the contribution of radiance from the layer centered at pressure p. So the weighting function responds to atm. transmission, which is a function of the absorb. of atm. constituents at that spectral band. The key is the selection of spectral bands. We choose the region surrounding a strong abs. band of CO_2, a well mixed gas. We choose each successive $\Delta\nu_i$ (corresponding to a separate channel on our satellite radiometer) to be further away from the central line. This takes advantages of the pressure broadening effect and allows us to calculate each retrieved temp. with a level in the atm. $\longrightarrow$ sounding!



Choose $\Delta\nu_i$ to corr. to wings of successive pressure-broadened lines.

Near center of abs. band : Here the weighting function has a peak which is symmetric about the maximum (near TOA)

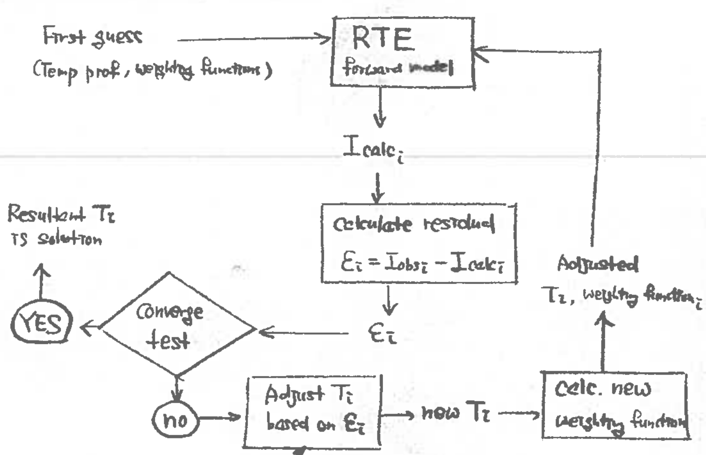
Away from center of abs. band : Here the peak of the weighting function is lower in the atm, and no longer symmetric about the max.
near wings

Retrieval for satellite pointing down : The shape of the weighting function depends on (1) increase in σT as $p \to 0$ (hence $\tau \to 1$), and (2) increase in pressure as path (τ) increases → pressure broadening effect. For $\Delta\nu$ near the center of the absorption band, the weighting function is sharply peaked near the level where the absorber conc. begins to become significant (in this case, CO_2, is mostly confined to the troposphere, so the first sample peaks near the tropopause). As we move further from the center, we are sampling the wings due to pressure-broad, where the peak of the weighting function is shifted near the sfc.

Retrieval for ground-based radiometer pointing up : The weighting functions will be very different than for the satellite case. Here the path originates at the sfc, so that the transmission function decrease as the path → 0. Hence, the weighting functions will all peak near the sfc! This makes sense in that pressure is max at sfc. Hence, each channel further from the central band samples deeper layers of atm, but all are peaked at sfc.

Recall pressure broad : $\alpha = \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right)^{1/2}\alpha_0$, $\alpha_0 = \alpha(STP)$
using CO_2 absorption line, which is well mixed → can calculate $\tau_{\Delta\nu}$ apriori!!

Physical Retrieval



Relaxation eq. used to adjust profiles → new profiles

Advantages
1) physically consistent
2) Does not require a large DB

Disadvantage
1) Computationally intensive
2) Relies heavily on accuracy of weighting functions
3) Solution may be influenced by first guess (depending on convergence criteria)
4) Contains no knowledge of atms statistical properties

Statistical retrieval

Collect K sets of data involving radiosonde temps and satellite radiances
Assume $i = 1, M$, M = # of sat. channels
$j = 1, N$, N = # of temp. layers.
Allow $\Delta T = T - \overline{T}$, and $\Delta I = I - \overline{I}$ → residuals
Hence the problem may be defined as

$\Delta T = C \Delta N$, ΔT + ΔN residual matrices from K cases.
C = matrix relating ΔT, ΔN, which contains least squares coeff.

Hence C is calculated from training data. The retrieval procedure is then
(1) Measure radiances, calc. ΔI residual.
(2) Multiply by coeff (C matrix) → ΔT
(3) add $\overline{T}$ → solution profile

Advantages
1) Once C is calculated, producing solutions is computationally fast.
2) Based on real statistics of atm
3) No initial guess required

Disadv.
1) No physics — statistical only
2) Results only as good as calibration (training) data →
3) can't be sure what specific prob. is described by statistics
4) Mindless, boring, no science ...

* Notes on temp. profile retrieval from satellite measurements.

Main points regarding temp. profile retrievals.

① assume uniformly mixed constituent gas.
② Transmission function can be pre-calculated at your different $\Delta\nu_i$ (wavenumber ban)
③ Selection of $\Delta\nu_i$ should not overlap strong absorption bands of other gases.
④ It is nice if the condition for LTE (local thermodynamic equilibrium) are met.
LTE assumes the following.
(i) mechanical equilibrium – no unbalanced force in the interior of the local system or between the system + the surroundings.
(ii) chemical equilibrium – local system does not tend to undergo a spontaneous change in its internal structure (no chemical reactions, no diffu
(iii) thermal equilibrium – the local system is isothermal with no flux at its boundary

In LTE, Kirchoff's law applies
$E_\lambda = a_\lambda B_\lambda(T)$
or in words

$\left(\begin{array}{l}\text{the emitted radiation}\\\text{at wavelength } \lambda\end{array}\right) = \left(\begin{array}{l}\text{the absorptance of}\\\text{the substance at } \lambda\end{array}\right) \times \left(\begin{array}{l}\text{radiation of a blackbody}\\\text{at temp. } T \text{ and wavelength } \lambda\end{array}\right)$

→ Kirchoff's law states that materials that are strong absorbers at a particular λ are also strong emitters at that λ. Similarly weak absorbers are weak emitters.

→ Kirchoff's law begins to fail in the upper atmosphere (≥ 50 to 60 km) becaus the mean free path is large enough that the frequency of molecular collisio is on the order of the frequency of individual absorption + scattering elements.

⑤ $\Delta\nu_i$ are selected to take advantage of collision broadening.

A temperature retrieval eq. takes the form

$$N_{\Delta\nu}(P_s) = B_{\Delta\nu}(T_s)\,\tau_{\Delta\nu}(P_s) + \int_{\ln P_s}^{\ln P_0} B_{\Delta\nu}(T_p)\frac{\partial \tau_{\Delta\nu}(p)}{\partial \ln p}\, d\ln p \quad (1)$$

where
- $B \equiv$ planck's function
- $\tau \equiv$ transmittance
- $P_0 \equiv$ pressure at satellite altitude
- $P_s \equiv$ sfc pressure
- $T_s \equiv$ sfc temperature.

We want to solve for $T_p \Rightarrow$ the temp. profile as a function of pressure. We measure $N_{\Delta\nu_i}(P_0)$ for $i = 1, 2, \ldots, M$ channels. We are faced with a highly nonlinear inversion problem.

We make 3 assumptions

① insert a climatological profile $\overline{T_p}$ and solve for the deviation profile

$$\delta T_p = T_p - \overline{T_p}$$

② We have a good independent estimate of T_s.

③ For a narrow band pass, $\Delta\nu$, we can expand the planck function using a Taylor series expansion about $\overline{T_p}$:

$$B_{\Delta\nu}(T_p) \simeq B_{\Delta\nu}(\overline{T_p}) + \delta T_p \frac{dB_{\Delta\nu}(\overline{T_p})}{dT} \qquad (2)$$

If we sub. (2) into (1) we get

$$N_{\Delta\nu}(P_0) = B_{\Delta\nu}(T_s)\tau_{\Delta\nu}(P_s) + \int_{\ln P_s}^{\ln P_0} \left(B_{\Delta\nu}(\overline{T_p}) + \delta T_p \frac{dB_{\Delta\nu}(\overline{T_p})}{dT}\right) \frac{\partial \tau_{\Delta\nu}(P)}{\partial \ln p}\, d\ln p.$$

We can write the preceding eq. such that on the RHS all we have is the unknown departure temp. profile. Doing so we get

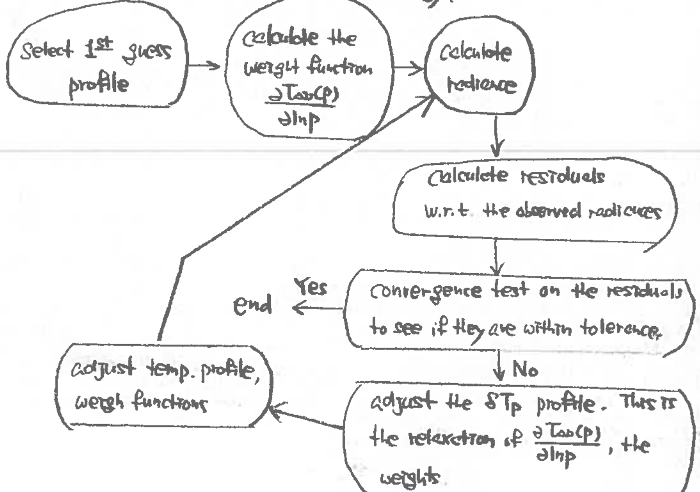
$$N_{\Delta\nu}(P_0) - B_{\Delta\nu}(T_s)\tau_{\Delta\nu}(P_s) - \int_{\ln P_s}^{\ln P_0} B_{\Delta\nu}(\overline{T_p}) \frac{\partial \tau_{\Delta\nu}(P)}{\partial \ln p}\, d\ln p$$

$$= \int_{\ln P_s}^{\ln P_0} \delta T_p \frac{\partial B_{\Delta\nu}(\overline{T_p})}{\partial T} \frac{\partial \tau_{\Delta\nu}(P)}{\partial \ln p}\, d\ln p$$

or

$$LHS_{\Delta\nu} = \underbrace{\int_{\ln P_s}^{\ln P_0} \delta T_p\, K(\Delta\nu, p)\, d\ln p}_{\text{Friedholm integral of the 1st kind}}$$

We must use numerical techniques to "solve" this inverse problem. Below is a flowchart of a solution strategy.



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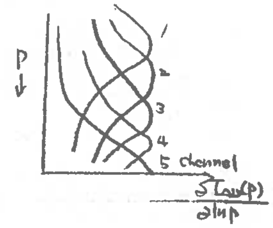
a)

We can design a set of narrow detectors where the spectral response is well behaved & peaks toward the center of the band. (ex. ⇒ 6 different frequencies for channels 1-6)

b)

$\frac{\partial \tau_{\Delta\nu}(P)}{\partial \ln p}$ is the transmission weighting function.

The general shape of the weighting function for various channels is shown below



c)

- $\frac{\partial \tau_{\Delta\nu}(P)}{\partial \ln p}$ is the weighting function. When multiplied by the planck function it represents the contribution of radiance from the layer at pressure p.
- When $\Delta\nu \to$ center of absorption band, $\frac{\partial \tau_{\Delta\nu}}{\partial \ln p} \to$ maximum value and is symmetric about this max.
  - → measured radiance mainly coming from the layer near the tropopause with little or no contribution from the lower troposphere.
- When $\Delta\nu$ shifts away from the center of an absorption band, $\frac{\partial \tau_{\Delta\nu}}{\partial \ln p} \to$ asymmetric because some of the radiance comes from the lower troposphere and thus its max is located in the lower troposphere
- For the purpose of temp retrieval $\Delta\nu$'s should be chosen to be near the centers of strong absorption bands.
- For a space borne radiometer looking down the shape of the weight function is governed by two factors.
  - ① the decrease in absorber gas with changing altitude.
  - ② the increase in transmission as the path decreases to zero as we approach the altitude of the satellite.

Together these combine to produce a familiar bell-shaped weighting function. The width of this curve characterizes the vertical resolution of the retrieval with the peak of the curve occurring at $\frac{\partial \tau_{\Delta\nu}}{\partial \ln p} = 0$. Thus measurements at different frequencies allow us to sample the emission from different layers in the atm. (layers centered around the peaks of the weighting function). Near the center line the weighting function is sharply peaked. As we move away from the line center and toward the wings the peak broadens and shifts toward the s

- For a sfc based radiometer looking up, the weighting function is very different from that of a satellite borne radiometer. The shape of these weighting function always maximizes at or near the sfc. The shape of the weighting function profile also varies with the change in the absorber with distance above the sfc and the decrease of transmission as altitude increases upward away from the instrument. An instrument looking up almost always receives most of its radiation from layers adjacent to the ground. In line centers this radiation almost always originates from layers near the sfc whereas radiation is received from higher up in the atm. for wavelengths located in the line wings. Max. emission occurs near the sfc where pressure is greatest & the line is broadest.

d)

Start with the retrieval eq. equated to the observed radiance

$$\text{measured } N = N_i - B_i(T_s)\tau_i(P_s)$$

$$= B_i[T(P_i)]\left[\frac{\partial \tau_i(P)}{\partial \ln p}\right] \Delta \ln P_i \qquad ①$$

Where P_i is the pressure at the max weight value for the given channel.
Let the P_i layer temp be $T'(P_i)$
Then consider

$$N_i' - B_i(T_s)\mathcal{T}_i'(P_s) = B_i[T'(P_i)]\left[\frac{\partial \mathcal{T}_i'(p)}{\partial \ln p}\right]\Delta \ln P_i \qquad ②$$

Assume the relationship between temp. & the Planck function is much larger/important than that between the temp. & weight function.
So

$$\frac{N_i - B_i(T_s)\mathcal{T}_i(P_s)}{N_i' - B_i(T_s)\mathcal{T}_i'(P_s)} \simeq \frac{B_i[T(P_i)]}{B_i[T'(P_i)]} \qquad ③$$

When sfc emission is the main source & small, then

$$\frac{N_i}{N_i'} \simeq \frac{B_i[T(P_i)]}{B_i[T'(P_i)]} \qquad ④$$

for channels $i = 1, 2, \cdots, M$.
Recall the planck function

$$B_i(T) = a\nu_i^3 / (e^{b\nu_i/T} - 1) \qquad a = 2hc^2, \quad b = hc/k$$

Steps to iteratively solve
(1) pass $n = 0$. Set $T_0'(P_i)$
estimate N_i' from eq ② for $i = 1, 2, \cdots, M$ channels.
(2) Compare N_i' and N_i.
If residual is small, stop → we have the temp. profile.
Otherwise go to (3)
(3) Use eq. (3) to solve for $T(P_i)$ and use this as our next guess $T_1'(P_i)$
(4) Increment pass counter. Goto (1)
Loop until we converge or exceed max # of iterations.
We obtain the final profile by linear interpolation between the retrieved temperatures.
Assumptions we make in this scheme
- know weight function
- CO_2 composition is uniform with height
- we measure in M separate channels.

Simplifications
- neglect sfc radiance
- $\Delta \ln P_i$ is not too large.

We want T_p (temp profile retrieval) from measurements $N_{\Delta\lambda}(P_0)$.
From the forward RTE

$$N_{\Delta\lambda}(P_0) = B_{\Delta\lambda}(T_s)\mathcal{T}_{\Delta\lambda}(P_s) + \int_{\ln P_s}^{\ln P_0} B_{\Delta\lambda}(T_p)\frac{\partial \mathcal{T}_{\Delta\lambda}}{\partial \ln p}\, d\ln p$$

where

$B_{\Delta\lambda}(T) = B_{\bar{\lambda}}(T) = \overline{B}_\lambda(T)$ ← broad band Planck function.
↳ highly nonlinear.

To obtain the temp. profile through inversion of the above eq. we assume
(i) Ignore scattering ($\tilde{\omega} = 0$) → use Schwarzchild's form of the RTE
(ii) design a set of narrow detectors.
(iii) precalculate $\mathcal{T}_{\Delta\lambda}$ & the weighting function $\frac{\partial \mathcal{T}_{\Delta\lambda}}{\partial \ln p}$
(iv) use a 1st guess profile of temp.
→ Climatology, radiosonde data, forecast product, ad hoc (isothermal atm., for example)
(v) We have a good, independent estimate of T_s.

(vi) Taylor series expansion of $B_{\Delta\lambda}(T_p)$ about our 1st guess $\overline{T}_p$ profile.

$$B_{\Delta\lambda}(T_p) \simeq B_{\Delta\lambda}(\overline{T}_p) + \underbrace{\delta T_p}_{T_p - \overline{T}_p}\frac{dB_{\Delta\lambda}(\overline{T}_p)}{dT}$$

The procedure for the retrieval is as follows.
Move everything to LHS of the retrieval eq. except

$$LHS = \int_{P_s}^{P_0} \underbrace{\frac{\partial \mathcal{T}_{\Delta\lambda}(p)}{\partial p}}_{\text{presumably known}} \frac{\partial B_{\Delta\lambda}(\overline{T}_p)}{\partial T} t(p)\, dp$$

$$LHS(\Delta\lambda) = \int_{P_s}^{P_0} \underbrace{K(\Delta\lambda, p)}_{\text{kernel, a matrix}} \underbrace{t(p)}_{\text{a vector}}\, dp$$

Given $i = 1, 2, \cdots, M$ pseudo-independent channels.

$$LHS_i = \int_{P_s}^{P} K_i(p)\, t(p)\, dp$$

In finite difference form

$$LHS_i = \sum_{j=1}^{N} K_i(P_j)\, t(P_j)\, \Delta_j P \quad \text{for } i = 1, 2, \cdots, M$$

we can use EOFs to determine $t(p)$

$$t(p) = \sum_{j=1}^{M} t_j \underbrace{W_j(p)}_{\text{basis function}}$$

Define a kernel function

$$A_{ij} = \sum_{j=1}^{M} K_i(P)\, \Delta_j P\, W_j(p) \quad \text{for } i = 1, 2, \cdots, M$$

we get

$$\widehat{LHS}_i = \sum_{j=1}^{M} \hat{A}_{ij}\hat{t}_j$$

Solve this for t_j ⇒ this will be your retrieval.

$$\hat{t}_j = \hat{A}_{ij}^{-1}\, \widehat{LHS}_i$$
↳ there is measurement noise in here.

We can solve the inverse problem using
(1) Direct linear inversion.
(2) Constrained linear inversion
(3) Statistical covariance method
(4) Backus-Gilbert inversion method.

MET5471: E. Smith (1 hour)

• The standard expression for the retrieval of surface temperature from split window measurements is given by the following regression equation:

$$T_s = a_0 + a_1 T_{11} + a_2 (T_{11} - T_{12})$$

where T_{11} and T_{12} are the measured clean and dirty window temperatures, and the a_i are best-fit regression coefficients developed from some training data set. Show how this basic formulation can be derived from first principles and in the process, interpret the role of the a_0, a_1, and a_2 coefficients in the above equation.

Sol)

$T_{11} = T(10.5-11.5\,\mu m \text{ band})$ = "clean" window

$T_{12} = T(11.5-12.5\,\mu m \text{ band})$ = "dirty" window → dirty due to H_2O vapor abs.

Recall notation: $N_\nu = I_\nu$ = monochrometic intensity / radiance

Write Schwarzchild's eq in transmission form

$$N_\nu = B_\nu(T_s)\tau_\nu + \int_{\tau_{sfc}}^{1=TOA} B_\nu(T(\tau'))\,d\tau'$$

where we define effective atm temp T_a as that which satisfies

$$\int_{\tau_{sfc}}^{1=TOA} B_\nu(T(\tau))\,d\tau = B_\nu(T_a)\int_{\tau_{sfc}}^{1} d\tau' = (1-\tau_\nu)B_\nu(T_a)$$

If the sfc/atm system behaves as a blackbody, we can relate the measured temp. at TOA to the above eq. →

(1) $N_\nu = B_\nu(T_m) = B_\nu(T_s)\tau_\nu + B_\nu(T_a)(1-\tau_\nu)$

we can now write eqs for the two spectral intervals in this problem.

→ (2) $N_{\nu_1} = B_{\nu_1}(T_{m_1}) = B_{\nu_1}(T_s)\tau_{\nu_1} + B_{\nu_1}(T_{a_1})(1-\tau_{\nu_1})$

(3) $N_{\nu_2} = B_{\nu_2}(T_{m_2}) = B_{\nu_2}(T_s)\tau_{\nu_2} + B_{\nu_2}(T_{a_2})(1-\tau_{\nu_2})$

As $\nu_1 \simeq \nu_2$ here, we can make the following assumptions:

a) Weighting functions are nearly equiv. → $T_{a_1} \sim T_{a_2}$ ($|T_{a_1} - T_{a_2}| < 1°C$)
→ T_a invariant

b) sfc emittance ~ invariant (not good assumption for land)

c) Diff. in τ due to differential H_2O vapor absorption.

So we can expend functions by Taylor series,

$$B_\nu(T) = B_\nu(T_a) + \frac{\partial B_\nu}{\partial T}(T-T_a)\cdots$$

Substitute and make lots of algebraic manipulations

→ (4) $B_{\nu_1}(T_s) = B_{\nu_1}(T_{m_1}) + \left[\frac{1-\tau_{\nu_1}}{\tau_{\nu_1}-\tau_{\nu_2}}\right][B_{\nu_1}(T_{m_1}) - B_{\nu_1}(T_{m_2})]$

we note $1-\tau_{\nu_1} = a_{\nu_1}$, $1-\tau_{\nu_2} = a_{\nu_2}$

→ (5) $B_{\nu_1}(T_s) = B_{\nu_1}(T_{m_1}) + \frac{a_{\nu_1}}{a_{\nu_2}-a_{\nu_1}}[B_{\nu_1}(T_{m_1}) - B_{\nu_1}(T_{m_2})]$

→ (6) $T_s = EBBT_1 + C\,[EBBT_1 - EBBT_2]$

From the preceding final form of the eq., we move to fitting real data. Training data sets are used with independent measured SST's and satellite multichannel data. When fit to the form we developed via linear regression, we get solution as

$$T_s = a_0 + a_1 T_{11} + a_2(T_{11} - T_{12})$$

where a_0 = bias

a_1 = slope (≈ 1)

a_2 = corresponds to diff. abs., and coeff. C defined earlier

Note: a_0 is sometimes used to convert SST to °C from T_{11}, T_{12} in K.
(So a_0 has -273.15 added to bias)

* Satellite SST estimates.

1/

MET5471: E. Smith (1 hour)

• Describe the following three satellite remote sensing techniques to obtain SST estimates:

a) 1-channel IR window technique

b) 2-channel split window technique

c) 3-channel split window + 3.7 μm window technique

In your descriptions, focus on the central assumptions and simplifications, as well as the main advantages and disadvantages of the techniques.

* rainfall retrieval

MET? (?): E. Smith (?)

- What are the relative advantages and disadvantages of retrieving rainfall from:

a) Raingauges
b) Radar
c) Infrared satellite imagery
d) Microwave satellite imagery

Sketch the climatological zonally averaged profile of precipitation for the globe and discuss how this profile could be expected to vary as a function of season, between land and ocean and as a function of longitude.

* Precipitation retrieval from satellite

5471

MET? (Satellite MET and Planetary atmospheres): E. Smith (?) *

- Precipitation retrieval from satellite

a) Describe the physical principles involved in precipitation retrieval based on the use of:
   i) the VIS-IR approach.
   ii) microwave brightness temperature approach.

b) Discuss the advantages and disadvantages of each technique.

c) Why is the microwave technique far more effective for the case of an ocean background as opposed to a land background?

d) For warm rain microphysics, which technique is more sensitive to the total liquid water content of the cloud? Explain.

Sol)

- Precip. retrieval – VIS/IR

physical principles : basic assumption – sfc rainfall related to depth of cloud and hence cloud top temp. This ~ holds for convective rainfall → deep, tall Cb towers. Field experiments have found high correlation of OLR for large areas (~100 x 100 km) to area-averaged rainfall.

One simple IR index is the GPI (GOES Precip Index), where

{ GPI (mm) ∝ $A_c \Delta t$, A_c = fractional area (0–100%) of cloud colder than threshold temp (235K) in a box ; Δt is the time duration.

This application has shown utility for results average in time & space → climo studies.

This approach works best in tropics, where convection accounts for most of rainfall.

Advantages of VIS/IR

1) Simple in concept (rain ∝ cloud top temp)
2) Measured from geostationary satellite → high temporal resolution. → easier to produce global datasets.
3) Easy to automate
4) Reasonable correlation for tropical convective rainfall.

Disadv. of VIS/IR

1) requires averaging ⟶ poor spatial resolution.
2) Basic assumption is flawed ; not directly sensing precip.
3) Poor for stratiform rain
4) Confuses upper cloud for deep convection.
5) No physics, no science ; basically, shit...

- Precip. retrieval – microwave

physical principles : First of all, the atm is quite transmissive at microwave freq (except for major obs. by O_2 at 60, 120 GHz, and minor obs. by H_2O vapor at 22 GHz). Moreover, $\lambda \gg D$ for cloud particles ⟶ retrieval not affected by most clouds !! However, as particle cross sections get larger (raindrops, ice), MW radiation is attenuated due to scattering, especially for higher frequencies (e.g. $f > 60$ GHz → O_2 band). Another aspect is that precip. is an active emitter at MW frequency (especially, low freq.). When viewed over a ~ homogeneous, cold ocean background, precip. appears in low freq as warmer (in TB) than the ocean. Hence this aspect allows sensing of low-level, stratiform precip. (Note : Over land, changes of TB can be due to changes in sfc emission, which is more variable and higher than over ocean → Hence it's hard to separate the precip. signature)

Retrieval schemes : (1) Physical – involve inversion of measured TB's through RT model ; (2) Statistical – correlate measured TB's to rainfall rate or other parameters.

Advantages : (1) Directly sensing hydrometeors ! (2) Attenuation problems are managable ; (3) Statistical retrievals easy to automate ; (4) more accurate than VIS/IR for wider range of precip. domains.

Disadvantages : (1) Data from polar orbiters → poor temporal, spatial coverage.
(2) large fields of view → poor horiz. resolution, beam filling problem.
(3) Poor contrast for warm rain over land → separate ocean & land algorithms
(4) RR (TB) is non-unique.

→ See the attached note ! (next page)

Note from Russ.

See the attached notes !

Technique	Measurement	Principle	Advantages	Disadvantages	Nature of the Sample
Infrared and/or Visible	Cloud detection	rainfall intensity inferred indirectly from cloud top temperature and/or cloud brightness	good resolution	not very quantitative	good coverage in space & time from geostationary platforms
Passive μ-wave scattering mode	Ice detection	rainfall intensity inferred indirectly from scattering by large ice particles aloft	works over both land & sea	not very quantitative	currently mounted on polar orbiters only
Passive μ-wave emission mode	Rain detection	rainfall intensity estimated from brightness temp. of rain column of known depth	useful accuracy over sea across a wide swath	poor spatial resolution and does not work well over land	Size of the instruments is such that they are restricted to polar orbiting platforms and thus provide poor coverage in space & time.
Active μ-wave (radar)	Rain detection	3D rainfall intensity estimated from attenuation and/or scattering by raindrops	potentially good horizontal & vertical resolution. works over land & sea	limited swath width, not yet tried in space	
		Table from Browning 1990 QJRMS vol 116 (p. 1025-1052)			(3 of 3)

Question: Precipitation retrieval from Satellite

Response: Sections 7.4 of Stephens' Remote Sensing text.

VIS/IR

Satellite infrared (IR) image data have been used extensively over the past two decades in an attempt to derive rainfall. The basic idea of this approach assumes that the rainfall data is related in some way to the depth of the cloud and thus to cloud top temperature. Since measurements of IR emissions @ window wavelengths provide a way of estimating this temperature, the ~~OLR~~ outgoing longwave radiation (OLR) is assumed to be related to rainfall.

Estimates of the OLR have been available since 1974 from the window channel measurements available from operational NOAA polar-orbiting satellites. It was quickly realized that OLR provides a ~~quantitative~~ qualitative index of tropical convection. Over large, fixed areas in excess of 100 km x100 km the OLR is found to be significantly correlated to rainfall. Arkin (1979) and Richards & Arkin (1981) estimate that 50 to 70% of the variance of areally averaged rainfall accumulations measured during GATE is explained by a linear function of the mean fraction of the area covered by cloud w/ equivalent black body temperatures colder than thresholds ranging betw 220 & 250K. Largest values of explained variance were found for a brightness temperature threshold of 235K and for areal averages defined by a grid ~~2.5 lat~~ 2.5° lat/lon on a side. These gross statistical correlations suggest ~~a~~ that OLR precipitation estimates may play some useful role in climatological studies of precipitation.

Arkin & Meisner (1987) applied this threshold method to IR imagery from the GOES satellite to arrive @ a rainfall estimate according to

$$GPI = 3A_c +$$

where GPI is the rainfall estimate (GOES Precipitation Index) ~~and is~~ expressed in mm. A_c is the fractional area (0% to 100%) of cloud colder than 235 K in a 2.5° lat/lon box. t is the length of the period (hours) for which A_c was the mean fractional cloudiness.

Studies suggest that OLR and rainfall are more highly correlated in the tropics than in the mid latitudes. This may be due to the ~~fact that most tropical precipitation~~ higher frequency of convective precipitation in the tropics. ~~Global~~ Tropical monthly mean rainfall maps produced using GPI show that the technique reproduces most features ~~of~~ in the tropical precipitation patterns ⇒ ITCZ, SPCZ, heavy precip over Indonesian maritime continent, Amazon Basin, Congo basin.

Microwave

Although retrieval of rainfall from microwave (μwave) emission measurements suffers from its own ambiguities, these measurements in principle offer a more direct way of estimating rainfall than do OLR measurements. The μwave technique exploits the direct consequences of the interaction betw μ-wave radiation & precipitation-sized hydrometeors. This interaction is characterized by the optical depth τ^* associated w/ the emitting rain drops. The basis for estimating rainfall lies in an assumed relationship between this optical depth and the rainfall rate and a suitable contrast between the emission from the surface & the emission from the raindrops. Emission from raindrops, when viewed against a cold ocean background, increases w/ increasing optical depth. This increased emission, measured as an increased brightness temperature, is then associated w/ the rain rate R.

We can explore a specific example of the relationship between R & τ^* by assuming a Marshall - Palmer size distribution

$$n(D) = N_0 e^{-\Lambda D}$$

where $N_0 = 0.16\ cm^{-4}$

$\Lambda = 81.56\, R^{-0.21}$

R is the rainrate in mm/hr.

It therefore follows ~~that the~~ by the definition that the volume extinction ~~of the extinction coefficient~~ coefficient is

$$\sigma_{ext} = \frac{N_0}{4}\int_0^\infty n(D)\pi D^2 Q_{ext}\, dD$$

Thus, it is straightforward in principle to derive the brightness temperature associated w/ upwelling radiation as a function of τ^* and thus as a function of R.

DUNING QJRMS 1990 Vol 116 pp 1025 to 1052

The μ-wave radiometers used for producing rainfall measurements generally operate @ window frequencies betw 5 & 200 GHz. These provide precip information based on two physical mechanisms. At frequencies below 20 GHz emission (absorption) predominates ($\therefore$ use low frequencies to sample warm cloud rain). At frequencies above 60 GHz scattering predominates. Rainfall intensity can be inferred directly from the emission from the column of rain itself and indirectly from the decrease in upwelling radiation caused by scattering from ice particles in the tops of convective clouds. Methods involving both emission & scattering can be employed over the sea. However, over land measurements of emissions from rain are confused by the high and variable emissivity of land surfaces. The use of a lower frequency enables heavier rain ~~rate~~ to be measured.

A. ~~an~~ real issue of concern is the highly variable in space (and time) nature of rainfall w/in an instrument's field of view. This problem is often refered to as the problem of beam filling. The transfer function relating rainfall rate to brightness temperature is highly nonlinear. ~~and~~ Therefore, small scale inhomogeneities w/in the instantaneous field of view can produce ~~a~~ biases in the rainfall estimate. ~~Further~~

Radiometers operating @ different μ-wave frequencies respond in different degrees to the precip @ different levels w/in the cloud. Thus, physically based μ-wave rainfall retrievals, if undertaken using a set of frequency channels, offer the important advantage of providing an indication of the vertical profile of precip. A byproduct of such a vertical precip profile is the surface rainfall intensity. The principle of the retrieval process is similar to that of temperature retrievals from satellite radiances.

Advantages of passive μ-wave

① based on physics ⇒ detecting hydrometeors.
② attenuation problems are manageable
③ fairly easy to automate on computer unless large cloud model involved
④ global dataset provided but at longer timescale than VIS/IR
⑤ TRMM

Disadvantages of passive μ-wave

① from low orbiters ∴ poor ~~spatial~~ sampling in time
② large fields of view,
③ cloud liquid water & ice attenuation
④ melting snow & ice
⑤ poor contrast over land
⑥ ↳ ∴ separate land & ocean algorithms
⑦ brightness temp to rainfall relationships are not generally unique

Rules of Thumb for μ-wave

① can't make measurement if land & ocean are together in field of view
② radar verifications suggest factor of two error in most algorithms
③ problem verifying raingauge (point measurements) w/ area averaged measurements.

* precip. retrieval

MET5471: E. Smith (1 hour, 1995)

• Explain the strengths and weaknesses of the following 2 instruments for estimating precipitation from a satellite nadir view:

1. a 4-channel passive microwave radiometer using frequencies at 10.7, 19, 37, and 85 GHz,
2. a dual-channel precipitation radar at 14 and 37 GHz.

Provide a brief explanation of how measurements from these two instruments could be used in combination to improve precipitation estimates over single instrument methods.

* microwave radiometer-bases / radar-based methods for precip. retrieval

MET5471: E. Smith (1 hour)

• Discuss the strengths and weaknesses of microwave radiometer-based and radar-based methods for precipitation retrieval. Describe the two proposed TRMM methods, i.e. (1) PIA constrained Hitshfeld and Bordan solution to radar equation and (2) tall vector inversion method, in which radiometer and radar measurements can be combined to make precipitation estimates. Explain why these two methods might improve upon sigle instrument methods. Use mathematical terms and precise quantitative and physical arguments in answering this question.

* Satellite-mounted radiometer

547

MET?: E. Smith (?)

• Imagine a satellite-mounted radiometer looking straight down at the earth's surface through an isothermal atmosphere at temperature 257°K. Assume that scattering is negligible and the surface temperature is 300°K. The optical depth of the atmosphere is given as a function of wavelength in the following table:

wavelength in μm	optical depth
4-10	10.0
10-12	0.0
12-14	1.0
14-20	10.0

Put x's on Fig. 1 indicating the approximate monochromatic radiance measured by the satellite at 4.1, 9.9, 10.1, 11.9, 12.1, 13.9, 14.1, and 20 μm.

Connect the points to graph $I\uparrow(\lambda, \tau = 0)$.

Repeat the procedure for a ground-based radiometer viewing the zenith sky on Fig. 2.

* global energy transport

MET? 5471 (Satellite MET and Planetary atmospheres): E. Smith (?) **

• Global Energy Transport

a) Diagram the mean annual zonal profiles of outgoing longwave radiation, reflected solar radiation, and net radiation (Q^*). Provide quantitative estimates of the minimum and maximum values of these terms (specify units).

b) Describe how to integrate the quantity Q^* so as to arrive at the required zonal transport of energy by the atmosphere-hydrosphere system. Diagram the resultant transport function.

c) Approximately how much of the required transport is accomplished by the atmosphere relative to the ocean?

d) What is the relationship between the latitude of maximum transport by the atmosphere to the latitude of maximum transport by the oceans (assume hemispherical symmetry)?

MET5471: E. Smith (1 hour)

- Given the expression:

$$C\frac{\partial T(\mu,\lambda,t)}{\partial t} = K(\mu,\lambda) - L(\mu,\lambda) - \nabla \bullet F(\mu,\lambda,z)$$

where C is thermal inertia for a column of atmosphere-ocean, t is time, z is height, μ is cosine of latitude, λ is longitude, K is absorbed solar flux density, L is outgoing infrared flux density, and $\nabla \bullet F$ is heat flux divergence within the atmosphere-ocean system:

a. Develop an expression for the required zonal energy transport using Green's theorem (Gauss's divergence theorem) for steady state conditions.

b. Provide schematic diagrams of the zonally averaged distribution (pole to pole) of solar radiation absorbed by the earth-atmosphere system, the infrared radiation emitted by the earth-atmosphere system, and the resultant zonal distribution of net radiation. Then illustrate the required poleward transport of heat for steady state climate. Finally, illustrate the zonally averaged distribution of planetary albedo. Label the axes of your diagrams with appropriate units and approximate magnitudes of the quantities in question.

a.

$C\frac{\partial T}{\partial t} = K - L - \nabla\cdot E = 0$ for steady state (Note: θ = lat, ϕ = lon)

Let $Q^* = K - L \longrightarrow Q^* = \nabla\cdot E$ Integrate over volume

$\iiint Q^* dV = \iiint \nabla\cdot E\, dV = \iint E\cdot\hat{n}\, dA$ for DIV theorem.

we note that Q^* is a volume integrated quantity $\longrightarrow$ Integral over volume reduces to area integral. Noting that in spherical coords

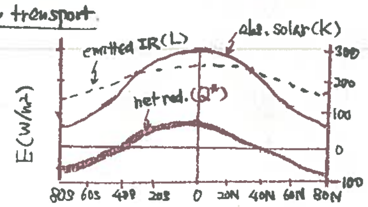
$$dA = r^2\cos\theta\, d\theta\, d\phi \Big|_0^{2\pi}$$

$$\rightarrow \overline{F}(\theta) = 2\pi \overline{R_e}^2 \int_0^{\pi/2} \overline{Q^*}(\theta)\cos\theta\, d\theta$$

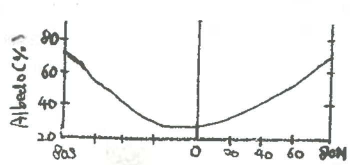
where overbar indicates zonal average.

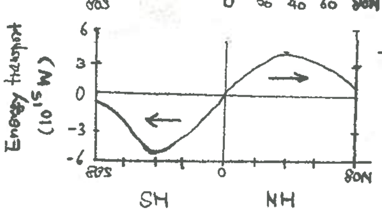
b.

- Zonal average of absorbed solar, emitted IR, net rad, albedo, meridional energy transport.



These diagrams are annual average. Seasonal values shift from hemisphere-hemisphere due to seasonal variation of incident solar radiation.





→ note: transport defined positive for northward. Hence poleward in both NH + SH.

→ see Liou Sec. 8.3.3

- How much of transport accomplished by atm. relative to ocean?

Energy transport (zonal) accomplished by

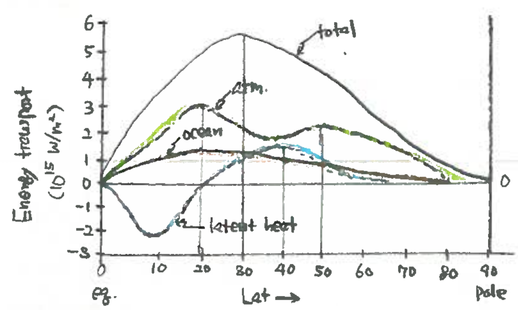
1) direct radiative heating/cooling (small wrt other terms when average over time)
2) Condensation of H_2O vapor
3) Horiz. flux on sensible heat carried by atm.
4) " " " " ocean

Percentage of transport accomplished by each mechanism:

Atm. transport	~ 50%
Ocean transport	~ 25%
Latent heat	~ 25%

) → Atm. trans ~ 2x ocean trans.

- Compare lat's of max transport for each mechanism



1) Both atm. + ocean transport have max near 20° lat, corr. to Hadley cell.
2) Latent heat has min near 10° lat due to excess precip in ITCZ.
3) Atm transport has a secondary peak in mid-lat, while latent heating contribution declines → both due to baroclinic zones/disturbances.

* large-scale water vapor-precipitation processes

MET5471: E. Smith (1 hour) → see Physics of climate, sec 3.6 p58, Sec 12.3 - p298

- This is a 4-part question on large-scale water vapor-precipitation processes.

a. Beginning with the assumption of water vapor balance and the velocity divergence form of the continuity equation, develop a general mass balance equation for water vapor (ignore liquid and frozen states). Assuming annual mean/steady state conditions, vertically integrated to show how water vapor transport is associated with the residual between evaporation and precipitation (E-P).

b. Diagram the mean zonal-averaged profile of E-P. Explain the underlying factors governing the behavior of these quantities, particularly addressing the respective roles of the atmosphere, hydrosphere, and lithosphere.

c. What is the relationship between surface salinity of ocean water and the mean annual profile of E-P? Explain.

d. Consider a mean value of precipitable water in the atmosphere of 2.5 g cm^{-2} and a mean global evaporation rate of 1 m yr^{-1}. If you assume global hydrological balance for an annual cycle, what is the mean residence time of water vapor in the atmosphere?

a. Mass balance eq. for water vapor : Liou sec. 4.4 ; Physics of climate Sec 3.6 + 12.22

Consider continuity eq. in two diff. forms:

(mass DIV form) $-\frac{\partial \rho}{\partial t} = \nabla \cdot \rho \underline{V}$; (velocity DIV form) $-\frac{1}{\rho}\frac{d\rho}{dt} = \nabla \cdot \underline{V}$

For water vapor (v), we can write $-\frac{1}{\rho_v}\frac{d\rho_v}{dt} = \nabla \cdot \underline{V}$

We know $q = \frac{\rho_v}{\rho} \rightarrow \rho_v = \rho q$ (substitute into cont. eq.)

$\rightarrow \frac{d(\rho q)}{dt} = -\rho q \nabla \cdot \underline{V} + S_v$

← sources/sinks of vapor

Linearize (allow quantities = mean + eddy term); subst. into cont. eq.; Solve for $\frac{d\bar{q}}{dt}$

$\rightarrow \frac{d\bar{q}}{dt} = \frac{S_v}{\bar{\rho}} + D$, D = eddy diffusion term

S_v = sources/sinks = Evap(E) − Cond(C)

An anal. of typical atm → diffusion term small wrt E + C.

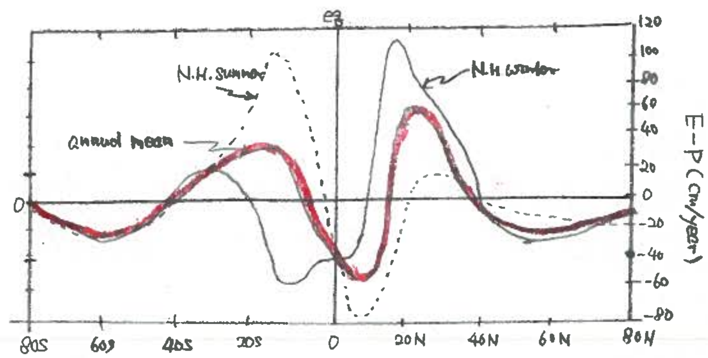
$\rightarrow \frac{d\bar{q}}{dt} \simeq E - C$

A typical assumption made in modeling is that vapor falls out as precip. after condensation → Precip (P) = C

Hence, $\boxed{\frac{d\bar{q}}{dt} \simeq E - P}$

Note: This eq. is integrated from sfc to TOA ($P_s \rightarrow P = 0$) to yield total column values. Hence E, q are column values, and P now corresponds to total P → precip at sfc!

b. Zonal average of (E−P) : Phys. of climate, fig 12.16, p298



- Roles of Atm., hydrosphere (ocean), lithosphere (land)

These 3 "spheres" are interrelated in their roles of water vapor - precip. cycle. The oceans provide a source of water vapor (through evap.), while both the oceans + land act as a sink for liquid water. Note that the process over land is a bit more involved. Rain which falls is not absorbed totally where it falls; it may be transported through the process of run-off (both above + below sfc) → River systems play a large role. The atm., through the general circ., provides a vehicle for global transport of water vapor. Let's now look at our E-P diagram.

E − P > 0 { constitutes source regions of vapor (DIV of vapor); more evap. than precip.

E − P < 0 { sink regions of vapor (CONV of vapor); more precip. than vapor

The meridional distribution of (E−P) in the diagram corresponds well to known aspects of the atm. general circulation and land/ocean processes.

- The sharp negative peak near the equator corresponds to the ITCZ, where its enhanced rainfall provides for a sink of vapor. Note the meridional progression north (south) in N.H summer (winter), and relative strength (stronger in N.H. summer) → known property of ITCZ.

- The positive peaks in the subtropics correspond well to the descending branch of the Hadley cell → suppresses precip. As the warm sub-tropical oceans provide for ready source of vapor through evap (replenished by ocean currents), this region is ~~there fore~~ a moisture source region. Seasonal change of peaks corresponds to behavior of subtropical highs.
- The negative peaks in mid-lat assoc. with transport baroclinic Lows (synoptic systems). With accompanying precip. → vapor sink.

c. Role of salinity

Large amount of rain in equatorial tropics (assoc. with ITCZ) will dilute ocean water ⟶ decrease salinity. Large evap. which occurs in subtropics will remove fresh water ⟶ increase salinity.
Hence we expect a correlation of sea surface salinity with water vapor divergence : high salinity ↔ DIV of water vapor (source region)
low " ↔ CONV " " " (sink ")
This is in fact what is observed (max salinity at sfc in sub-trop. oceans)

d. Mean residence time (in terms of PW)

we know $PW(kg/m^2) = \rho(kg/m^3)\,PW(m)$

So mean $PW = \overline{PW} = 25\ kg/m^2$

$Evap\,(kg/m^2\,yr) = \rho(kg/m^3)\,Evap\,(m/yr) = (10^3\,kg/m^3)(1\,m/yr)$
$= 10^3\ kg/m^2\,yr$

Global hydrological balance ⟶ Precip = Evap.

So $10^3\,kg/m^2 yr$ falls out as precip. Hence the mean residence time ($\overline{T}_{Res}$) of the amount of water present in the atm ($\overline{PW}$) is given by

$$\overline{T}_{Res} = \frac{25\,kg/m^2}{10^3\,kg/m^2\,yr} = 0.025\,yrs \simeq 9\ days.$$

* 1-layer atmospheric water budget.

MET5471 (Planetary Atmospheres): Smith (1.5 hour, 1998)

• Write system of equations for 1-layer atmospheric water budget of closed ocean basin. Explain how such a system could be evaluated using: (a) conventional measurements; and (b) satellite measurements. Be sure to distinguish between the terms that can be diagnosed directly from measurements and which terms (if any) would have to be obtained by residue.

* relaxation method for solving a remote sensing inversion problem

MET5471 (Planetary Atmospheres): Smith (1 hour, 1998) *

• Explain the basis for and the implementation of the relaxation method as a means for solving a remote sensing inversion problem. Outline the derivation of a relaxation-based solution for the retrieval of atmospheric temperature using radiance measurements from a multispectral grating radiometer.

MET5471: E. Smith (1 hour, 1995)

• Explain conceptually, using mathematical terms and concisely worded statements, the differences between:

a) classical statistical-based inversion
b) physical inversion using iterative relaxation
c) physical inversion using forward RTE model in conjunction with variational control technique.

Discuss the main advantages and disadvantages of three techniques. Fell free to use specific examples in this discussion.

* thermal weighting function

MET5421 (Radiative Transfer): Smith (1 hour, 1997, 1998) **

- Derive an expression for a thermal weighting function from first principles within the context of radiative transfer. If one measures thermal radiances at successively greater spacings away from the Q-branch of the CO_2 fundamental around 15 μm, why do the altitudes of the weighting function maxima monotonically decrease?

MET5471: E. Smith (1 hour)

- This is a 4-part question on thermal weighting functions.

a. Define a thermal weighting function by developing a solution to the non-scattering version of the 1-dimensional radiative transfer equation (Schwarzchild form) and identifying the term normally referred to as the weighting function.

b. Explain in mathematical terms why a thermal weighting function contains a localized peak at some distinct vertical level in the atmosphere.

c. Explain the physics of why a sequence of radiometer frequencies extending away from the Q-branch in the 15 μ CO_2 fundamental would enable retrieving the temperature profile if pointed down at the atmosphere from a space platform.

d. If the same radiometer were placed on the ground and pointed up at the atmosphere, would the weighting functions be similar or different. Could the temperature profile be retrieved in this fashion or not. Explain.

* Eric Smith — Russ's note

- Flux or irradiance (luminosity) is power per unit area.

$$F = \frac{P}{4\pi r^2} \qquad F_\lambda = \int_0^{2\pi}\int_0^{\pi/2} I_\lambda(\theta,\phi)\cos\theta\,(\underbrace{\sin\theta\, d\theta\, d\phi}_{d\omega})$$

↳ $F \propto r^{-2}$

- Intensity or radiance is given as $N = \frac{E}{\Delta\Omega}$; $\Delta\Omega = \frac{A_c}{r^2}$ so $N = \frac{F r^2}{A_c}$

↳ (luminence or brightness) ↳ solid angle

N is indep. of r

- Radiant power is irradiance integrated over an area A

$$P_{\Delta\lambda} = \int_A F_{\Delta\lambda}\, dA \quad \leftarrow \text{units of watts} = \frac{J}{s}$$

- Radiant energy is radiant power integrated over time

$$E_{\Delta\lambda} = \int_t P_{\Delta\lambda}\, dt \quad \leftarrow \text{units of Joules.}$$

- Relationship between frequency (f), wavelength (λ) & the speed of light (c) & wavenumber (ν)

$$c = f\lambda \ , \quad \lambda = \frac{1}{\nu}$$

consider

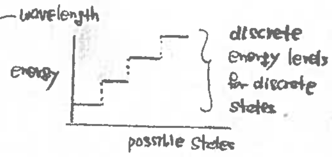
$$\frac{dF_\lambda}{d\lambda} = \frac{dF_\nu}{d\nu}\frac{d\nu}{d\lambda} \qquad \frac{d}{d\lambda}\left(\frac{1}{\lambda}\right) = -\frac{1}{\lambda^2}$$

so

$$(-\lambda^2)\frac{dF_\lambda}{d\lambda} = \frac{dF_\nu}{d\nu} \Rightarrow -\lambda^2 I_\lambda = I_\nu \quad \text{and} \quad \int F_\nu\, d\nu = -\int F_\lambda \frac{d\lambda}{\lambda^2}$$

- $c = f\lambda = \frac{f}{\nu}$ ← frequency , $\nu = \frac{1}{\lambda}$ ← wavelength (wavenumber)

↑ speed of em radiation ~3×10^8 m/s in vacuum



em spectrum

x ray 0.01μm — UV — 0.4μm VIBGYOR visible — 1.0μm near IR — IR 10μm — μ-wave 10^3–10^5 μm (1 to 10cm)

high frequency, high energy ... low frequency, low energy

- line spectrum — quantum theory maintains that atoms can change their energy levels only thru discrete amounts of energy (photons). Thus molecules can only interact with molecules with certain energy amounts.

- absorption/emission associated with orbital changes → Xray, uv, visible (most energy)
  - " " " " vibrational " → IR
  - " " " " rotational " → μ-wave (least energy)

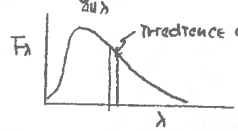
- UV & visible radiation can provide energy for photochemical reactions.
  → photons provide or take away energy necessary for change in molecular structure.

- Xray & UV can provide enough energy to strip electrons from atoms → ionize atom.

- Width of absorption lines may be increased by
  (a) Doppler broadening ⇒ random molecular motions leads to a range of velocities ⇒ the effect is proportional to K.E. of the molecules which in turn is proportional to $\sqrt{T_{absolute}}$
  (b) Collision broadening ⇒ results from interactions between electrostatic force fields of individual molecules during collisions. This effect depends upon the frequency of collisions which is proportional to the pressure of the gas.

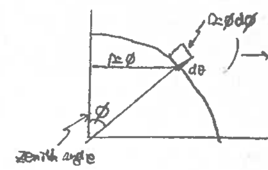
- Radiant flux – rate of energy transfer by em radiation (J/s = Watts)
- Irradiance (F) is radiant flux/area → W/m^2
- monochromatic irradiance (F_λ) is irradiance per unit wavelength

note that $F = \int_{\Delta\lambda} F_\lambda\, d\lambda$ → $W/m^2 \cdot \mu m$



unit solid angle $d\omega = \sin\theta\, d\theta\, d\phi$

- Radiance is irradiance per unit solid angle → W/m^2 steradian
- For irradiance the radiant flux may come from any direction. With radiance we specify irradiance from a particular solid angle direction.
- element solid angle = unit area on a sphere.



$d\omega = \sin\phi\, d\phi\, d\theta$

$$\text{Irradiance} = \int_0^{2\pi} \text{radiance } d(\text{solid angle})$$

$$\Rightarrow F = \int_0^{2\pi} I\cos\phi\, d\omega = \int_0^{2\pi}\int_0^{\pi/2} I\cos\phi\sin\phi\, d\phi\, d\theta$$

(ϕ = zenith angle)

irradiance $\propto r^{-2}$ is dependent on distance to source
radiance is independent of distance to source

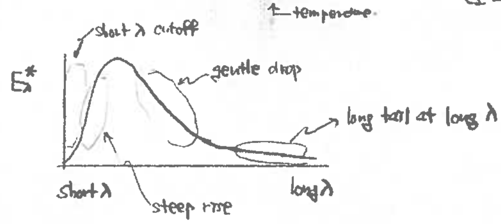
- diffuse radiation → radiation coming from a source which subtends a finite arc of solid angle.
- parallel beam radiation → radiation from a concentrated source such that all radiation comes from same direction ⇒ parallel beam
  → We can treat solar radiation as parallel beam.

- Black body – hypothetical body comprised of sufficient molecules absorbing and emitting em radiation in all parts of the em spectrum so that
  (1) all incident radiation is completely absorbed (hence black)
  (2) in all wavelengths bands and in all directions the maximum possible emissi[on] is realized.
  ⇒ Black body radiation is isotropic → radiance is independent of directio[n]
  → absorption = emission = 1 at all wavelengths.

- The amount of radiation emitted by a blackbody is uniquely given by Plank's la[w]

$$E_\lambda^* = C_1/\lambda^5 \exp(C_2/\lambda T - 1) \quad \text{where} \quad C_1 = 2\pi hc^2,\ C_2 = \frac{hc}{k}$$

T ← temperature; k ← Boltzmann constant



- The wavelength of max emission ⇒ this gives Wien's displacement law

$$\lambda_{max} = \frac{2897}{T} \quad (\text{where } \lambda_{max} \text{ is in } \mu m)$$

- The blackbody irradiance is given by

$$F^* = \int_0^\infty E_\lambda^*\, d\lambda = \sigma T^4$$

σ ← Stefan Boltzmann const.

$F^* = \sigma T^4 \propto T^4$ ← blackbody irradiance. $\sigma \sim 5.67\times10^{-8}\ W/m^2 K^{-4}$

- Radiative equilibrium → no net gain or loss of radiative energy
  → gain = loss
- Planetary albedo (α) = that fraction of total incident solar radiation that is reflected back to space.
  ↳ $(1-\alpha)$ is that fraction of total incident solar radiation that is not reflected back to space
- Blackbody radiation is an upper limit.
- We define the emissivity

$$\varepsilon_\lambda = \frac{F_\lambda}{F_\lambda^*} \equiv \frac{\text{actual irradiance at } \lambda + T}{\text{blackbody irradiance at } \lambda + T} \le 1 \qquad \varepsilon_\lambda \in [0,1]$$

This measures how strongly a given body radiates as a function of λ

- We also define absorptivity

$$a_\lambda \equiv \frac{\text{actual absorption at } (\lambda, T)}{\text{blackbody absorption at } (\lambda, T)} \qquad a_\lambda \in [0,1]$$

- Grey body ⇒ material for which emissivity + absorptivity < 1 independent of wavelength.

- Kirchhoff's law

Materials which are strong absorbers at a particular wavelength are also strong emitters at that wavelength. Similarly weak absorbers are weak emitters. Thus, $a_\lambda = \varepsilon_\lambda$.

This result applies independent of thermal/radiative equilibrium, It applies to solids + gases.

Many substances are selective emitters/absorbers. That is, if they strongly absorb/emit at λ_1, they may only weakly absorb/emit at λ_2.

ex ⇒ snow → weak absorber of visible and near IR
→ strong " of IR.

In the atm → weak absorber of solar SW radiation but good absorber of terrestrial (LW) radiation ⇒ to maintain radiative equilibrium. sfc temp of earth ↑

- Monochromatic radiation incident upon an opaque sfc is either absorbed or reflected. That is

$$E_\lambda(\text{absorbed}) + E_\lambda(\text{reflected}) = E_\lambda(\text{incident})$$

from which follows

$$a_\lambda + r_\lambda = 1$$

↳ reflectivity ≡ fraction of incident irradiance which is reflected

For a non-opaque sfc there may be transmission of some of the incident irradiance. Thus

$$a_\lambda + r_\lambda + T_\lambda = 1$$

↳ transmissivity
↳ that fraction of incident irradiance that is transmitted thru the sfc (without absorption)

In the absence of scattering, the absorption of parallel beam radiation as it passes downward through a horizontal layer of gas of thickness dz is proportional to the number of molecules per unit area that are absorbing radiation along the path. That is,

$$da_\lambda = -\frac{dE_\lambda}{E_\lambda} = -k_\lambda \rho \sec\phi \, dz \qquad \phi \text{ is zenith angle.}$$

k_λ is the absorption coefficient ⇒ it measures the fraction of the gas molecules per unit wavelength interval that are absorbing radiation at wavelength λ.

Integrate from z to TOA ($z \to \infty$)

$$\int_z^\infty d\ln E_\lambda = \int_z^\infty k_\lambda \rho \sec\phi \, dz$$

$$\ln \frac{E_\lambda(\infty)}{E_\lambda(z)} = \sec\phi \int_z^\infty k_\lambda \rho \, dz$$

$$\ln \frac{E_\lambda(z)}{E_\lambda(\infty)} = -\sec\phi \int_z^\infty k_\lambda \rho \, dz$$

so

$$E_\lambda(z) = E_\lambda(\infty) \exp\left[-\sec\phi \int_z^\infty k_\lambda \rho \, dz\right]$$

$$= E_\lambda(\infty) \exp[-\tau_\lambda]$$

where $\tau_\lambda = +\sec\phi \int_z^\infty k_\lambda \rho \, dz$

is referred to as the optical depth or optical thickness

It is a measure of the cumulative depletion that a beam of radiation experiences as it passes through a layer from ∞ to z.

The relationship

$$E_\lambda(z) = E_\lambda(\infty) e^{-\tau_\lambda}$$

⇒ statement that monochromatic irradiance decreases exponentially with increasing path length through a layer.
→ is often referred to as Beer's Law.

The transmissivity of the layer above z is

$$T_\lambda = \frac{E_\lambda(z)}{E_\lambda(\infty)} = e^{-\tau_\lambda}$$

If we can ignore scattering/reflection

$$a_\lambda = 1 - T_\lambda = 1 - e^{-\tau_\lambda}$$

That is, absorption → 1 exponentially as path length ↑

If the absorption coefficient k_λ is independent of z we often define a densi[ty] weighted path length

$$u = +\sec\phi \int_z^\infty \rho \, dz \quad \text{then} \quad \tau_\lambda = k_\lambda u.$$

The nonlinear relationship between absorptivity + optical depth,

$$a_\lambda = 1 - e^{-\tau_\lambda}$$

is linear for sufficiently small path lengths, that is

$$a_\lambda \sim \tau_\lambda = k_\lambda u.$$

As $u\uparrow$, $a_\lambda\uparrow$ linearly then nonlinearly until eventually as $u \to \infty$, $a_\lambda \to 1$ for all λ ⇒ widening of an absorption band.

- Chapman's model for the vertical profile of absorption

In an isothermal atm.

$$\rho = \rho_0 e^{-\frac{z}{H}}$$

← scale height
↳ sea level density

Thus we may substitute for ρ in the expression for the optical depth to get

$$\tau_\lambda = \sec\phi \int_z^\infty k_\lambda \rho_0 e^{-\frac{z}{H}} dz$$

Assume the sun is directly overhead ($\phi = 0 \Rightarrow \sec\phi = 1$)

$$\tau_\lambda = \rho_0 \int_z^\infty k_\lambda e^{-\frac{z}{H}} dz$$

Assume k_λ is independent of height

$$\tau_\lambda = \rho_0 k_\lambda \int_z^\infty e^{-\frac{z}{H}} dz$$

Integrate this to get

$$\tau_\lambda = \rho_0 k_\lambda (-H)\left(\cancelto{0}{e^{-\frac{\infty}{H}}} - e^{-\frac{z}{H}}\right)$$

$$\tau_\lambda = \rho_0 k_\lambda H e^{-\frac{z}{H}}$$

Now the incident radiation absorbed within any differential layer of the atm is given by

$$dE_\lambda = E_{\lambda\infty} T_\lambda \, da_\lambda$$

Now $T_\lambda = e^{-\sigma_\lambda}$ & $da_\lambda = k_\lambda \rho \sec\phi \, dz$ (1 here)

So $dE_\lambda = E_{\lambda\infty} e^{-\sigma_\lambda} k_\lambda \rho_0 e^{-\frac{z}{H}} dz$

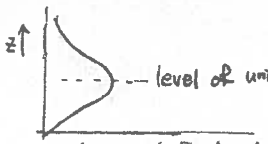
or $\frac{dE_\lambda}{dz} = E_{\lambda\infty} \underbrace{(\rho_0 k_\lambda e^{-\frac{z}{H}})}_{\text{This is } \frac{\sigma_\lambda}{H}} e^{-\sigma_\lambda}$

or $\frac{dE_\lambda}{dz} = \frac{E_{\lambda\infty}}{H} \sigma_\lambda e^{-\sigma_\lambda}$

At the level of strongest absorption

$$\frac{d}{dz}\left(\frac{dE_\lambda}{dz}\right) = 0 = \frac{E_{\lambda\infty}}{H} \frac{d}{dz}(\sigma_\lambda e^{-\sigma_\lambda})$$

It can be shown using the above expression for σ_λ that the max absorption occurs at $\sigma_\lambda = 1 \Rightarrow$ the level of unit optical depth.



The result is qualitatively true for earth atm. (most absorption takes place along that portion of the ray path for which optical depth is unity)

- Terrestrial IR is diffuse, and so we must speak of radiance L_λ rather than irradiance.

Schwarzchild's eq

$$dL_\lambda = -k_\lambda (L_\lambda - L_\lambda^*) \rho \sec\phi \, dz$$

L_λ^* → blackbody monochromatic radiance

k_λ → absorption coefficient for radiance.

This equation describes the net contribution of a layer of thickness dz to the upward passing monochromatic radiance of radiation.

It can be shown that for an isothermal atm

$$L_{\lambda z} - L_\lambda^* = (L_{\lambda 0} - L_\lambda^*) \exp(-\sigma_\lambda)$$

So that $L_{\lambda z}$ exponentially approaches L_λ^* as optical thickness $\sigma_\lambda \uparrow$.

- Scattering of solar radiation

Size parameter $\alpha = \frac{2\pi r}{\lambda}$ r = uniform radius of spherical particles.

for $\alpha \ll 1 \Rightarrow$ Rayleigh scattering

- Scattered radiation is equally divided between forward & reverse directions
- Scattering amount is proportional to $\alpha^4 \Rightarrow \frac{1}{\lambda^4}$ (as blue light λ < λred light ⇒ blue light is scattered more and hence blue sky)
- Rayleigh scattering occurs in earth atm for solar radiation
- Scattering of μ-wave by rain drops is largely Rayleigh scattering → principle used in radar.

for $\alpha \geq 50$ we must use geometric ray optics.

- Scattering of visible light by cloud drops, rain drops, & ice crystals falls in the range ⇒ halos, rainbows, coronas.

for $0.1 \leq \alpha \leq 50 \Rightarrow$ Mie scattering

- angular distribution of scattering is very complicated and varies rapidly with α (but forward scattering does predominate over backward scattering)
- Scattering of sunlight by haze, dust, smoke falls in realm of Mie scatter

- Relative importance of various radiative transfer processes in global energy balance. (1 ≡ highest, 3 ≡ least).

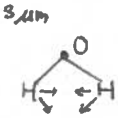
	Solar (SW) Absorption	Solar (SW) Scattering	Terrestrial (LW, IR) Absorption emission	Terrestrial (LW, IR) Scattering
Air molecules	1	2	1[c]	3
Aerosols	2[b]	2	2	3
Clouds	2	1[c]	1[c]	3

∵ b ≡ essential roles in production of photochemical smog.
c ≡ application to remote sensing.

We see the following important processes.

① Scattering of solar radiation by clouds (and affect on planetary albedo)
② absorption of " " by gases (O_3 in stratosphere, H_2O in tropos
③ " " terrestrial radiation by gaseous constituents (CO_2, H_2O, O_3) ← greenhouse ga
④ " " " " by clouds.

- water vapor — very strong vibration at 6.3 μm → bending



Ozone — very strong vibration at 9.6 μm symmetric band stretching

Carbon dioxide — very strong vibration at 4.3 μm

O --- C --- O
asymmetric band stretching.

- Passive detection — detecting naturally emitted or reflected radiation
- active detection — " radiation emitted by the sensor and reflected scattered back to the sensor by objects

- Solid angle ~ $\frac{\text{area}}{(\text{distance})^2}$ $d\omega = \frac{r \sin\theta d\phi \cdot r d\theta}{r^2}$ → $d\omega = \sin\theta \, d\phi \, d\theta$

θ ≡ zenith angle (colatitude)
φ ≡ azimuth angle (longitude)

For a complete sphere $\int d\omega = 4\pi$ steradians
For hemisphere $\int d\omega = 2\pi$ sr

- Irradiance ≡ $\frac{\text{radiant power}}{\text{area}}$ → r^{-2} dependence but independent of origin of power

Radiance ≡ $\frac{\text{radiant power}}{\text{area} \cdot \text{solid angle}}$ → independent of r but dependent on direction of radiant power.

monochromatic irradiance (E_λ)

$$E_\lambda = \int_{\omega} \underbrace{L_\lambda}_{\text{Radiance}} \cos\theta \, d\omega \Rightarrow E_\lambda = \underbrace{\int_0^{2\pi}\int_0^{\frac{\pi}{2}}}_{\text{hemisphere}} L_\lambda(\theta,\phi) \cos\theta \underbrace{\sin\theta \, d\phi \, d\theta}_{d\omega}$$

For isotropic radiance, $E_\lambda = \pi L_\lambda$

The integrated band flux (irradiance) is given by

$$E_{\Delta\lambda} = \int_{\lambda_1}^{\lambda_2} E_\lambda \, d\lambda$$

A detector measures power $P = \int_A E_{\Delta\lambda} \, dA$ (be careful here, satellite measures relative to some solid angle)

More specifically

$$P = \overline{N}\, A_c\, \Delta\omega_c$$

- $\Delta\omega_c$ → solid angle subtended by collector plate of satellite.
- A_c → area of collector plate
- $\overline{N}$ → average radiance measured by satellite
- P → power measured by collector

A transducer converts power to voltage. We quantize the voltage to produce counts.

• Terms

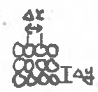
IGFOV ≡ instantaneous geometric field of view → aperature angle

Nadir footprint ≡ area subtended by the aperture angle
↳ resolution is defined in terms of the nadir foot print.

Sampling rate ≡ rate at which scan is sampled

Scan rate ≡ rate at which collector is moving

pixel size ≡ separation between pixel ceters in x & y.



$$PS = (\Delta x)(\Delta y)$$

where Δx = scan rate * sampling time * height

Δy = height of satellite * step angle * height.

• Brightness temp. may be obtained from Planck's law

Assume isotropic radiance

$$\pi N_\lambda^{s} = \left(\frac{2hc^2}{\lambda^5}\right)\left[\exp\left(\frac{hc}{k\lambda T}\right)-1\right]^{-1}$$

Then solve for T ⇒ the brightness temperature

We may simplify this by making the Rayleigh – Jeans approximation

Note that for large λ and not too small T, $\frac{hc}{k\lambda T}$ is very small

In this case

$$e^{\frac{hc}{k\lambda T}} \sim 1 + \frac{hc}{k\lambda T}$$

so

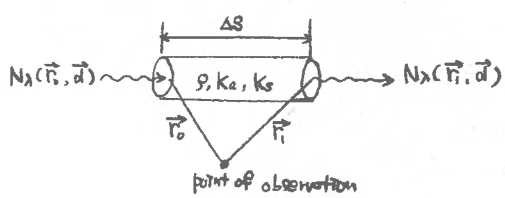
$$\pi N_\lambda \simeq \left(\frac{2hc^2}{\lambda^5}\right)\left[\frac{hc}{k\lambda T}\right]^{-1}$$

so

$$T_B \sim (\pi N_\lambda)\left(\frac{\lambda^4}{2kc}\right)$$

↳ brightness temp.

• Radiative transfer



The radiance at $\vec{r}_1$ is given by

$$N_\lambda(\vec{r}_1,\vec{d}) = N_\lambda(\vec{r}_o,\vec{d}) + \frac{dN_\lambda}{dS}\Delta S$$

where

$dN_\lambda(\theta,\phi) = -N_\lambda(\theta,\phi)k_{a\lambda}\rho\, dS$ ⇝ loss due to absorption by medium

$\quad - N_\lambda(\theta,\phi)k_{s\lambda}\rho\, dS$ ⇝ " " " scattering by "

$\quad + \mathbb{P}_\lambda[N_\lambda(\theta,\phi)]k_{s\lambda}\rho\, dS$ ⇝ gain due to scattering by "

$\quad + B_\lambda(T)k_{a\lambda}\rho\, dS$ ⇝ gain " " emission " "

↑ Planck function

scattering integral function

$$\mathbb{P}_\lambda = \frac{1}{4\pi}\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} P_\lambda(\theta,\phi;\theta',\phi')\, N_\lambda(\theta',\phi')\, \cos\theta'\, d\theta'\, d\phi'$$

↳ phase function → that fraction of radiance N_λ from direction (θ',ϕ') that is scattered into the direction (θ,ϕ) (include $\frac{1}{4\pi}$)

We define the volume absorption and scattering coefficients

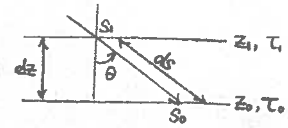
$$\beta_{a\lambda} = k_{a\lambda}\rho \qquad \beta_{s\lambda} = k_{s\lambda}\rho \longrightarrow \beta_{e\lambda} = \beta_{a\lambda}+\beta_{s\lambda}, \quad k_{e\lambda} = k_{a\lambda}+k_{s\lambda}$$

↑ extinction coefficients

• Define the optical depth τ_λ

$$d\tau_\lambda(S_0,S_1) = -k_{e\lambda}\rho\, dS = -\beta_{e\lambda}\, dS$$

$$= -k_{e\lambda}\rho\frac{dz}{\mu} = -\beta_{e\lambda}\frac{dz}{\mu}$$



$\mu = \cos\theta$

$dz = \cos\theta\, dS$

Thus we may rewrite the RTE eq. as

$$dN_\lambda(\theta,\phi) = -N_\lambda(\theta,\phi)\left[(\beta_{a\lambda}+\beta_{s\lambda})\right]dS + B_\lambda(T)\,\beta_{a\lambda}\, dS + \mathbb{P}_\lambda[N_\lambda(\theta,\phi)]\,\beta_{s\lambda}\, dS$$

$(\beta_{a\lambda}+\beta_{s\lambda})$ → can replace with $\beta_{e\lambda}$

dS → can replace with $\frac{dz}{\mu}$

Divide thru by $-\beta_{e\lambda}dS$ and recognize $-\beta_{e\lambda}dS = d\tau$ then

$$\frac{\mu\, dN_\lambda(\theta,\phi)}{d\tau} = N_\lambda(\theta,\phi) - \left(\frac{\beta_{a\lambda}}{\beta_{e\lambda}}\right)B_\lambda(T) - \left(\frac{\beta_{s\lambda}}{\beta_{e\lambda}}\right)\mathbb{P}_\lambda[N_\lambda(\theta,\phi)]$$

Define the single scatter albedo $\tilde{\omega} = \beta_{s\lambda}/\beta_{e\lambda} = k_{s\lambda}/k_{e\lambda}$

$$1-\tilde{\omega} = \beta_{a\lambda}/\beta_{e\lambda}$$

Then

$$\frac{\mu\, dN_\lambda(\theta,\phi)}{d\tau} = N_\lambda(\theta,\phi) - \underbrace{(1-\tilde{\omega})B_\lambda(T) - \tilde{\omega}\,\mathbb{P}_\lambda[N_\lambda(\theta,\phi)]}_{\text{represent source terms}}$$

The simple forms of the RTE above do not consider

(a) polarization (b) refraction (c) diffraction (d) Raman scattering

When there are no source terms we obtain Beer's law

$$\frac{\mu\, dN_\lambda(\theta,\phi)}{d\tau} = N_\lambda(\theta,\phi)$$

When there is no scattering we obtain Schwarzchild's eq.

$$\frac{\mu\, dN_\lambda(\theta,\phi)}{d\tau} = N_\lambda(\theta,\phi) - (1-\tilde{\omega})B_\lambda(T)$$

with $\tilde{\omega} = 0$ ⇒ no scattering

$$\frac{\mu\, dN_\lambda}{d\tau} = N_\lambda - B_\lambda(T)$$

Integrate to get radiation at level z from z_s (sfc.) to z

$$N_\lambda(\tau,\mu,\phi) = N_\lambda(\tau_s,\mu,\phi)\, e^{-\tau(z_s,z)/\mu} \quad \text{← sfc term}$$

$$+\int_{\tau(z)}^{\tau(z_s)} B_\lambda(T(\tau'))\, e^{-\tau'(z',z)/\mu}\, d\tau'/\mu \quad \text{← atm term}$$

$Tr(0)$ ———— z_0, $\tau_0(z_0)=0$

$Tr(\tau)$ ———— z, $\tau(z)$

$Tr(\tau')$ ———— z', $\tau'(z')$

$Tr(\tau_s)$ ———— z_s, $\tau_s(z_s)$

↑ transmissivity ⇒ $dTr_\lambda = d[e^{-\tau}] = -e^{-\tau}d\tau$

In terms of transmission

$$N_\lambda(\tau,\mu,\phi) = N_\lambda(\tau_s,\mu,\phi)\, Tr_\lambda(\tau_s) + \int_{Tr(\tau_s)}^{Tr(\tau)} B_\lambda(T(\tau'))\, dTr'(\tau')$$

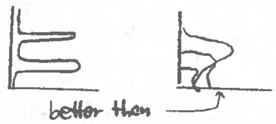
Recall the integrated RTE in qualitative terms

$$N_\lambda(TOA) = \underset{\uparrow \text{ sfc term}}{N_\lambda(sfc)} + \underbrace{\int_{sfc}^{TOA} (\text{atm emission})_\lambda \, dz}_{\text{integral term}}$$

Satellites can perform imagery, sounding, radiation budget.

① Imagery – select frequencies where integral term is much smaller than surface term

② Sounding – select frequencies in which the integral term dominates use sharpest possible weighting functions.



③ Radiation budget – measure broad band flux without worrying if from sfc to atm.

- Limb scanning

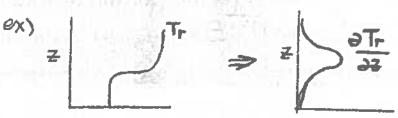
↳ scan thru atm and not directly at sfc

+ · better vertical resolution
  · avoids variable sfc effects
  · large opacity due to long path length (good for trace gases)
  · able to azimuthally scan

– · poor horizontal resolution
  · longer paths increase likelihood of cloud interference
  · violates basic assumption that state parameters are not changing along paths.

. If $Tr(z)$ is the transmission function for a given medium (gas), then we call $\frac{\partial Tr}{\partial z}$ the weighting function

ex)



We can rewrite the RTE in the form

$$N_{\bar\lambda}(z_0) = B_{\bar\lambda}(T_s)\, Tr_{\bar\lambda}(z_s) \quad \longleftarrow \text{sfc contribution}$$

$$+ \int_{z_s}^{z_0} B_{\bar\lambda}(T(z)) \underbrace{\frac{\partial Tr_{\bar\lambda}}{\partial z}}_{\text{weighting function}} dz \quad \longleftarrow \text{atm contribution}$$

satellite measures broad band flux (between λ_1 + λ_2) not monochromatic flux

- Temperature retrieval

Take Schwarzchild's eq in the form

$$N_\lambda(0) = B_\lambda(T_s)\, Tr_\lambda(P_s) + \int_{P_s}^{P} B_\lambda(T_{p'}) \frac{\partial Tr_\lambda(p')}{\partial p'} dp'$$

Inversion means solving for the temperature profile T_p based on $N_\lambda(0)$ from channels $i = 1, 2, \cdots, M$

We generally make 3 assumptions in "solving" the inverse problem

① We can insert a climatological profile $\overline{T}_p$ + essentially solve for the perturbation temperature $\Rightarrow \delta T_p = T_p - \overline{T}_p$

② We can measure T_s by using IR window channels. That is, somehow we can compute the sfc term $B_\lambda(T_s)\, Tr_\lambda(P_s)$

③ We can expand $B_\lambda(T_p)$ as a Taylor series and ignore higher order terms, that is,

$$B_\lambda(T_p) = B_\lambda(\overline{T}_p) + \delta T_p \frac{dB_\lambda(\overline{T}_p)}{dT} + \text{higher order terms}$$

Then the RTE to solve becomes

$$\underset{\uparrow \text{ measure}}{N_\lambda(0)} - B_\lambda(T_s)\, Tr_\lambda(P_s) - \int_{P_s}^{P} \overset{\text{assume we know}}{B_\lambda(\overline{T}_p)} \frac{\partial Tr_\lambda(P)}{\partial p} dp$$

→ leading term in Taylor series expansion

$$= \int_{P_s}^{P} \delta T_p \underbrace{\left(\frac{dB_\lambda(\overline{T}_p)}{dT} \frac{\partial Tr_\lambda(P)}{\partial p} \right)}_{\text{call this a kernel function}} dp$$

- Physical retrieval

① Start with 1st guess temperature profile

② feed T_p into forward RTE to get radiances

③ compare computed + obs radiances

④ if convergence ⇒ done, else, adjust T_p based on residuals

must d... this f... n chan...

Advantages

① physically consistent at every step to within the approx made in the forward model.

② does not require a large data set

Disadv.

① Computationally expensive

② relies heavily on accuracy of weighting functions $\frac{\partial Tr_\lambda}{\partial z}$

③ Influence from 1st guess T_p profile.

④ Contains no knowledge of the statistical properties of the atm.

· Statistical retrievals

Collect many pairs of radiances + temperature profiles.

↳ in layers $j = 1, \cdots, N$

↳ in channels. $i = 1, \cdots, M$.

Assume mapping

$$(\underset{\text{mean value}}{T - \overline{T}})_j = C_{ji}\,(N - \overset{\text{mean value}}{\overline{N}})_i$$

↑ transform matrix to go from radiances to temperature profile based on large training dataset comprised of K pairs.

Rewrite as

$$\underset{N\times K \text{ matrix}}{\Delta T} = \underset{M\times K \text{ matrix}}{C\,\Delta N}$$

C: $N \times M$ matrix

Solve for C

$$\Delta T \Delta N^T (\Delta N \Delta N^T)^{-1} = C \Delta N \Delta N^T (\Delta N \Delta N^T)^{-1}$$

$$= C \Delta N \underbrace{\Delta N^T \Delta N^{T-1}}_{I} \Delta N^{-1}$$

$$= C \Delta N I \Delta N^{-1}$$

$$= C \Delta N \Delta N^{-1}$$

$$\boxed{\Delta T \Delta N^T (\Delta N \Delta N^T)^{-1} = C}$$

→ Once we have C, ① we measure N, ② from ΔN, ③ evaluate $C\Delta N$, and ④ add $\overline{T}$ to the result to obtain T.

Advantages

① Once C is computed, the method is fast + easy.

② senses statistics of relationship between T + N.

③ No initial guess required.

no physics in algorithm

② requires large training data set.

→ issues of generalization to
→ issues of daily, seasonal, interannual variability.

③ you can not guarantee that C embodies all the important statistical properties.

The above is a statistical regression approach.

we can expand T & N in terms of their covariance structure using EOFs ⇒ statistical eigenvector approach.

We have not yet considered the effect of error ⇒ this can be incorporated into statistical methods. But then we need to make assumptions about error covariances.

Minimum information method

① assume temp. errors are uncorrelated between different layers
→ a poor assumption.

② assume radiance errors are uncorrelated between channels.
→ reasonable assumption.

• Perturbation technique is good for VAS, however it requires a first guess which is close to the true solution. → unlike physical method which can start from an arbitrary 1st guess.

• Split window technique.

• Rainfall retrieval

Two "flavors" → passive & active sensing.

→ many problems of which one concerns the many scales of motion (mesoscale, synoptic, etc.)

advantages of VIS/IR (passive)

① Simple in concept → rain ∝ cloud top temp.

② made from geostationary platform ∴ high resolution temporal coverage.

③ produce global datasets (due to ②)

④ easily automated for operational use.

⑤ good for convective precip.

Disadv. of VIS/IR (passive)

① Poor spatial resolution

② Poor physics → not directly sensing rain, rather infer it from cloud top temperatures.

③ not easily transferable.

④ poor with stratiform precip.

⑤ no general solution ⇒ each technique is "engineered".

⑥ results based on correlation between cloud top properties & radar echo properties.

Generally speaking, any algorithm may be tuned to yield good estimates of volumetric rainfall over large enough space scales & long enough time scales ⇒ but then we are doing little more than reproducing climatology.

VIS/IR techniques

· GPI ⟶ $GPI = \left(3\ \frac{mm}{hr}\right)\left(\begin{matrix}\text{fractional area with } T_B < 235\,K \\ \text{over } 2.5° \text{ cell}\end{matrix}\right)\Delta t$

· Life history schemes

$$R_v = a_0 A_c + a_1 \frac{dA_c}{dt}$$

↳ time rate change in fractional cloud coverage

→ fractional cloud coverage for cloud type.

· Area-time integral (ATI)

— based on high correlation between cloud area (echo area) & total volume of falling rain

— define $ATI = \int_{time} A(t)\,dt$ then $R_v = \alpha (ATI)^{\beta}$

· Height Area rainfall threshold (HART)