Krish

1) Derive isentropic Pot. vort. eq. Conservation of Pot. Vort. implies changes among abs. vort. + static stability. How would you use that partitioning to describe lee cyclogenesis. Will it work equally well in the lee of mountains when the basic flows are westerly or easterly?

2) Time rate of change of angular momentum in local cylindrical coord. system. Discuss various torques. Describe how to interpret max. intensity of a hurricane taking into account sources, sinks + boundary values of parcel's angular momentum.

3) Nondimensionalize the zonal E.O.M in the PBL + discuss what condition you expect Ekman, Advective + Stokes boundary layers. Give an example of forcus for the advective boundary layer + explain its role in cross equatorial flow of Somali Jet.

4) Given closed domain equations for the rotational + divergent K.E + for internal plus potential energy, discuss using inequality arguments, the maintenance of a statistically steady state monsoon from differential heating. Explain the role of all salient covariances. Explain significance of orientation of the velocity potential w.r.t. streamfunction in this problem.

5) Derive spectral form of the barotropic, nondivergent vort. eq. Discuss how transform method is used to handle linear + a non-linear terms. Show how you would make one time step forecast starting from u, v over the globe at 500 mb.

6) Spectral form of the semi-implicit shallow water eq.

a) Explain clearly why ~~the~~ we use semi-implicit algorithm.

b) Starting from u, v + z at single level as func. of lat/lon. draw detailed flow chart showing spectral method for solution.

c) One always encounters a Helmholtz eq. in this eq. how it is solved spectrally.

7) Discuss Computational stability of the semi-implicit time differencing scheme applied to a simple linear wave eq. Discuss advantages over an explicit method. In application to shallow water show seperation of terms for fast + slow modes. Indicate how Helmholtz eq. pops up.

8) Derive stencil for 4th order accurate Laplacian or describe method for implementing it. Using this, describe how one constructs 4th order accurate Jacobian which satisfies quadratic invariance.

9) Write down the shallow water equation in semi-implicit form and describe the Helmholtz eq. for the free surface height. Discuss the invariants of the problem for a closed domain. What may be appropriate boundary conditions for this problem.

10) Seminar Title / speaker, Approach, Main results, Limitations.

11) Spectral shallow water. Outline each of the following.

a) Spectral closed system

b) treatment of Helmholtz eq. for free surface height

c) " " nonlinearities

d) " " semi-implicit time differencing

(b–d: put together spectrally.)

12) Using scale anal. (time/space), show balance of forces for Ekman, Advective + Stokes boundary layers. Derive appropriate frequency equations.

13) Sketch wind + pressure fields for Somali Jet over the Arabian Sea during summer at the 1 km level. Discuss balance of forces from south to north following the jet.

14) Zonally + meridionally propagating waves on the Madden-Julian time scale 30 - 50 days. Passage of these waves have been related to occurrence of intra-seasonal climate variability. Describe that clearly i.t.o. motion field, clouds + weather. What are vertical + horizontal scales of these waves.

15) Sequentially describe features of an El Niño episode + its atmospheric impact. Describe physical aspects of initiation of warm water of the El Niño in a coupled ocean-atmos. context. Sketch global east-west divergent circulation during an El Niño + an El Viejo + its implications on tropical climate.

16) Place following events in logical time sequence : (1) convection (2) PNA pattern (3) SST anom. (4) Trade wind anom. (5) Tropical rain + droughts (6) West. wind anom. (7) Oceanic Kelvin wave.

17) Barotropic transfer of energy for a zonal mean flow to eddies on the scale of African waves. Discuss : synoptic aspects horiz./vert. scales. What computation would you carry out to estab. importance of the barotropic process. Energetics, inflection point instability, growth rate vs. scales of motion.

18) From first principles, formulate the stability, heating + rainfall rate estimates. Use simple Q.G. omega eq. as framework for discussion. Also discuss ellipticity of the ω-eq.

19) Write down two angular momentum eqs.

a) one using spherical coord.

b) one relevant to hurricane in local cylindrical coords.

Discuss relevance for maintenance of the tropical gen. circ. + of hurricanes. Incl: role of torques, transports, + turbulent eddy motion.

20) Clearly show spectral transform method formulation of barotrop. vort. eq. Start from vort. eq. + show Legendre-Fourier transform of the spectral eq. + clearly portray the one time step solution procedure for each of the terms. Use any standard time differencing scheme.

21) Describe African easterly wave (locations, season, altitude, intensity, frequency, extent). Relation to local Hadley cell using sketch. Discuss role of barotropic + combined barotropic-baroclinic instability of this environment + possible growth of African waves.

22) Adiabatic - inviscid pot. vort. eq. Describe formation of mid-lat. lee trough using conservation principle.

$$\frac{d}{dt}\left\{\zeta_a \frac{\partial\theta}{\partial p}\right\} = 0$$

23) One-paragraph on each :

a) Ozone hole
b) Nuclear Freeze
c) Sahelian drought
d) EL Niño
e) VAS soundings
f) Equatorial Kelvin waves
g) Rossby gravity waves
h) Wave CISK
i) PNA pattern
j) Madden-Julian wave
k) Heat low.

* Seasonal variability of Hadley cell

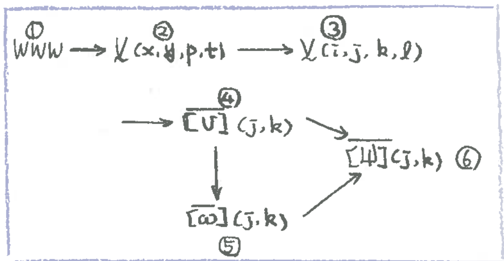
IMI

MET5533: T. N. Krishnamurti (?)

• If you are to investigate the seasonal variability of Hadley cell how would you start from the world weather watch (WWW) data sets and proceed sequentially to calculate an map of the Hadley cell and discuss its variability?

my note p.1

Sol) To investigate the seasonal variability of the Hadley cell, we could procede as shown below



① World Weather Watch data set contains various weather related variables such as u, v, p, ϕ, RH etc at irregularly spaced points. Data sources for this data set include surface observation stations, upper air stations, satellites, ships, commercial aircraft, etc.

② From WWW data read in data sets needed for objective analysis

③ Perform objective analysis to obtain wind field on a regular space–time grid → OI, Barnes analysis etc

④ Perform time and zonal average to obtain $[\bar{v}]$ where [ ] denotes a zonal average, $\bar{\ }$ denotes a time average. Since we wish to examine seasonal variability, the time average should extend over 3 months (June to August, for example)

⑤ Once we have $[\bar{v}]$ we use the kinematic method to obtain $[\bar{\omega}]$. (Dines' compensation rule) vertically integrate the zonally averaged continuity eq.

$$\frac{\partial[\bar{v}]}{\partial y} + \frac{\partial[\bar{\omega}]}{\partial p} = 0$$

⑥ The final step is to obtain $[\bar{\psi}]$, the time mean, zonally averaged streamfunction. The stream function should satisfy the relation

$$\frac{\partial[\bar{\psi}]}{a\partial\phi} = -[\bar{\omega}]\frac{2\pi a\cos\phi}{g}$$

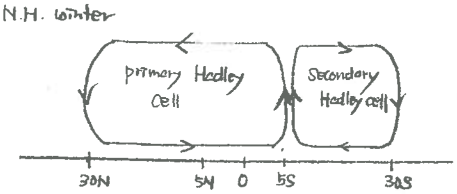
$$\frac{\partial[\bar{\psi}]}{\partial p} = [\bar{v}]\frac{2\pi a\cos\phi}{g}$$

By solving these two eqs and mapping the resulting $[\bar{\psi}]$ we can study the Hadley cell and its seasonal variation.

Seasonal variability

During the N.H. winter, the Hadley cell dominates. There is a primary (huge) meridional circulation cell which extends from 5°S to 30°N. A secondary meridional circulation is located between 5°S and 30°S.

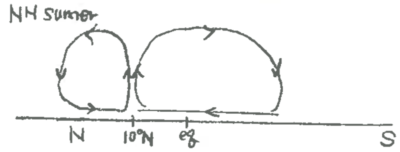
N.H. winter



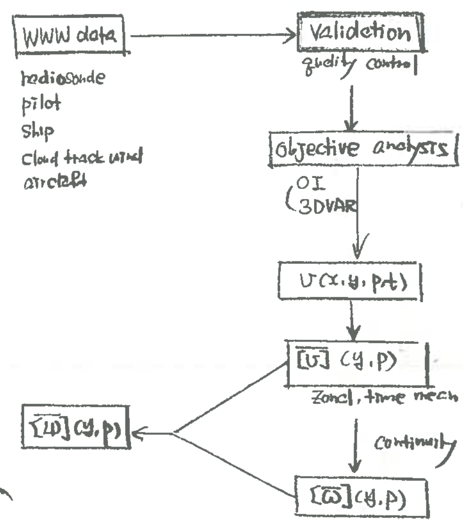
In the fall & spring the Hadley cell is almost of the same magnitude both north & south of the equator where the rising motion is concentrated. The pattern during the N.H. summer reverses that of the NH winter with the primary circulation in the Southern hemisphere.

Note that the rising branch of the Hadley cell follows the sun.

NH summer



* Picture of the Hadley cell



* Hadley cell + Cross-equatorial transport.

MET5534 (3) (Tropical Meteorology): T. N. Krishnamurti (1 hour, 1997)*

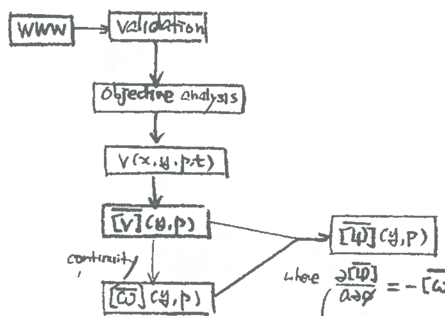
- a) Define the Hadley Cell. b) If you are to map the Hadley Cell from real data, show the sequence of steps (including equations for a stream function) you would need to map it. ✓ c) Explain how the Hadley Cell transports momentum and moisture across the equator from the winter to the summer hemisphere, however it transports kinetic energy the opposite ways. d) How do transports of heat and moisture typically vary among El Nino and La Nina years.

Sol)

a) Hadley cell : the meridional circulation between equator and the subtropical high.

(rising motion : ITCZ
(sinking motion : subtropical high

b) look at the answer of another question.

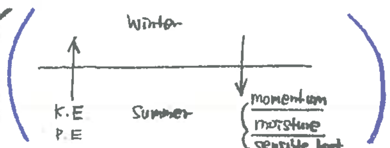


where

$$\frac{\partial[\psi]}{a\partial\phi} = -[\omega]\frac{2\pi a\cos\phi}{g}$$

$$\frac{\partial[\psi]}{\partial p} = [v]\frac{2\pi a\cos\phi}{g}$$

c)

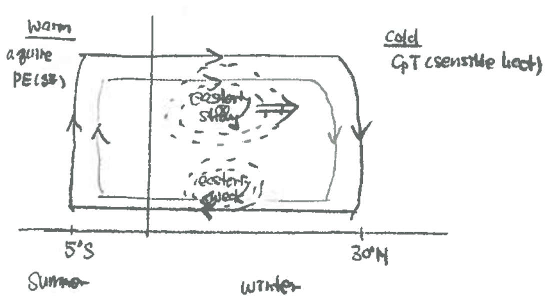


For any variable Q, (the mean meridional transport) is

$$[\overline{QV}] = [\overline{Q}][\overline{V}] + \overline{[Q'][V']} + [\overline{Q^* v^*}]$$

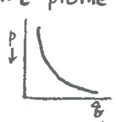
(total transport) = (transport by mean meridional circulation) + (transport by transient eddies) + (transport by standing eddies)

Let's look at the below diagram. for Hadley cell.



According to the above figure, westerly momentum is transported from winter to summer

The moisture profile looks like



So, moisture is also transported to summer hemisphere by mean meridional Hadley circulation.

K.E always increase with height, therefore it will be transported from the summer to winter.

d) usually the sensible heat and moisture are transported from the winter to summer hemisphere.

Think about it ? → zonal shift ?

* major zonal asymmetries

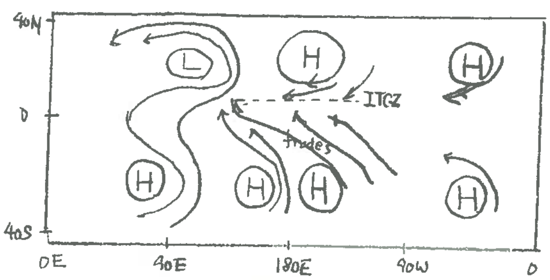
MET5533: T. N. Krishnamurti (?)

- Describe the major zonal asymmetries of the time mean motion field during the northern summer over the tropics at 850 and 200 mbs.

→ look at my note p4

Sol) The principle asymmetric features at 850mb in the NH summer are

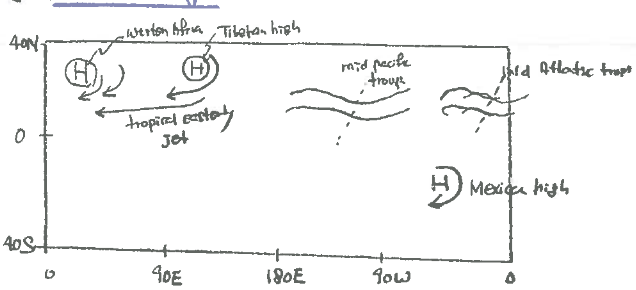
(1) Subtropical high pressure areas of N + S Atlantic oceans
(2) " " " " " Pacific oceans
(3) Trade wind system of major oceans (N+S Indian ocean) ✓
(4) Cross equatorial flow off the Somalia coast over the Arabian sea
(5) Southeast monsoons. Monsoon trough over India,
(6) Australia high, Mascarene high.
(7) ITCZ is located north of the equator
(8) Heat lows over deserts. ✓



<850mb NH summer>

The salient features of the 200mb N.H flow field are

(1) Tibetan high pressure area
(2) West African high pressure area
(3) Mid-Pacific trough
(4) Mid-Atlantic trough
(5) Tropical easterly jet over Asia & equatorial Africa
(6) Mexican high



In south east Asia, there is a branch of rising motion in the Walker cell. This coupled with the upper level jet stream can bring heavy precipitation. Over North Africa is the decending branch of the Walker cell. The southern hemisphere jet stream is reduced in its intensity and moves to about 27°S

The velocity potential field shows strong gradient of χ over the South Indean ocean and in the vicinity of 200mb troughs

※ look at p4 figure!

* Hadley + Walker circulation

MET5533: T. N. Krishnamurti

• Describe how you would go about finding the intensity of Hadley and Walker circulations. Why are these circulations considered important, what are their different roles in the tropical general circulations. How are these circulations maintained.

Sol) The horizontal wind can be written as

$$\underline{V} = \underline{V}_\psi + \underline{V}_\chi$$

($\underline{V}_\psi$ → rotational part; $\underline{V}_\chi$ → divergent part.)

where
$$\begin{cases} \underline{V}_\psi = -\mathbf{k} \times \nabla\psi \\ \underline{V}_\chi = -\nabla\chi \end{cases}$$

$$\nabla^2\overline{\chi} = -\nabla \cdot \overline{\underline{V}} \rightarrow \text{obtain } \overline{\chi}$$

poisson

$$\overline{V_\chi} = -\nabla\overline{\chi}$$: time mean velocity potential $\overline{\chi}$

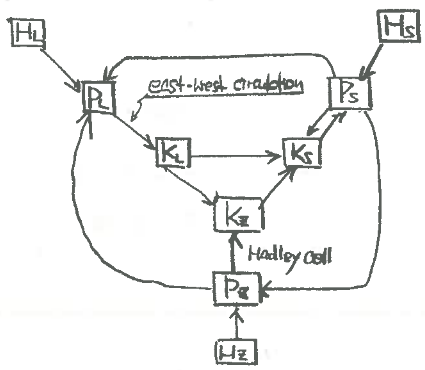
⇒ Intensity of the Hadley and east/west circulations (Walker)

$$I_H = -\frac{1}{L}\oint \frac{\partial\overline{\chi}}{\partial y}\,dx \quad \leftarrow \text{Hadley}$$

$$I_E = -\frac{1}{y_2 - y_1}\int_{y_1}^{y_2} \frac{\partial\overline{\chi}}{\partial x}\,dy \quad \leftarrow \text{Walker.}$$

These circulation is important in understanding tropical energetics.

⇒ look at my note p5



* ENSO scenario

MET5533 (Tropical Meteorology): T. N. Krishnamurti (1 hour)

• In the El Nino – Southern Oscillation scenario the following elements appear to have a sequential role:

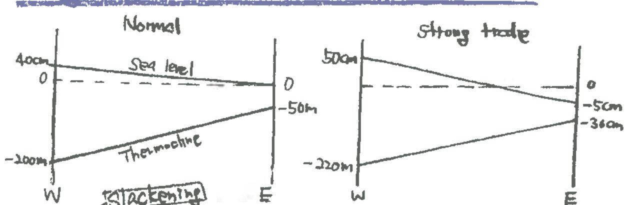
(1) Convection (4)
(2) PNA patterns (6)
(3) SST anomalies (3)
(4) Trade wind anomalies (1)
(5) Tropical rain and droughts (7)
(6) Westerly wind anomalies (5)
(7) Oceanic Kelvin waves. (2)

Place these events in a logical time sequence and describe this scenario giving physical arguments.

Sol) Sequence of the scenario

1. Strong trades (4)

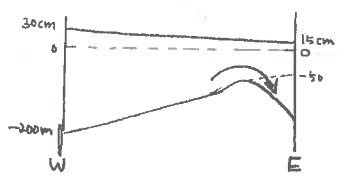
2. Piling of water over the western equatorial ocean.



3. Slacking of the trades

4. Easterward propagating (in the ocean) of a trapped equatorial Kevin waves (7)

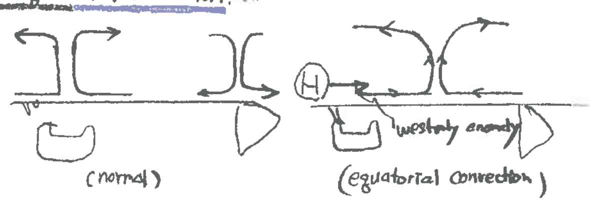
5. Thermocline overturning.



6. Initiation of warm water (Coastal east Pacific) (3)

7. Spreading of warm water toward central Pacific ocean

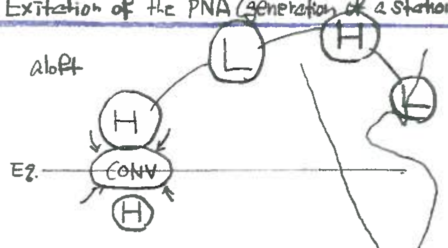
8. Equatorial convection. (1)



9. Further slackening of trades

10. Westerly wind anomaly (lower troposphere) (6)

11. Excitation of the PNA (generation of a stationary Rossby wave) (2)



<upper level height anomalies>

12. Global westerly wind anomaly (200mb)
— meandering in tropical belt.
— leads to heavy rainfall and droughts

13. Climate anomalies (Drought and heavy rains) (7)

* ENSO scenario → related phenomena.

5533

MET? (Tropical): T. N. Krishnamurti (?)

• Discuss the El Nino scenario in the context of Indian Monsoon rainfall and south-east Asia and Australian droughts. Include in your discussion the role of elements such as trades, ocean temperatures, oceanic convection, tropical waves, 200 mb flow patterns and divergent circulations.

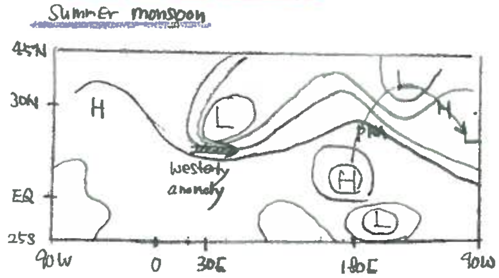
Sol) First we discuss the sequence of the El Nino phenomena

1. Strong trades ✓
2. Piling up of water over the western equatorial ocean ✓
3. Slackening of the trades ✓
4. Eastward propagation of a trapped equatorial Kelvin wave (in the ocean) ✓
5. Thermocline overturning in the oceans ✓
6. Initiation of warm water in coastal east Pacific ✓
7. Spreading of warm water westward to central Pacific (oceanic Rossby wave) ✓
8. Shift in equatorial convection from maritime continent to central Pacific ✓
9. Further slackening of the trades ✓
10. Westerly wind anomaly in lower troposphere ✓
11. Excitation of PNA pattern ⇒ generation of stationary Rossby wave ✓
12. Global westerly wind anomaly (200mb) ✓
13. Climate anomalies (drought + heavy rain) ✓

Toward understanding the EN event, it is very helpful to discuss the difference field of the streamfunction (ψ) and velocity potential (χ) between EN + non EN years at 200mb. The difference field of ψ shows that there are two main features

(a) the familiar PNA pattern over the central Pacific ocean the two upper anticyclones (a high and a low on either side of the equator)

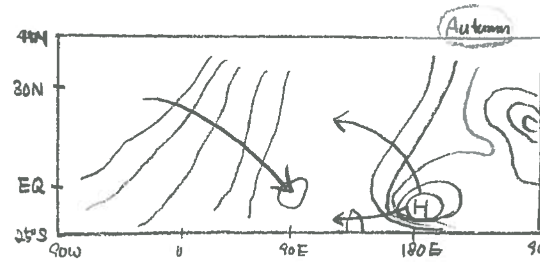
(b) a strong westerly anomaly over the region of the Asian summer monsoon



<ψ – difference field>

It is not altogether clear that the origin of this pronounced anomaly over the monsoon region can be traced to events over the warm equatorial Pacific. It is conceivable that antecedents that contribute to the strengthening of the trades over the Pacific ocean may play a role in producing these strong westerly wind anomalies over the monsoon region.

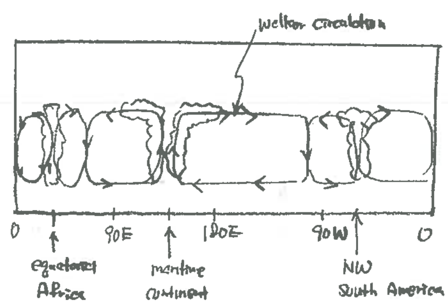
The difference pattern of the velocity potential χ in the autumn months reveals a prominent center over the central Pacific ocean.



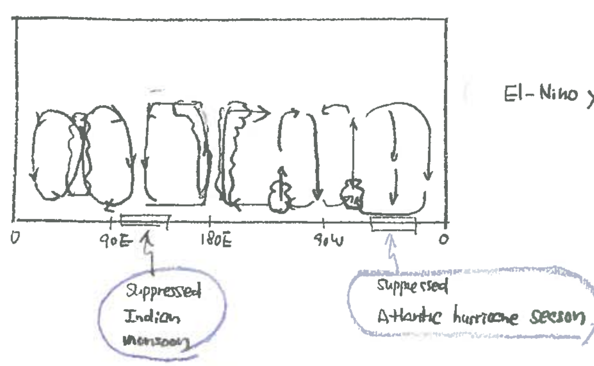
<EN – non EN 200mb χ>

The strongest gradient of the χ is found to the west of the max value. This contributes to strong downward motion over Indonesia, New Guinea, and Northern Australia ← possibly leading to drought over these regions

* The long lasting westerly wind anomaly and an associated low pressure area to its north over the monsoon region form in the upper troposphere. A southward + eastward shift of the Tibetan high during the EN years contributes to this lowering of pressure. It is worth noting that EN years are years of negative departures (or extremely deficient rainfall). This aspect of interannual variability is also a pronounced feature of the winter monsoon rainfall over regions such as Northern Asia, Indonesia, Malaysia and southern India.



non El-nino year



El-Nino year

* El Nino scenario

MET5533: T. N. Krishnamurti (?)

- Question

a) Describe in sequence the elements of the El Nino.

b) Provide a detailed analysis of the evolution of the westerly wind anomaly; the oceanic mixed layer overturning; and the warming of the equatorial ocean.

look at other answers.

* El Nino scenario

MET5533: T. N. Krishnamurti

• Question

a) Describe in sequence the salient features of an El Nino episode and its atmospheric impact.

b) Describe the physical aspects of the initiation of warm water of the El Nino in a coupled ocean-atmosphere context.

✓ c) Sketch the global east west divergent circulation during an El Nino and Elviejo and its implication on tropical climate.

look at other answers.

* El Nino

MET5533 (Tropical Meteorology): T. N. Krishnamurti (1 hour, 1995)

• Describe in some detail with sketches the sequence of the elements of a southern oscillation starting from a strong trade wind phase and culminating in the same phase. Describe within it the comings and goings of the cold and warm phases of the El Nino. How would a feature such as an east-African drought be a part of his scenario, discuss.

look at other answer

→ African drought appears to be characterized by phenomena on two time scale

① relatively short "episodes" of serious widespread drought which usually last a year or two, sometimes a little longer, but rarely more than 4~5 years.

② long dry "period" spanning a decade or more which may include several very dry episodes.

* 30 to 50 day oscillations (Madden-Julian time scale)

5533

MET?(Tropical): T. N. Krishnamurti (1 hour) → note P9 + P10

• Describe the zonally and meridionally propagating waves on the Madden-Julian time scale of 30 to 50 days. The passage of these waves have been related to the occurrence of intra seasonal climate variability. Describe that clearly in terms of motion field, cloud and weather. What are the horizontal and vertical scales of these waves.

Sol) • Spectrum analysis from nearly 10 years of daily rawinsode data for Canton Island (3S, 172W) revealed the following in the 30-50 day mode. (Madden + Julian, 1972)

* ① Strong peakes in the zonal wind in the lower and upper troposphere. Weak to non-existent in the midtroposphere.
* ② Station pressure had a peak in phase with zonal wind oscillation in the lower troposphere and out of phase with that in the upper troposphere.
* ③ The tropospheric temperatures.
  ⊖ temperature anomalies ~ ⊕ pressure anomalies

• Observational aspects of the Indian summer monsoon in the context of 30-50 day mode.

1. Meridional propagation → scale : 3000 km. (low)
   - ① Zonally oriented cloud lines
   - ② trains of cyclonic and anticyclonic flow patterns in the lower troposphere.
   - ③ precipitation anomalies
   - ④ geopotential and wind fluctuations in the lower and upper troposphere.
2. Zonal propagation → wave # 1 + 2 (upper)
   - ① tropical OLR fields
   - ② Divergent wind field at 200 mb
   - ③ sea level pressure field.
3. Air-Sea interaction
   - ① an oscillation of SST
   - ② latent and sensible heat fluxes.
4. Energy exchange (low frequency mode)
   - ① LFM receive a substantial amount of K.E from the high frequencies
   - ② LFM gain KE from the long term mean flow.
   - ③ LFM receive a small amount of energy from the PE on the same frequencies.

• Past results

Over India the meridional passage of the cyclonic + anticyclonic flow modulates the monsoon in the 30-50 day time scale.

Over China, the 30-50 day modulation (of the oceanic components of the flow towards the land mass) enhances the onshore circulation of the subtropical highs.

Over Australia it is the enhancement of the monsoon trough between 10°S and 15°S which is the ITCZ over this region that undergoes a modulation from the passage of low frequency modes.

※ Low frequency modes (associated with the monsoon)

- An important time scale of the monsoon (around 40 days)
- Identifies (wet / dry) spells of the monsoon
- exhibits both the (summer / winter) monsoon
- Slow meridionally propagating waves
  - Scales : of the order of 3,000 km
  - Speed : around 1° lat / day
- originate over the ocean
- propagate meridionally over the monsoon land masses
- driven by cumulus convection and air-sea coupling via moisture fluxes

* maintenance of monsoon (inequality arguments) : ψ-χ interaction

MET5533: T. N. Krishnamurti (1 hour) * ⟶ note p8+9

- Given closed domain equations for the rotational and divergent kinetic energy and for the internal plus potential energy. Discuss, using inequality arguments, the maintenance of a statistically steady state monsoon from differential heating. Explain the role of all the salient covariances. Explain the significance of the orientation of the velocity potential with respect to the stream function in this problem.

Sol) We have the following system of eqs using ψ and χ.

$$\frac{\partial K_\psi}{\partial t} = \langle K_\chi \cdot K_\psi \rangle + F_\psi$$

$$\frac{\partial K_\chi}{\partial t} = -\langle K_\chi \cdot K_\psi \rangle + \langle E_{P+I}, K_\chi \rangle + F_\chi$$

$$\frac{\partial E_{P+I}}{\partial t} = -\langle E_{P+I}, K_\chi \rangle + G + F_{I+P}$$

$\therefore V_H = V_\psi + V_\chi$

where $V_\psi = k \times \nabla\psi$, $V_\chi = -\nabla\chi$

So $\nabla \cdot \vec{V}_H = -\nabla^2\chi$, $k \cdot \nabla \times V_H = \nabla^2\psi$

Now the inequality argument. We first assume a steady state circulation. This means all $\frac{\partial}{\partial t}(\,)$ terms above are zero. Next, starting with the ψ eq., we note that frictional dissipation is always negative $\Rightarrow \therefore \langle K_\chi, K_\psi \rangle$ is positive. Next on to the χ eq. With $\langle K_\chi, K_\psi \rangle$ positive, F_χ negative, Thus we must have $\langle E_{P+I}, K_\chi \rangle$ be positive. Finally onto the E_{P+I} eq. With $\langle E_{P+I}, K_\chi \rangle$ positive and F_{I+P} negative, we must have $G > 0$.

That is, $G \propto \overline{H'T'}$ ≡ diabatic heating drives the monsoon system

↳ warming where warm, cooling where cool $\Big\}$ $\overline{H'T'} > 0$

In greater detail, the area averaged energy eqs for a closed domain are

$$\frac{\partial \overline{K_\psi}}{\partial t} = B_\psi + \left[\overline{f\nabla\psi \cdot \nabla\chi} + \overline{\nabla^2\psi\nabla\psi \cdot \nabla\chi} + \overline{\nabla^2\chi(\nabla\psi)^2/2} + \overline{\omega J(\psi, \tfrac{\partial\chi}{\partial p})}\right] + \overline{F_\psi}$$

$$\frac{\partial \overline{K_\chi}}{\partial t} = B_\chi - \overline{\chi\nabla^2\phi} - \boxed{\psi\text{-}\chi \text{ interaction terms}} + \overline{F_\chi}$$

└ conversion between E_{P+I} + K_χ

$$\frac{\partial \overline{E_{P+I}}}{\partial t} = B_{P+I} + \overline{\chi\nabla^2\phi} + \overline{G_{P+I}} + \overline{D_{P+I}}$$

For a closed system the boundary flux terms are identically zero since there is no net increases or decreases in the total domain energy. Therefore, $B_\psi = B_\chi = B_{P+I} = 0$.

The term $\chi\nabla^2\phi$ represents the conversion of potential plus internal energy into K.E. of the divergent flow. Note that

$$\overline{\chi\nabla^2\phi} = \overline{\phi\nabla^2\chi} = -\overline{\phi\frac{\partial\omega}{\partial p}} = \overline{\omega\frac{\partial\phi}{\partial p}} = -\overline{\omega\alpha} = -\frac{R}{p}\overline{\omega T}.$$

This term denotes the ascent of relatively warm air and the descent of relatively cold air within the domain. It is interesting to note that $-\overline{\omega T}/p > 0$ supplies energy to drive the divergent circulations (Hadley as well as Walker cells) while the nondivergent circulation can only receive energy from the divergent motions.

The total K.E. of the nondivergent motions $\overline{K_\psi}$ can only increase (in a closed domain) via ψ-χ interactions. In the absence of dissipation and ψ-χ interactions, $\frac{\partial\overline{K_\psi}}{\partial t} = 0 \Rightarrow$ as in the case of barotropic nondivergent dynamics, the rotational K.E. is conserved in an area averaged sense.

Consider the ψ-χ interaction terms

*(1) $\overline{f\nabla\psi \cdot \nabla\chi}$: magnitude depends on the orientation of the vectors $\nabla\psi$ and $\nabla\chi$. If in the N.H. they are nearly parallel, then energy exchanges go from the divergent to rotational modes. The converse holds true if $\nabla\psi$ and $\nabla\psi$ are antiparallel. If $\nabla\psi$ and $\nabla\chi$ are orthogonal there is not energy exchange between the rotational + divergent motion fields.

(2) $\overline{\nabla^2\psi\nabla\psi \cdot \nabla\chi}$: Note that as defined the Laplacian of the streamfunction is the vertical component of relative vorticity. Thus this term is similar to term (1) except that it is scaled by the relative vorticity, $\nabla^2\psi$ instead of the planetary vorticity. In regions of cyclonic vorticity $\nabla^2\psi > 0$ so KE goes from χ to ψ.

(3) $\overline{\nabla^2\chi\frac{(\nabla\psi\cdot\nabla\psi)}{2}}$: This represents a covariance between the horizontal divergence and the K.E. of the nondivergent component. At a single point the contribution of this term is of the form,

$\frac{\partial}{\partial t}\left(\frac{\nabla\psi\cdot\nabla\psi}{2}\right) = \nabla^2\chi\left(\frac{\nabla\psi\cdot\nabla\psi}{2}\right)$ and leads to exponential growth of the nondivergent KE whenever the horizontal convergence ($\nabla^2\chi > 0$) is positive.

(4) $\overline{\omega J(\psi, \frac{\partial\chi}{\partial p})}$: not simple to interpret covariance between vertical motion and the negative of the advection of the vertical gradient of χ by the rotational component of the wind. Regions of large $\frac{\partial\chi}{\partial p}$ are generally associated with regions of large vertical variations of convergence. Following streamfunction downstream from these regions, energy exchange from irrotational to the nondivergent flows can occur if there is general upward motion.

Order of magnitude calculations shows that in the K_ψ + K_χ eqs, the dominant term are $\overline{f\nabla\psi\cdot\nabla\chi}$ and $\overline{\nabla^2\psi\nabla\psi\cdot\nabla\chi}$. The magnitude of these terms depend on the orientation of $\nabla\psi$ and $\nabla\chi$ and the sign of f and $\nabla^2\psi$. The geometry of the low level monsoon flow is anticyclonic just north of the equator as $\nabla^2\psi < 0$ while $f > 0$. Thus the first two terms oppose each other, although $\overline{f\nabla\psi\cdot\nabla\chi}$ is the largest term.

* dry and moist static stability (trade wind)

5533

MET? : T. N. Krishnamurti (?)

- Discuss how the following processes stabilize and/or destabilize the dry and moist static stability of the trade wind environment.

a) cloud top cooling (what clouds and where and how)
b) large scale divergence → stabilize.
c) air sea interaction (surface fluxes)
d) cloud top evaporation
e) large scale advective processes (horizontal and vertical)

look at my note p10~12

$$\frac{d\Gamma_d}{dt} = -\Gamma_d \frac{\partial \bar{\omega}}{\partial p} + \frac{\partial^2 \overline{\omega'\theta'}}{\partial p^2} - \frac{1}{C_p}\left(\frac{p_0}{p}\right)^{R/C_p} \sum_i \frac{\partial H_i}{\partial p}$$

↑ (b)

* dry static stability + maintenance of trade wind inversion

5593
MET? : Krish? (?) → note p10 + 11

- Derive (or write down) an equation for the time rate of change of dry static stability. (Include effects of heat sources and sinks). Discuss qualitatively the maintenance of the trade wind inversion using the above framework.

Sol) Start w/ the thermodynamic eq. in the form

$$C_p \frac{T}{\theta}\frac{d\theta}{dt} = \sum_i H_i \qquad (1)$$

where $\sum_i H_i = H_R + H_{con} + H_{evap} + H_{sen}$

(H_R → radiation, H_{con} → condensation, H_{evap} → evaporation, H_{sen} → sensible heat)

Expand (1) as (※ look at my note p10) $\qquad \frac{\theta}{T} = \left(\frac{P_0}{P}\right)^{R/C_p}$

$$\frac{\partial\bar\theta}{\partial t} + \bar{V}\cdot\nabla\bar\theta + \bar\omega\frac{\partial\bar\theta}{\partial p} = -\frac{\partial\overline{\omega'\theta'}}{\partial p} + \frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\sum_i \overline{H_i} \qquad (2)$$

where we have decomposed all variables into a mean and deviation(′), we have only retained the vertical eddy flux of eddy potential energy.

Define the dry adiabatic lapse rate $\bar\Gamma_d = -\frac{\partial\bar\theta}{\partial p}$ and take $-\frac{\partial}{\partial p}$ of (2)

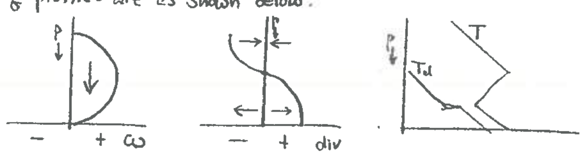
$$\frac{\partial\bar\Gamma_d}{\partial t} + \bar{V}\cdot\nabla\bar\Gamma_d + \bar\omega\frac{\partial\bar\Gamma_d}{\partial p} = -\bar\Gamma_d\frac{\partial\bar\omega}{\partial p} + \frac{\partial^2\overline{\omega'\theta'}}{\partial p^2} - \frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\sum_i\frac{\partial H_i}{\partial p}$$

where we have used the fact that $\frac{\partial \bar{V}}{\partial p}\cdot\nabla\bar\theta = 0$ (thermal wind relation says $\frac{\partial V}{\partial p} \perp \nabla\theta$).

We rewrite the above as

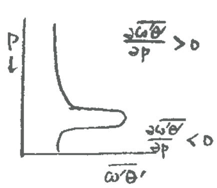
$$\frac{d\bar\Gamma_d}{dt} = -\bar\Gamma_d\frac{\partial\bar\omega}{\partial p} + \frac{\partial^2\overline{\omega'\theta'}}{\partial p^2} - \frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\sum_i\frac{\partial H_i}{\partial p} \qquad (3)$$

* In the trade wind inversion environment, the typical ω, $\nabla\cdot V$ and T, q profiles are as shown below.



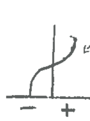
- The largest source term in (3) for maintaining $\bar\Gamma_d$ in the trade wind inversion is $-\bar\Gamma_d\frac{\partial\bar\omega}{\partial p}$. Since $\bar\Gamma_d > 0$ and from above $-\frac{\partial\bar\omega}{\partial p} > 0$, $-\bar\Gamma_d\frac{\partial\bar\omega}{\partial p} > 0$. This term (the divergence term, since $-\bar\Gamma_d\frac{\partial\bar\omega}{\partial p} = \bar\Gamma_d(\nabla\cdot V)$) stabilizes the atmosphere.

- The $\frac{\partial^2\overline{\omega'\theta'}}{\partial p^2}$ term (→ lifting of the inversion layer.): The observed normal distribution of the covariance $\overline{\omega'\theta'}$ is as shown



we see that $\frac{\partial\overline{\omega'\theta'}}{\partial p} > 0$ above inversion base

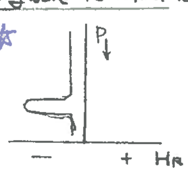
$\frac{\partial\overline{\omega'\theta'}}{\partial p} < 0$ below 〃 〃



∴ $\frac{\partial^2\overline{\omega'\theta'}}{\partial p^2} < 0$

The $\frac{\partial^2\overline{\omega'\theta'}}{\partial p^2}$ term destabilizes the trade wind inversion partly. No → look at my note p11.

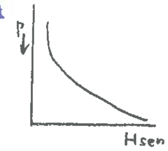
- Longwave radiation is a cooling effect. $\left(\because \frac{d\bar\Gamma_d}{dt} \propto -\frac{\partial H_R}{\partial p}\right)$



$\frac{\partial H_R}{\partial p} > 0$ below the inversion base → destabilize

$\frac{\partial H_R}{\partial p} < 0$ above 〃 〃 → stabilize

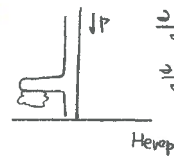
- Sensible heat flux



$\frac{\partial H_{sen}}{\partial p} > 0$ and $-\frac{\partial H_{sen}}{\partial p} < 0$

The sensible heat effect destabilizes trade wind inversion

- evaporative cooling



$\frac{\partial H_{evap}}{\partial p} < 0$ above ⇒ $-\frac{\partial H_{evap}}{\partial p} > 0$ → stabilize above inversion

$\frac{\partial H_{evap}}{\partial p} > 0$ below ⇒ $-\frac{\partial H_{evap}}{\partial p} < 0$ → destabilize below inversion

- H_{con} : non-precipitating clouds set to 0.

<Trade wind inversion>

* moist static stability

5583

MET? : T. N. Krishnamurti (?) ⟶ note p11 + 12

- Derive an equation for the time rate of change of moist static stability and discuss the various mechanisms that can contribute to the stabilizing and destabilizing of a sounding.

√ How is the conditionally instability of the large scale tropics restored continuously.

look at my note p11 + 12 + 16

* moist stability eq.

MET5533: T. N. Krishnamurti (?)

- Given the moist stability equation

moist static energy

$$\frac{\partial \overline{\Gamma_m}}{\partial t} = -\overline{\mathbf{V}} \cdot \nabla \overline{\Gamma_m} - \overline{\Gamma_m} \frac{\partial \overline{\omega}}{\partial p} - \overline{\omega} \frac{\partial \overline{\Gamma_m}}{\partial p} + \frac{\partial^2}{\partial p^2} \overline{E_m' \omega'} - \frac{\partial}{\partial p} \sum_i \overline{H_i}$$

where $$\sum_i \overline{H_i} = L\overline{E}_B + \overline{H}_R + \overline{H}_{SEN}$$

Discuss the role of 'Radiative destabilization' and of the 'vertical eddy flux term'.

look at my note p11 & p12

* African easterly jet

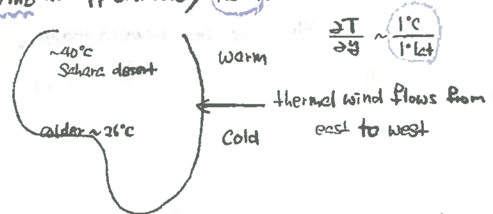
5533

MET?: T. N. Krishnamurti (?)

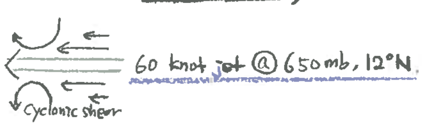
• Describe the African easterly jet (location, seasons, altitude, intensity, extent). Discuss its location in reference to the local Hadley cell with illustration (label all coordinates). Discuss the role of barotropic and combined barotropic-baroclinic instability of this environment and the possible growth of African waves.

Sol) Intense surface heating over North Africa during summer results in an easterly thermal wind which sets up a strong jet maximum around 650 mb at approximately 12°N.

$\frac{\partial T}{\partial y} \sim \frac{1°C}{1° lat}$

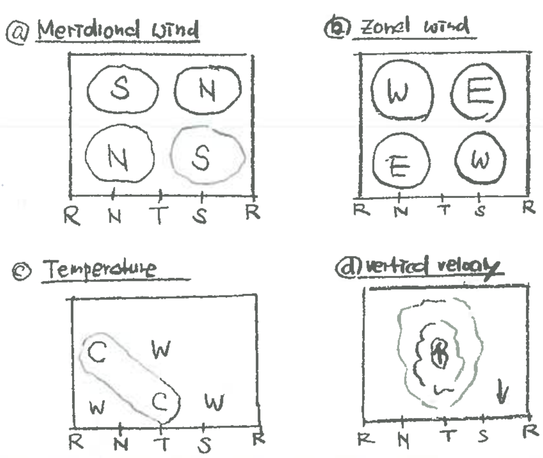


Synoptic-scale disturbance (~2500km) develop on the cyclonic shear side of the jet. These disturbances typically have phase speed of 8 m/s (5° to 7° lon/day) and develop at a period of 4 to 6 days. We call this disturbances African easterly waves



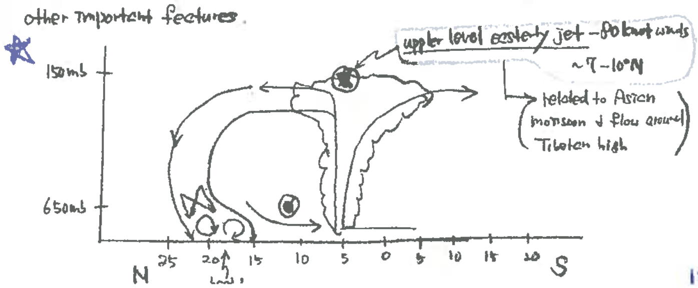
Composite analysis of easterly waves using GATE data reveal the following structural characteristics.

- R = ridge – fair wx
- T = trough – rain
- N = northerlies – pretrough
- S = southerlies – post trough

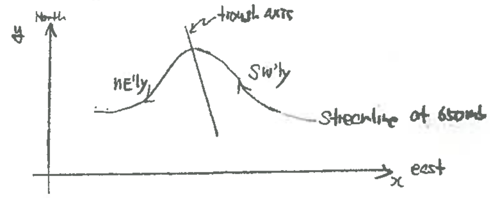


The above diagrams are anomaly fields.

• A meridional cross-section through the low level jet reveals several other important features.



The meridional wind in easterly waves that move off the west African coast has the largest amplitude ~650mb and is strongly correlated w/ the zonal wind anomalies.



There is strong cyclonic vorticity at 650mb and strong anticyclonic vorticity at 200mb. Note that easterly waves generally form on the cyclonic shear side of the jet (at 650mb) and on the anticyclonic shear side of the upper level (150mb) easterly jet. This structure in vorticity from cyclonic at low levels to anticyclonic in the upper level suggest a vertically deep disturbance which exists throughout the troposphere. The upward vertical motion is correlated w/ the cold temperature anomaly. The disturbance (African waves) is a cold core low up to ~700mb and warm core above it.

The convergence of westerly momentum flux into the easterly jet weakens the jet and amplifies the wave as $K_Z \rightarrow K_E$. This is the barotropic instability mechanism ⇒ Shear of the horizontal zonal flow is extracted and realized as K.E. of the eddy (the wave). Burpee also found that baroclinic effects play a role in easterly waves. In particular he found that $\overline{v'T'} < 0$ ⇒ heat is transported equatorward while $\frac{\partial \overline{T}}{\partial y} > 0$ (temp. increases poleward). This is heat flow down the temperature gradient and is consistent with what we observe in mid-latitude baroclinic waves.

Thus the African wave is thought to be maintained by both horizontal (barotropic) and vertical (baroclinic) shear and not by convection.

This was what Charney's initial value approach suggested.

The necessary condition for the barotropic instability mechanism to be operative is that the meridional gradient of the absolute vorticity must change sign over some specified meridional distance. The necessary condition for this to occur (inflection point instability) is that

$$\frac{d}{dy}(\zeta + f) = 0 \quad \text{for some } y \in [y_{min}, y_{max}]$$

* African waves → barotropic process

MET5533: T. N. Krishnamurti (60 minutes) *

- The barotropic transfer of energy for a mean zonal flow to eddies on the scale of African waves is considered important. Discuss the following in the context:
  - (a) Synoptic aspects of African waves and its broad scale environment. → look other notes
  - (b) Observed scale (horizontal and vertical) and frequency of these waves. → " " "
  - ✓(c) What computation would you carry out to establish the importance of the barotropic process. Present a detailed framework: e.g. energetics, inflection point instability, growth rate versus scales.

* Note for Barotropic growth of wave

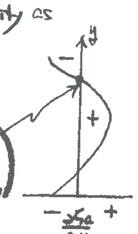
a) Inflection point instability

: The meridional gradient of absolute vorticity of basic flow changes sign with the region

Let ζ_a be absolute vorticity as

$$\zeta_a = f - \frac{\partial U}{\partial y}$$

(At this point the shear vorticity is going fast to curvature vorticity)

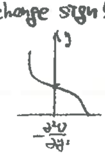


$$\int_{-d}^{d} \left(\frac{\partial \zeta_a}{\partial y}\right) dy = 0$$

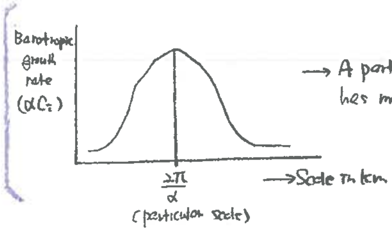
→ this term should change sign ??

$$\frac{\partial \zeta_a}{\partial y} = \beta - \frac{\partial^2 U}{\partial y^2}$$

(sign does not change in tropics)



b) Barotropic growth rate diagram



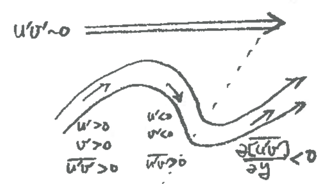
→ A particular scale wave has maximum growth rate

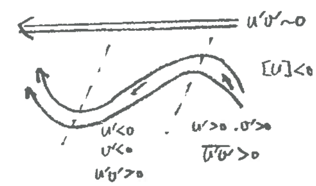
c) Barotropic energetics

The barotropic growth of eddy kinetic energy is the decay of zonal kinetic energy

$$\frac{\partial \overline{K}_E}{\partial t} = \overline{[U]\frac{\partial}{\partial y}[u'v']} = \langle K_Z \cdot K_E \rangle = -\frac{\partial \overline{K_Z}}{\partial t}$$

The eqs states that convergence of westerly momentum flux in a westerly current increases the zonal K.E.





Convergence of westerly momentum flux tends to weaken the low level African jet.

⇓

Easterly jet weakens the easterly wave amplifies.

$$-\frac{\partial \overline{K_E}}{\partial t} = [\bar{u}]\frac{\partial [\overline{u'v'}]}{\partial y} > 0$$

→ look my note p14?

* Tropical instability mechanisms

3 major types of instability considered → horizontal shear, vertical shear, convection

① barotropic
② baroclinic
③ CISK → look others

① horizontal shear – barotropic instability

- As earth vorticity is small in tropics, conservation of absolute ζ implies conversion between shear & curvature vorticity.
- the necessary condition for barotropic instability is that the gradient of ζ_a change sign over the domain of interest.
- Barotropic instability only appears to be important off west coast of Africa
- 1D process

② vertical shear

- We do not find free baroclinic instability in the tropics
- the necessary condition for combined barotropic – baroclinic instability is that the NS gradient of "potential vorticity" must change sign
- barotropic/baroclinic stability can occur off west coast of Africa and NW Australia.
- 2D process

* African wave → barotropic energy exchange, combined barotropic–baroclinic instability

MET5533: T. N. Krishnamurti (?)

- Question

1. Given the following expression for the barotropic energy exchange from zonal to the eddy motion:

$$\langle K_z \cdot K_E \rangle = \overline{\overline{[u]\frac{\partial}{\partial y}[u'v']}}$$

   Discuss its applicability to the west-African easterly wave and the west-African low level jet. → look other notes
2. a) What is meant by the term 'combined barotropic-baroclinic instability'. → look my note p13.
   b) What parameter do you examine for studying the necessary condition for the combined instability. What is that supposed to tell us about the African wave. G_a, G_{pa}
   c) Provide a short summary (1/2 page) on the main findings by RENNICK on the initial value approach to the combined instability for the growth of African waves.

* Rennick paper (The generation of African waves)

abstract

A linearized, pseudo-spectral, primitive eq model is used to simulate the response of the low-level easterly jet over the northern Africa to perturbations on the scale of African waves. The model results show that the jet is unstable due to both its horizontal and vertical shears. The most unstable wave supported by the jet has a wavelength of 3000km and a period of 2–2.5 days. It attains its maximum intensity at the 700mb level, near 14°N. This compares favorably with the characteristics of the observed waves.

The K.E. of the waves grows at the expense of the K.E of the mean jet. Energy is transferred at approximately equal rates by the horizontal and vertical Reynolds stresses. Energy conversions involving APE are nearly an order of magnitude smaller, reflecting the fact that the K.E of the waves accounts for about 90% of the total wave energy.

→ also look at my note p13'

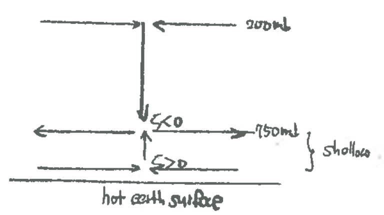
* heat lows

1

MET5533: T. N. Krishnamurti (?) *

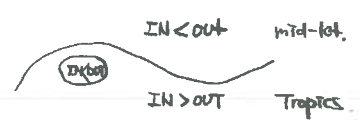
- This question relates to tropical/subtropical "heat lows" over deserts.
  - a) What are the typical vertical distributions of vertical motion, divergence and relative vorticity in these systems.
  - b) What do the satellite signatures of net incoming minus the net outgoing radiation look like at the top of the atmosphere in these systems.
  - √c) Describe the framework for the heat budget of the heat low emphasizing relative contributions from the role of adiabatic descent, radiative components, lateral import/export of energy and the maintenance of the thermal stratifications.
  - d) What are the plausible teleconnections of the heat lows with the rain producing systems around it.

(a)



the lower, shallow cell has surface convergence in the heat low, rising air and divergence at 850mb (cyclonic vorticity)
the upper, deeper cell, starting with divergence at 850mb, has descending air with weak convergence above this level

b) In general, the incoming minus the outgoing radiation is positive over the tropics and negative over the higher latitudes. A major exception occurs over the desert regions where satellite measurements show that the outgoing radiation exceeds the incoming radiation



* c) The first law of thermodynamics,

$$\frac{d\theta}{dt} = \frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p} \sum_i H_i$$

↓ domain average, continuity eq

$$\overset{①}{\frac{\partial[\theta]}{\partial t}} = -\overset{②}{[\underset{\sim}{V}]\cdot[\nabla\theta^*]} - \overset{③}{[\omega]\frac{\partial[\theta]}{\partial p}} + \overset{④}{\frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\left[\sum_i H_i\right]} - \overset{⑤}{[\nabla\cdot\underset{\sim}{V}^*\theta^*]} - \overset{⑥}{\frac{\partial}{\partial p}[\omega^*\theta^*]}$$

where [ ] : average over the domain
* : deviations from the mean value

Term②: average horizontal advection of heat.
Term③: average adiabatic heating.
Term④: net radiative heating
Term⑤: mean horizontal convergence of eddy heat flux
small, neglect
Term⑥: sensible heating due to vertical variation of the mean convective and turbulent heat flux

↳ term⑥ is calculated as a residual.

⇒ adiabatic descent (Term③)
Adiabatic warming is associated with the downward motion.
this adiabatic motion acts to stabilize the potential temperature profile

⇒ radiative components (Term④)
There is a net radiative heat loss during a 24h periods at all levels except for a slight gain around 550-450mb

⇒ lateral import/export of energy (Term②)

* During the pre-sunrise hours, the horizontal advective and longwave radiative processes contribute a cooling which is nearly balanced by the heating due to vertical advective processes.

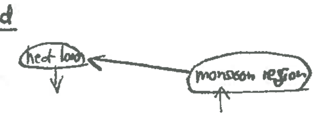
* During the midday hours, there is net radiative heating and heating due to vertical advective processes. The horizontal advective processes give a cooling up to 400mb and a heating above. Approximately half of sensible heat estimated to leave the earth's surface is transformed to the atmosphere between the surface and 975mb and half 975mb and 650mb.

* A crucial element in the maintenance of stratification over the desert region is the importation of heat into the upper troposphere coupled with subsidence above this region

* Energy supply most likely comes from planetary-scale divergent circulation.

(d) The downward motions are in fact part of a large scale vertical overturning which is associated with the Hadley and east-west circulations.

W field



★ Several terminology in TM
(concepts.

5533 or 5534
MET?: T. N. Krishnamurti (60 minutes)

• Try to write a small paragraph, e.g. for the Glossary of Meteorology, for the following:

(a) Equatorial Kelvin waves
(b) Rossby gravity waves → waves that resembles an inertio-gravity wave for long zonal scale ($k \to 0$) and resembles a Rossby wave for zonal scale's characteristic of synoptic-scale disturbances.
dispersion eq.
$$\nu = k\sqrt{gH}\left[\frac{1}{2} \pm \frac{1}{2}\left(1+\frac{4\beta}{k^2\sqrt{gH}}\right)^{1/2}\right]$$
+ root : an eastward-propagating equatorial inertio-gravity wave
− root : a westward- 〃 〃 〃 〃
(c) Wave Cisk
(d) PNA pattern
(e) Madden Julian wave
(f) Heat low.

(a) Equatorial Kelvin waves, (b) Rossby gravity wave

• Consider the linearized momentum + continuity eqs for a fluid system of mean depth h_e in a motionless basic state on a beta plane.

$$(1a)\quad \frac{\partial u'}{\partial t} - \beta y v' = -\frac{\partial \phi'}{\partial x} \qquad \phi = gh$$
$$(1b)\quad \frac{\partial v'}{\partial t} + \beta y u' = -\frac{\partial \phi'}{\partial y}$$
$$(1c)\quad \frac{\partial \phi'}{\partial t} + gh_e\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0$$

Assume
$$(2)\quad \begin{pmatrix} u' \\ v' \\ \phi' \end{pmatrix} = \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\phi}(y) \end{pmatrix} e^{ik(x-ct)}$$

• For the equatorial Kelvin wave the meridional velocity perturbation vanishes. Set $v' = 0$ and subst. (2) into (1)

$$k(-ic\hat{u}) = -ik\hat{\phi}$$
$$\beta y \hat{u} = -\frac{\partial \hat{\phi}}{\partial y}$$
$$-ikc\hat{\phi} + gh_e ik\hat{u} = 0$$

($c\hat{u} = \hat{\phi}$)

$$-ikc^2\hat{u} + gh_e ik\hat{u} = 0$$
$$\boxed{c^2 = gh_e}$$

dispersion relation same as that for shallow water gravity waves

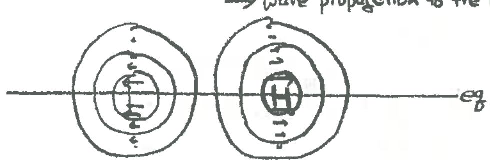
$$-ikc\frac{\partial \hat{u}}{\partial y} = -ik\frac{\partial \hat{\phi}}{\partial y}$$
$$-ik\beta y \hat{u} = +ik\frac{\partial \hat{\phi}}{\partial y}$$
$$-ikc\frac{\partial \hat{u}}{\partial y} - ik\beta y\hat{u} = 0$$
$$-c\frac{\partial \hat{u}}{\partial y} = \beta y \hat{u}$$
$$\frac{\partial \ln \hat{u}}{\partial y} = -\frac{\beta y}{c}$$
$$\ln\frac{\hat{u}}{\hat{u}_0} = -\frac{\beta}{c}\frac{y^2}{2}$$
$$\hat{u}(y) = \hat{u}_0(y)\, e^{-\left(\frac{\beta y^2}{2c}\right)}$$

(y; eq. y=0)

for bounded solution $c > 0$ in which case $\hat{u}(y) \to 0$ as $y \to \infty$
The waves are "trapped"

Thus, Kelvin waves are eastward propagating w/ zonal velocity + geopotential perturbations that vary in latitude (y) as Gaussian functions centered on the equator. In the zonal direction the balance of forces is exactly that for a eastward propagating shallow water gravity wave. The meridional force balance of a Kelvin wave is an exact geostrophic balance between the zonal velocity + perturbation meridional pressure gradient. It is the change of sign at the equator that permits this special type of equatorial mode to exist.

⇒ wave propagation to the east.



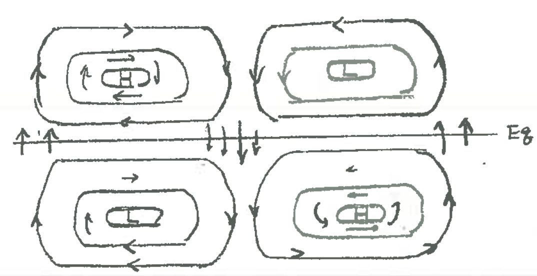
• If we retain v' and repeat the above analysis we find a cubic dispersion relation of the form

$$\frac{\sqrt{gh_e}}{\beta}\left(-\frac{k}{\nu}\beta - k^2 + \frac{\nu^2}{gh_e}\right) = 2n+1, \quad n = 0, 1, 2, \cdots$$

→ dispersion relation for equatorially trapped, free oscillation for zonal wave number k and meridional mode number n.

The 3 solutions are
① eastward propagating gravity wave } equatorially trapped
② westward 〃 〃 〃
③ Westward propagating Rossby wave

When $n=0$ the meridional velocity perturbation is centered on the equator with a Gaussian distribution in latitude. Only two modes are retained in this case. One is an eastward propagating gravity wave. The other is a westward propagating gravity wave. This mode resembles an inertio-gravity wave for long zonal scales ($k \to 0$) and resembles a Rossby wave for synoptic scales. We call this a Rossby-gravity wave.
The structure for a Rossby gravity wave is shown below.



(c) Wave CISK (Conditional Instability of the Second kind) → look next page also!

ordinary conditional instability (of the 1st kind) produces maximum growth rates for motions on the scale of individual cumulus. It cannot be used to explain the synoptic scale organization of the motion. Observations indicate that the mean tropical atmosphere is not saturated (even in the PBL). Thus a parcel must undergo a considerable amount of forced ascent before it becomes positively buoyant. Such forced ascent will occur in an organized manner only in regions of low level convergence. The cumulus convection + large scale motion must then be viewed as cooperatively interacting. The cumulus supplies the heat necessary to drive the large scale disturbance, and the large scale disturbance produces the moisture convergence necessary to drive the convection.
When the cooperative interaction between the cumulus convection and a large scale perturbation leads to unstable growth of the large scale system, the process is referred to as CISK.

In wave-CISK, the low level convergence is simply the convergent velocity field associated with the wave itself.

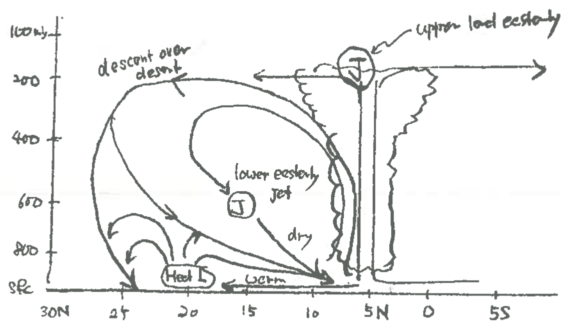
(d) PNA pattern → look at other notes.

(e) Madden-Julian wave → look at other notes and below

(f) heat low

The heat low is a very shallow feature often found over large deserts (Sahara, Arabian, desert region of SW US etc.) Near the sfc, there is a strong warming of the air due to large fluxes of LW radiation, reflected SW radiation and sensible heat flux during the daytime hours. There is large-scale ascending motion of the warm air in the lower atmosphere. The heat low is a region of an anomalous heat sink as the OLR exceeds incoming SW radiation. Large scale descent above the shallow heat low develop to maintain a thermal balance.

Dry convection rapidley removes superadiabatic lapse rates. The warm tropospheric columns call for an upper anticyclone while large scale descent calls for upper level convergence. The upper anticyclone which forms is a dynamic anticyclone which is most likely maintained by lateral input of anticyclonic relative vorticity in the upper troposphere. This lateral input can come from the Asian monsoon region where the poleward side of the tropical easterly jet contains large amounts of anticyclonic relative vorticity which is steadily advected downstream.



<Cross section over Africa>

(e) Madden-Julian wave

(I) Nearly 10 years of daily rawinsonde data for Canton Island (3S, 172W) were subjected to spectral analysis which revealed the following in the 30-50 day mode (Madden + Julian, 1979)

(1) Peaks in the zonal wind were stronger in the lower troposphere, weak to non-existent in the 700-400mb layer + strong again in the upper troposphere.

(2) Station pressure possessed a peak which was in phase w/ the lower tropospheric zonal wind oscillation and out of phase w/ that in the upper troposphere.

(3) Tropospheric temperatures exhibited a similar peak: positive station pressure anomalies were associated w/ negative temperature anomalies throughout the troposphere.

This kind of variability in the 30-50 day mode has been analyzed in many other places in the tropics, one such area being the monsoon region.

(II) Observational aspects of the Indian Summer Monsoon in the context of the 30-50 day mode.

① meridional propagation

(1) meridional propagation of zonally oriented cloud lines over Indian subcontinent from equatorial latitudes to Himalayas.

(2) meridional propagating trains of cyclonic + anti-cyclonic flow patterns in the lower troposphere

(3) precipitation anomalies w/ periodicities ~ 50 days

(4) geopotential + wind fluctuations in the lower + upper troposphere with periodicities ~ 40 days.

② Zonal propagation

(1) zonal propagation of tropical OLR

(2) eastward propagating divergent wind field @ 200mb.
↳ planetary wave #1 which exists all year with this timescale

(3) in-phase propagation of sea level field accompanying upper level divergent wave.

③ Air sea interaction

(1) oscillation in SST over western Pacific + Bay of Bengal.
↳ 0.3 to 1.0°C

(2) latent + sensible heat fluxes → changes due more to fluctuations in wind speed + SSTs. Moisture and temperature oscillation did not seem to be important
↳ 30 ± 3 W/m²
↳ 200 ± 30 W/m²

④ Energy exchange

K.E. of low frequency mode (LFM) on 30-50 day timescale is primarily maintained by the following processes.

(1) They recieve a substantial amount of K.E from the high frequencies

(2) They gain K.E from the long term mean flow

(3) They recieve a small amount of energy from the potential energy on the same frequencies.

(III) Some past results from Krish

Over India it is the meridional passage of the cyclonic and anti-cyclonic flow which modulate the monsoon on the 30 to 50 day timescale. Dry spells occur when the LFM is out of phase w/ the climatological flow. Wet spells occur when the LFM flow is in phase w/ the climatological flow.

Over China the 30-50 day modulation of the oceanic components of the flow towards the land mass enhances the onshore circulation of the subtropical highs.

LFM in phase w/ onshore climatological flow brings monsoon rains.
LFM out of phase (anti-parallel) to climatological flow brings dry spell

Over Australia it is the enhancement of the monsoon trough between 10° + 15°S (which is the ITCZ over this region) that undergoes a modulation from the passage of the LFM

again → when LFM flow + climo. flow are in phase → wet spell
→ " " " " " " anti-parallel → dry spell

(c) Wave CISK again

Short for conditional instability of the second kind usually considered the primary factor for tropical development and sustenance by Charney + Eliassen (1964)

The cooperative interaction of cumulus convection and the large-scale vertical motion.

"The cumulus convection supplies the heat necessary to derive the large-scale disturbance, and the large-scale disturbance produces the moisture convergence necessary to derive the cumulus convection".

★ Several concepts in TM

5533

MET? : T. N. Krishnamurti (?)

• Define the following terms and describe them as well:

a) Conditional instability
b) Combined barotropic-baroclinic instability
c) PNA pattern
d) Equatorial Kelvin wave
e) Mixed Rossby gravity wave
f) Madden-Julian oscillation

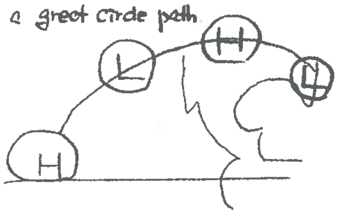
Sol)

a) Conditional instability → look at other notes

b) Combined barotropic-baroclinic instability → look at my note (p13')

c) PNA pattern

The observed upper tropospheric height anomalies during NH winter. This pattern suggests a train of stationary Rossby waves that emanates from the equatorial source region and follows a great circle path.



d) Equatorial Kelvin wave

waves that are eastward propagating and have zonal velocity and geopotential perturbations that vary in latitude as Gaussian functions centered on the equator.

1) phase speed : $c = \pm\sqrt{gH}$

→ identical to that for shallow-water gravity waves.

2) Perturbation zonal velocity

$$\hat{u} = u_0 e^{-\beta y^2/2c}$$

→ decaying away from the equator ($c > 0$)

3) The meridional force balance :

$$\beta y \hat{u} = -\frac{\partial \hat{\Phi}}{\partial y}$$

→ extract geostrophic balance between the zonal velocity and the meridional pressure gradient.

e) look at the previous note.

f) Madden-Julian oscillation

The 30-50 day mode revealed from the spectrum analysis of nearly 10 years of daily rawinsonde data for Canton Island (3S, 172W).

1) Peaks in the zonal wind were strong in the lower and upper troposphere, weak to non-exist in the 700-400 mb layer.

2) Station pressure has a peak

- in phase —— lower level zonal wind oscillation
- out of phase —— upper " " " "

3) Temperature

⊖ anomaly — ⊕ pressure anomaly

Observed in many other places in the tropics (monsoon region)

★ Several concepts in TM

MET5533: T. N. Krishnamurti (?) *

- Describe the following in one paragraph each:
  - ✓ a) Ozone hole,
  - ✓ b) Nuclear freeze,
  - c) Sahelian drought,
  - d) El Nino and
  - ✓ e) VAS soundings.

a) Ozone hole → look at Ruscher's note

b) Nuclear freeze → " " "

✓ c) Sahelian drought

Strong persistence of the Sahel drought can be understand if a strong positive feedback mechanism is operating, partly driven by changes in surface properties. The key factors in the mechanisms thus far studied are the surface albedo and soil moisture, both of which affect the radiation balance at the surface, the first directly, the second directly through its influence on the latent heat flux.

In deserts, there are three primary land surface influences;

1) high surface albedo
2) lack of soil moisture and evapotranspiration
3) low surface drag due to the absence of vegetation

It was found that the influences of soil moisture and surface albedo tend to mitigate each other, i.e., an increase in surface albedo produces subsidence and moisture divergence near the surfaces, whereas, a reduction in evapotranspiration produces a thermal low, rising motion and moisture convergence.

d) El nino → look at other notes.

✓ e) VAS soundings

The Visible Infrared Spin Scan Radiometer Sounder has eight visible channel detectors and six thermal detectors to sense IR in 12 spectral bands. Retrieval of accurate temperature and moisture profiles from these radiances can be accomplished by using the radiative transfer equation, the solution of which relies upon the physical properties of the measurements.

Clouds are delineated by white areas, high water vapor concentrations by blue + green, dry, cloud free areas by dark brown + black.

*

* Several concepts in TM

5533

MET?: T. N. Krishnamurti (?)

- Explain the following concepts very clearly

a) Inflection point instability ✓ ⟶ already answered
b) Conditional instability (barotropic/baroclinic) ↘
c) Radiative destabilization (cloud-radiative) ⟶ dry + moisture static stability (trade wind inversion)
d) Non-convective rain ⟶ look at TM II or NWP
e) Pacific North American Patterns
f) East-West circulation

look at other notes ?

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* Some notes for expecting questions.

- Available Potential energy
  - The total potential energy that can be converted to K.E
  - Explosive cyclone (1mb/1hr) deepening cyclone
    - over ocean (latent heat release, CISK, APE
    - over land — strong upper-level forcing.

- Potential vorticity
  - It is important because it indicates the relationship between vorticity and stability as a conservative traces for the study of parcel motion.

- Hadley circulation (J. Bjerknes, 1966)
  - refers to the rising motion at the thermal equator and simultaneous sinking motion in the belt of subtropical highs.

- Condition for baroclinic Instability
  - The horizontal temperature gradient (or equivalently thermal wind - vertical wind shear) must be strong to overcome stabilizing β effect

- Energy conversions during baroclinic instability
  - ZAPE ⟶ EAPE ⟶ EKE ⟶ frictional dissipation + ZKE

- Condition for barotropic instability ↙ inflection point instability
  - Somewhere in the domain, $\frac{\partial}{\partial y}(f-\frac{\partial u}{\partial y})=0$
  - ZKE ⟶ EKE

- Wave interaction
  - The exchange of energy from one to another wave number
  - Includes the interactions w/ all other wave number
  - Conversion of energy — through nonlinear term
  - Involve at least three wave number

- Enstrophy
  - Global integral of one-half of vorticity squared related to rotational KE : $\int_m \frac{1}{2}\zeta^2 dm$
  - Conserved in nondivergent frictionless barotropic flow.

- baroclinic instability
  - the process by which the baroclinic wave, induced mainly by horizontal advectio[n], amplify APE mainly due to planetary-scale temperature gradient converts to KE.
  - Energy transformation of baroclinic instability.
    1. ZAPE is generated by differential heating
    2. ZAPE ⟶ EAPE as a result of eddy advection of temperature.
    3. EAPE ⟶ EKE by slantwise convection.
    4. EKE ⟶ TE : dissipation
       ↘ ZKE : small dissipation by turbulence.

- Energy cascade
  - This is a consequence of the 2nd law of thermodynamics, which can be expressed as a statement to the effect that,
    In the absence of external forcing, any system must tend toward a state of greater and greater randomness and disorder.
  - maximum entropy corresponds to energy dissipation
    ⟶ random molecular motion ⟶ radiation.

- Kuo-Eliassen eq.
  - The thermodynamic effect of heating and heat flux convergence are differentiated meridionally.
  - The momentum effects of friction and momentum flux convergence are differentiated vertically.
  - Both effects act as torques.

- Slantwise convection
  - the unstable motion that occurs when air parcels are displaced along a slope that lies between horizontal and the slope of θ surface.

* large-scale condensation

MET5534: T. N. Krishnamurti (40 minutes) * → also look at NWP question

- In tropical and middle latitude weather system we have to deal with the large scale condensation process.

From first principles formulate the stability, heating and rainfall rate estimates. Use a simple quasi-geostrophic omega-equation as a frame work for your discussion. Discuss also in ellipticity of the omega-equation in this context.

Sol) Stable heating is invoked when the following three conditions are met.

- $\omega < 0$ (ascent) : dynamic ascent.
- $-\frac{\partial \theta}{\partial p} > 0$, $-\frac{\partial \theta_s}{\partial p} > 0$: stable
- $\frac{q}{q_s} > 0.8$ (to account for subgrid-scale saturation) $\simeq 1.0$ (saturated)

Non-convective heating

$$H_{NC} = -L\frac{dq_s}{dt} \simeq -L\omega\frac{\partial q_s}{\partial p}$$

where, $q_s = \frac{0.622 e_s}{p - 0.378 e_s}$

$e_s = 6.11 \exp\left[\frac{a(T-273.16)}{T-b}\right]$ "Teten's formula"

$$\Rightarrow \frac{\partial q_s}{\partial p} = f\left(\frac{\partial T_s}{\partial p}\right) \quad \text{——①}$$

and $C_pT + gz + Lq_s = E_s$

Taking $\frac{\partial}{\partial p}$ into E_s, that is, moist adiabat.

$$C_p\frac{\partial T_s}{\partial p} + g\frac{\partial z_s}{\partial p} + L\frac{\partial q_s}{\partial p} = 0 \quad \text{——②}$$

eliminate $\frac{\partial T_s}{\partial p}$ between ① and ② to get $\frac{\partial q_s}{\partial p}$

Thus the rainfall can be written as

$$R = -\frac{1}{g}\int_{P_B}^{P_T} \frac{H_{NC}}{L}\,dp$$

Meanwhile

- $\frac{\partial^2 u}{\partial t^2} = \nu\frac{\partial^2 u}{\partial x^2}$ → hyperbolic
- $\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right)$ → parabolic
- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ → elliptic

QG ω-eq. is

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right] + \frac{1}{\sigma}\nabla^2\left[V_g\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right]$$

which is elliptic

$$aU_{xx} + 2bU_{xy} + cU_{yy} = F(x, y, u, u_x, u_y)$$

- Hyperbolic type if $b^2 - ac > 0$ → wave eq.
- Parabolic 〃 if $b^2 - ac = 0$ → heat (diffusion) eq.
- ✓ Elliptic 〃 if $b^2 - ac < 0$ → Laplace eq.

In the ω-eq., $b = 0$

Therefore

- if $\sigma > 0$; $c > 0$: $b^2 - ac = -ac < 0$: elliptic ✓
- if $\sigma = 0$: no Ω-eq. : inelliptic ✓
- if $\sigma < 0$: $c < 0$: $b^2 - ac = -ac > 0$: inelliptic ✓

* Comprehensive note !

Large scale condensation is usually invoked in a NWP model of dynamic ascent of stable, saturated air occurs at any level of the model atmosphere. The ascent is usually a consequence of large scale dynamics such as differential vorticity advection or thermal advection. Other sources of stable ascent are those due to orography or even buoyancy driven ascent from the lower troposphere. In the latter case both convective + nonconvective (↳ symmetric instability) clouds could coexist.

The conditions for invoking large scale condensation are

(1) $\omega < 0$ (ascent)

(2) $-\frac{\partial \theta}{\partial p} > 0$, $-\frac{\partial \theta_e}{\partial p} > 0$ (stable)

(3) $\frac{q}{q_s} > 0.8$ (saturated)

In a multilevel PDE NWP model we usually check for and remove large scale supersaturations each time step. Below we outline, in general, how this is done.

We compute the difference $\Delta q = q - q_s$. If $\Delta q > 0$, then at this level of the atm. we set the contributions of large scale condensation to heating using the 1st law of thermodynamics and the moisture vapor continuity eq.

$$C_p\frac{T}{\theta}\frac{\partial \theta}{\partial t} = \frac{L\Delta q}{\Delta t} \quad \text{→ increase in temperature from condensation}$$

and

$$\frac{\partial q}{\partial t} = -\frac{\Delta q}{\Delta t} \quad \text{→ loss of water vapor to condensation}$$

Thus the supersaturation is simply condensed out with an equivalent heat release in the thermal equation at that level of the atmosphere. This is usually done at the end of each timestep after other processes here contributed to a positive Δq.

How do we calculate q_s?

Various formulae exist to compute q_s. One such approach is to start with Teten's eq. to obtain the saturation vapor pressure, e_s.

(1) $e_s = (6.11\,mb)\exp\left[\frac{a(T-273.16)}{T-b}\right]$

a, b are constants which vary for saturation over ice + water

	Ice	Water
a	21.87	17.26
b	7.66	35.86

Given e_s and the pressure, the saturation specific humidity q_s is

(2) $q_s = 0.622 e_s / p - (1 - 0.622) e_s$

The simplest formulation for nonconvective heating can be described as follows

$$H_{NC} = -L\frac{dq_s}{dt}$$

and can be approximated as (expand $\frac{dq_s}{dt}$ and drop all terms except vertical advection)

(3) $H_{NC} = -L\omega\frac{\partial q_s}{\partial p}$

We measure $\frac{\partial q_s}{\partial p}$ along the moist adiabat constructed by applying Teten's formula at various model levels. (Note $\frac{\partial q_s}{\partial p}$ is the slope of a saturated moist adiabat)

We note that the total moist static energy of the affected layer is conserved. This implies

$$\text{const} = h_{s_{initial}} = h_{s_{final}} = gZ_s + C_pT_s + Lq_s$$

We differentiate this with respect to p.

$$0 = g\frac{\partial Z_s}{\partial p} + C_p\frac{\partial T_s}{\partial p} + L\frac{\partial q_s}{\partial p} \qquad (4)$$

Assuming hydrostatic balance

$$\frac{\partial Z_s}{\partial p} = -\frac{1}{g}\frac{RT_{vs}}{p}$$ ← use T_v instead of T because the air is saturated.

This may be rewritten using the fact that $T_{vs} = (1+0.61 q_s)T_s$

Thus (4) becomes

$$0 = -\frac{R(1+0.61q_s)T_s}{p} + C_p\frac{\partial T_s}{\partial p} + L\frac{\partial q_s}{\partial p} \qquad (5)$$

We can use (1) and (2) to obtain an expression for $\frac{\partial q_s}{\partial p}$ as a function of T_s and p. This equation may then be solved for $\frac{\partial T_s}{\partial p}$. We substitute this into (5) to get an eq. for $\frac{\partial q_s}{\partial p}$

$$\frac{\partial q_s}{\partial p} = C_1C_2 / 1 + \frac{L}{C_p}C_1C_3$$

where $C_1 = \frac{0.622}{p}e_s$, $C_2 = \frac{1}{p}C_3\left[\frac{RT}{C_p p}(1+0.61 q_s)\right]$

$$C_3 = \frac{a}{T-b} - \frac{a(T-273.16)}{(T-b)^2}$$

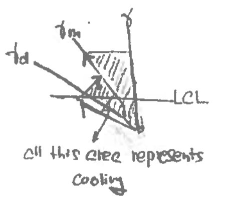
Once we have $\frac{\partial q_s}{\partial p}$ we simply compute the nonconvective heating from (3).

Stable rainfall is represented by the integral

$$R_s = \frac{1}{g}\int_0^{p_s}\frac{H_{NC}}{L}dp = -\frac{1}{g}\int_0^{p_s}\left(\omega\frac{\partial q_s}{\partial p}\right)dp$$

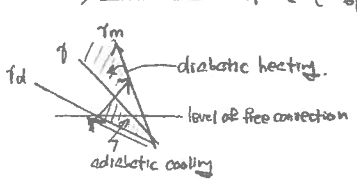
✓ Why does large scale, nonconvective precip. cool the atmosphere whereas convective precip warms the atmosphere?

→ For nonconvective precip. ascent occurs in an absolutely stable ($-\frac{\partial\theta}{\partial p} > 0$, $-\frac{\partial\theta_e}{\partial p} > 0$) atmosphere.



The large scale rising motion is associated with large amounts of adiabatic cooling. This cools the atmosphere. The condensation of water vapor releases latent heat which warms the rising air. However, because the ascent is occurring in a stable atmosphere, the adiabatic cooling of ascent > diabatic warming due to condensation. Thus, in the net the atmosphere cools.

→ For convective precip, ascent occurs in a conditionally (possibly absolutely) unstable atmosphere ($-\frac{\partial\theta}{\partial p} > 0$, $-\frac{\partial\theta_e}{\partial p} < 0$)



① The rising motion occurs in a conditionally stable atm. Dry adiabatic ascent to the LFC does cool the atm. but above the LFC the latent heat release associated w/ condensation overwhelms the adiabatic cooling. Thus in the net there is warming.

② The rising motion occurs over a very small fraction of the grid box. Over the remainder of the grid box, a large scale subsidence is found. This adiabatic warming associated with the subsidence is more than the cooling associated w/ ascent due to the fact that subsidence occurs over a much larger area than the convective ascent.

✓ How do we achieve dynamically forced large scale ascent?

We turn to the QG ω-eq.

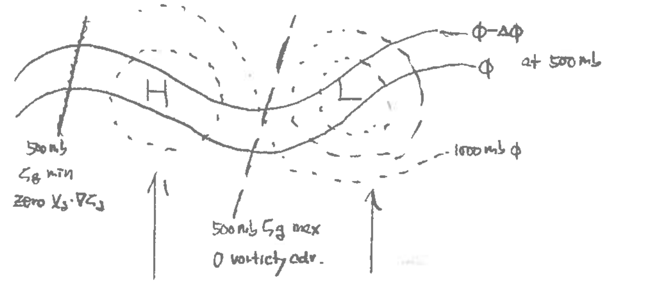
$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{ⓐ} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V_g\cdot\nabla\left(\frac{1}{f_0}\nabla^2\phi + f\right)\right]}_{ⓑ} \underbrace{- \frac{1}{\sigma}\nabla^2\left[-V_g\cdot\nabla\left(-\frac{\partial\phi}{\partial p}\right)\right]}_{ⓒ}$$

Term ⓐ is an elliptic operator which spreads the forcing on the RHS both horizontally (∇^2) and vertically ($\frac{\partial^2}{\partial p^2}$). For oscillatory ω fields

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega \propto -\omega$$

Term ⓑ : differential vorticity advection.

Vorticity advection in typical developing mid-lat. baroclinic waves is ~0 near the surface and increases moving up in the atmosphere. This is illustrated below.

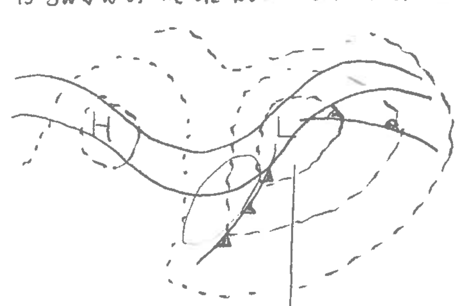


region of 500mb NVA at the sfc weak vort adv.	region of 500mb PVA weak vort. adv. at sfc
$-\frac{\partial}{\partial p}$(vort. adv.) < 0	$-\frac{\partial}{\partial p}$(vort. adv.) > 0
as $-\omega \propto -\frac{\partial}{\partial p}$(vort. adv.)	as $-\omega \propto -\frac{\partial}{\partial p}$(vort. adv.)
$-\omega < 0$	$-\omega > 0$
or $\omega > 0$	or $\omega < 0$
sinking motion	rising motion

The vertical motion field maintains a hydrostatic temperature field (temp. and thickness are proportional) in the presence of a differential adv. Without the compensating vertical motion either the 500mb vort. changes could not remain geostrophic or the temp. change in the 500–1000mb layer could not remain hydrostatic.

Term ⓒ : localized temperature advection.

In a developing mid-lat. baroclinic wave the temperature advection is most important at the sfc. There is warm adv E & NE of the sfc low. Cold adv. is SW & W of the sfc low. This is illustrated below



region of low level cold air advection (decreasing thickness)	region of warm air adv (increasing thickness)
$-V_g\cdot\nabla(-\frac{\partial\phi}{\partial p}) < 0$	$-V_g\cdot\nabla(-\frac{\partial\phi}{\partial p}) > 0$
so $-\frac{1}{\sigma}\nabla^2($ " $) < 0$	so $-\frac{1}{\sigma}\nabla^2($ " $) > 0$
$-\omega < 0$	$-\omega > 0$
$\omega > 0$	$\omega < 0$
sinking motion	rising motion

geopotential heights are falling, ζ_g is ↑ since $\zeta_g = \frac{1}{f_0}\nabla^2\Phi$ with little vort. adv. ζ_g↑ requires horizontal convergence. Upper level convergence implies sinking motion through mass continuity.

geopotential heights are rising at the ridge. Since $\zeta_g = \frac{1}{f_0}\nabla^2\Phi$ an increase in Φ implies a decrease in ζ_g. From the vorticity eq. we see that for ζ_g to decrease there must be horizontal divergence in the absence of vorticity adv. Continuity of mass implies rising motion

✓ The QG ω-eq. is elliptic. Why? Is it always elliptic?

Consider the general PDE in ψ

$$A\frac{\partial^2\psi}{\partial x^2} + B\frac{\partial^2\psi}{\partial x\partial y} + C\frac{\partial^2\psi}{\partial y^2} = \text{forcing}.$$

define $\delta^2 = AC - B^2$

If $\delta^2 > 0$ we have an elliptic eq.
$\delta^2 < 0$ we have a hyperbolic eq.
$\delta^2 = 0$ " " a parabolic eq.

In the QG ω-eq. we have

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \left(\frac{f_0^2}{\sigma}\right)\left(\frac{\partial^2}{\partial p^2}\right)\right]\omega = \text{forcing}$$

We have no mixed partials so $B = 0$

Let $A = 1$, $C = \frac{f_0^2}{\sigma}$

Then $\delta^2 = AC - B^2 = \frac{f_0^2}{\sigma}$

The sign of δ^2 depends on the sign of σ. σ is the stability defined as $\sigma = -\frac{\alpha}{\theta}\frac{\partial\theta}{\partial p}$. In a statically stable atmosphere $-\frac{\partial\theta}{\partial p} > 0$ so that the QG ω-eq. is an elliptic eq. However, in an atmosphere that is statistically unstable, $-\frac{\partial\theta}{\partial p} < 0$. The QG ω-eq. is then hyperbolic. (previous discussion of the QG ω-eq does not apply in this case)

So

Statically stable atm. $\left(-\frac{\partial\theta}{\partial p} > 0\right) \Rightarrow$ QG ω-eq is elliptic
" unstable atm. $\left(-\frac{\partial\theta}{\partial p} < 0\right) \Rightarrow$ QG " " hyperbolic.

* Shallow non-precipitating cloud

→ Elementary cloud model.

MET5534: T. N. Krishnamurti (?)

• In the model of the shallow non precipitating cloud (FRANK MURRAY NOT NICKERSON) we discussed three physical processes: condensation, evaporation and associated heating (or cooling). Describe how these processes were handled in the cloud model.

Sol) Stream function ψ

$$\mathbf{V} = -\frac{\partial\psi}{\partial z}\mathbf{i} + \frac{\partial\psi}{\partial x}\mathbf{k} = -\mathbf{j}\times\nabla\psi .$$

z, x: $u = -\frac{\partial\psi}{\partial z}$, $w = \frac{\partial\psi}{\partial x}$

$$\Rightarrow \frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi) + \underbrace{\frac{g}{T_M}\frac{\partial T'}{\partial x}}_{\text{buoyancy term}} + \underbrace{\nu_M\nabla^4\psi}_{\text{diffusion term}}$$

→ temperature eq.

$$\frac{dT}{dt} = \underbrace{-w\frac{g}{C_p}}_{\text{adiabatic term}} + \underbrace{\left(\frac{dT}{dt}\right)_{ph}}_{\text{phase change term (liquid} \leftrightarrow \text{vapor)}} + \underbrace{\nu_T\nabla^2 T}_{\text{diffusion term}}$$

→ mixing ratio

$$q = \underbrace{q_l}_{\text{liquid water}} + \underbrace{q_v}_{\text{water vapor}}$$

$$\frac{dq}{dt} = \nu_q\nabla^2 q \quad (\because \text{assume } \nu_q = \nu_T)$$

$$\frac{dq_l}{dt} = \boxed{\left(\frac{dq_l}{dt}\right)_{ph}} + \nu_q\nabla^2 q_l \quad \rightarrow \text{generation of liquid water from microphics}$$

$$\frac{dq_v}{dt} = \boxed{\left(\frac{dq_v}{dt}\right)_{ph}} + \nu_q\nabla^2 q_v \quad \rightarrow \text{generation of water vapor from microphysics}$$

where $\left(\frac{dq_l}{dt}\right)_{ph} = -\left(\frac{dq_v}{dt}\right)_{ph}$

$$\frac{dQ}{dt} = L\left(\frac{dq_l}{dt}\right)_{ph}$$

In this model, it was assumed that all the latent heat goes to change the temperature of the dry air.

$$\left(\frac{dT}{dt}\right)_{ph} = \frac{1}{C_p}\frac{dQ}{dt} = \frac{L}{C_p}\left(\frac{dq_l}{dt}\right)_{ph}$$

It is assumed that whenever supersaturation occurs there will be immediate condensation down to the point of exact saturation. Conversely, whenever subsaturation occurs in the presence of liquid water there will be immediate evaporation up to the point of exact saturation unless the water is all evaporated before this point is reached.

→ require that saturation mixing ratio be known

$$q_{vs} = \frac{\varepsilon e_s}{p - e_s} \qquad \text{where } \varepsilon = \frac{m_v}{m_d}$$

$$e_s = 6.11\exp\left\{\frac{a(T-273.16)}{T-b}\right\} \quad \text{Teten's formula}$$

⇒ microphysics

q_v, q_l, T → interplay

① $\left.\frac{dq_l}{dt}\right|_{ph} = \frac{q_v - q_s}{\Delta T}$ if $q_v > q_s$, if other processes make supersaturation

② 〃 $= 0$ if $q_v \le q_s$

③ $\left.\frac{dq_v}{dt}\right|_{ph} = \frac{q_s - q_v}{\Delta T}$ if $q_l > (q_s - q_v)$

④ 〃 $= q_l$ if $q_l < (q_s - q_v)$

⑤ 〃 $= 0$ if $q_v \le q_s$

→ Corresponding heating/cooling terms

① $\left.\frac{dT}{dt}\right|_{ph} = L\frac{q_v - q_s}{\Delta T}$

② $\left.\frac{dT}{dt}\right|_{ph} = 0$

③ $\left.\frac{dT}{dt}\right|_{ph} = -L\frac{(q_s - q_v)}{\Delta T}$

④ $\left.\frac{dT}{dt}\right|_{ph} = -L\frac{q_l}{\Delta T}$

⑤ $\left.\frac{dT}{dt}\right|_{ph} = 0$

MET5534 Krish.

★ Structure of hurricane?

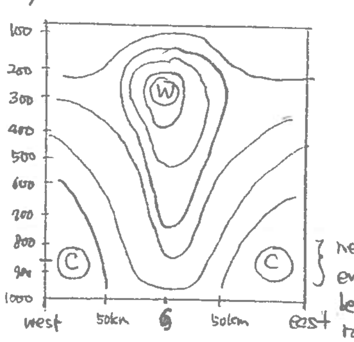
Sol) The hurricane is a strong, very intense low pressure system occuring over ocean areas (Pacific + Atlantic) during the summer season. In its mature stage, its pressure, wind, temperature and geopotential fields are almost symmetric but precipitation is not. It is believed that Hurricanes are the result of the interaction between the large scale environment and cumulus scale. CISK theory, proposed by Oayama and Cherney is one popular theory for the development of a hurricane. According to CISK, if there is an initial disturbance embedded in a large scale system, the large scale system may provide moisture convergence through frictional ekman pumping. At the same time, the release of latent heat by the convection will continue to make the sfc pressure fall and the disturbance will develop. The two, large scale convergence and cumulus convection work cooperatively to strengthen the storm.

A new theory for hurricane development is the air-sea interaction method.

Below we discuss temperature, geopotential, and wind anomalies for a mature hurricane. Anomalies are defined as (mean hurricane − mean undisturbed tropics)

① Temperature anomalies.

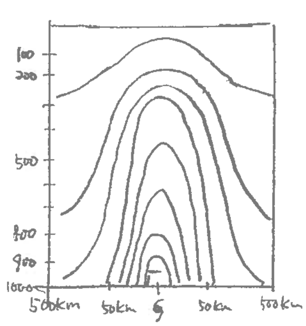
The main feature of the temperature field is the warm core, especially pronounced at around 250mb. Typical anomalies are on the order of 16°C. This warm core is due to latent heat release, maximized in the upper portion of the deep convection, and the compensating subsidence in the eye.



} negative anomalies due to evaporation of rainwater below cloud base (near storm) radiative cooling (away from storm)

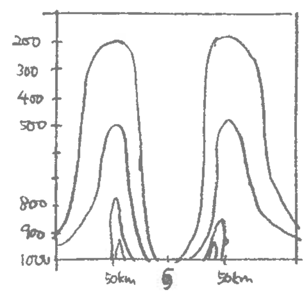
② Geopotential

As the hurricane is a strong low pressure system, the max negative geopotential anomalies are located right @ the center of the storm near the ground. It is due to strong divergence @ 200mb (primarily due to latent heat release?) The max difference is ~2000 feet above the surface.

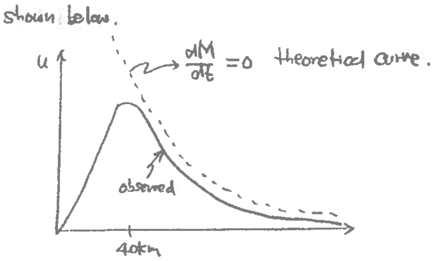


③ Wind speed

The wind speed structure is rather interesting. The wind speed increases as the air parcel moves in towards the storm center up to a radial distance of about 40km. The wind speed increases, almost linearly in radial distance, from the storm to the eyewall. Max winds of ~100 kts are located near the storm center, near the ground.



The tangential wind profile of air following radially into the storm is shown below.



The frictional torque (especially that due to cumulus convection) accounts for the decrease in $u(r)$ for $r < 40km$.

* Atmospheric angular momentum

5534

MET?: T. N. Krishnamurti (?) *

- This question is on atmospheric angular momentum. Write down two angular momentum equations for a parcel of air: one for the atmospheric angular momentum (using spherical coordinates) and the other relevant to a hurricane (using a local cylindrical coordinate).

Discuss the relevance of the angular momentum principle for the maintenance of the tropical general circulation and of a hurricane (respectively). Your discussions should include the role of different torques, transports and of turbulent eddy motions.

Sol) In spherical coords. the momentum eq takes the form

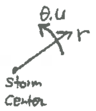
$$\boxed{\frac{\partial M}{\partial t} + u\frac{\partial M}{a\cos\phi\,\partial\lambda} + v\frac{\partial M}{a\partial\phi} + \omega\frac{\partial M}{\partial p} = -g\frac{\partial z}{\partial\lambda} - a\cos\phi\,F_\lambda} \quad (1)$$

where $M = (\Omega a\cos\phi + u)\,a\cos\phi$

$$\therefore \frac{dM}{dt} = -a\cos\phi\left[g\frac{\partial z}{a\cos\phi\,\partial\lambda} + F_\lambda\right]$$

In local cylindrical coords the tangential momentum eq is

$$\frac{\partial u}{\partial t} + \frac{u}{r}\frac{\partial u}{\partial\theta} + V\frac{\partial u}{\partial r} + \omega\frac{\partial u}{\partial p} + \frac{uv}{r} = -\frac{g}{r}\frac{\partial z}{\partial\theta} + F_\theta \quad (2)$$



u is the tangential wind component. V is the radial wind component.

To derive the corresponding momentum eq. we make use of the contr. eq.

$$\frac{\partial u}{r\partial\theta} + \frac{\partial rV}{r\partial r} + \frac{\partial\omega}{\partial p} = 0 \quad (3)$$

Multiply (2) by r and use (3) to obtain

$$\boxed{\frac{dM}{dt} = -g\frac{\partial z}{\partial\theta} + F_\theta r} \quad (4)$$

→ x look at my note p.20-21

where $M = ur + \frac{f}{2}r^2 \equiv$ angular momentum in local cylindrical coord.

$\frac{f}{2}r^2$ → planetary A.M

ur → local or relative AM

In the following discussion we will find it more useful to use the momentum eq.(4), the one in local cylindrical coords. From (4) we see that angular momentum M is conserved following the motion if

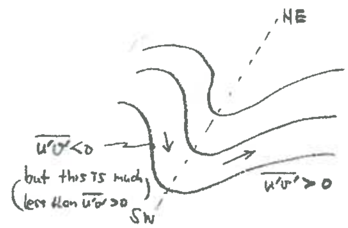
$$-g\frac{\partial z}{\partial\theta} + F_\theta r = 0$$

$F_\theta r$ → frictional torque

$-g\frac{\partial z}{\partial\theta}$ → pressure gradient torque.

① First, how does the A.M. concept play a role in the general circulation?

The tangential stress exerted upon the earth by the atmosphere is in the direction of the sfc wind. Conversely the earth exerts a stress upon the air in the direction opposite to that of the sfc wind. Corresponding to this stress, there exists a flow of absolute angular momentum whose intensity depends upon the magnitude of the eastward component of the stress on the atmosphere and upon the distance from the polar axis. The atmosphere gains westerly momentum in the tropics and loses it in higher latitudes. The poleward transfer of AM is accomplished by the meridional circulation of the axially symmetric Hadley cell and mid-lat. baroclinic eddies. The eddies are most important in the mid-lats. where the typical flow @ a short distance from the sfc is westerly w/ superimposed troughs in the vicinity of sfc cyclones. These troughs often assume a NE to SW tilt of the trough line.



This gives a positive correlation between northward + eastward velocities and so enables the northward transport of A

Often the departure from a sinusoidal wave pattern and tilt of the trough line are small @ higher latitudes near the Northern border of the sfc westerly belt. Thus in this area the northward transport of air is small. The poleward transport of air is much more pronounced @ lower latitudes.

We have discussed the role of eddies in the poleward transport of AM. Recall that the absolute AM eq. has two forcing terms ⇒ pressure torques and frictional torques. The pressure torque acts to transfer AM from the atmosph to the ground provided that the sfc pressure and the slope of the ground are positively correlated. Observations indicate that this is generally the case in the mid-lats where there is a slight tendency for the sfc pressure to be higher on the western side of mountains as compared to lower pressure on the eastern sides.

In the tropics and SH the exchange of AM is dominated by turbulent eddy stresses.

② Now we turn our attention to AM + hurricane.

The AM principle has great relevance to the maintenance of a hurricane. To see this consider an parcel @ r_0 outside the hurricane. Suppose this parcel moves into the hurricane and moves toward the center of the storm. If we assume conservation of AM. then @ radius r we must have

$$M(r) = M_0$$

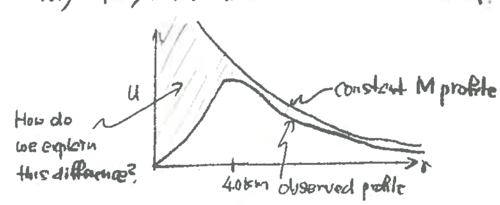
$$ur + \frac{f_0}{2}r^2 = u_0 r_0 + \frac{f_0}{2}r_0^2 \quad (r < r_0)$$

Solve for u

$$u = \left(u_0 r_0 + \frac{f_0}{2}r_0^2\right)\frac{1}{r} - \frac{f_0}{2}r \quad (r < r_0)$$

or $\boxed{u = \frac{M_0}{r} - \frac{f_0}{2}r}$

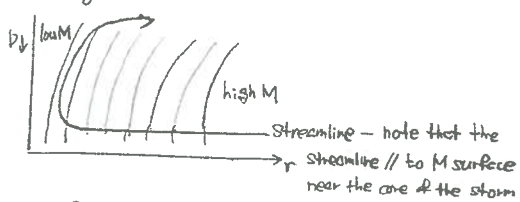
In the limit as $r \to 0$, $u \to \infty$! Clearly something is wrong. Assuming conservation of AM has lead to the conclusion that as the parcel moves toward the storm center, its tangential speed will increase very rapidly. This differs from what is observed.



There is obviously a difference between the two curves, especially as we move closer to the storm center. Why this difference?

⇒ we assumed $\frac{dM}{dt} = 0$. We now see that the pressure + friction torques must play a prominent role in the mature hurricane.

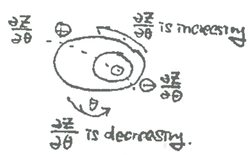
If we plot contours of AM computed from observed data we find that AM → 0 as r → 0. This must be due to the pressure and/or frictional torques.



Let's consider frictional + pressure torques, respectively.

(a) Pressure torque ⇒ $r\frac{g}{r}\frac{\partial z}{\partial \theta}$ term

The pressure torque term can be viewed in terms of β-gyres. This describe the non-zeroness of the pressure gradient term. With an asymmetric hurricane, the horizontal pressure gradient (and ∴ the wind field) is not symmetric.



If a parcel of air spiraling into a hurricane encounters a region of $-\frac{\partial z}{\partial \theta} < 0$, the parcel AM will decrease.

Conversely, in a region of $-\frac{\partial z}{\partial \theta} > 0$, parcel AM will increase.

Do note that observations indicate that the inner rain band regions of mature hurricanes are generally symmetric and so the pressure torque term is generally small. However, the presence of any asymmetrics can and does affect the AM of parcels in the storm environment.

(b) The frictional term $F_\theta r$ → compare to my note!

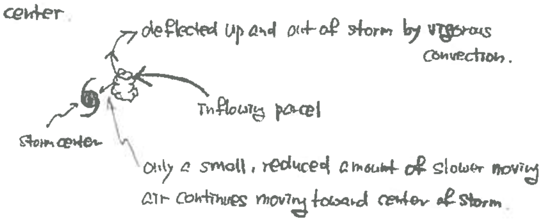
Frictional torque is important in the budget of momentum in the hurricane. The frictional torque can be decomposed into two terms: surface friction + cloud friction (turbulence)

$$F_\theta = F_{\theta s} + F_{\theta c}$$

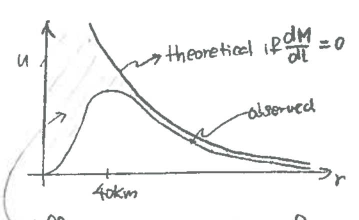
The most important factor is in the cloud turbulence component. In particular the important term is the vertical gradient of the covariance between u' and ω'

$$-g\frac{\partial}{\partial p}(\overline{u'\omega'}) \rightarrow \text{vertical eddy flux of AM}$$

Areas of convection + substantial rising motion act in such a way as to block the AM inflow in the inflow channel and therefore deflect into the vertical plane AM trying to get to the storm center.



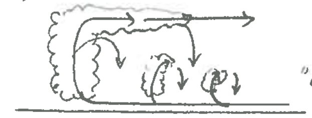
Recall the below diagram



If we consider only the cumulus friction torque we see that in order to have the observed difference between the two curves we must have

$$F_{\theta c} = -g\frac{\partial \overline{u'\omega'}}{\partial p} < 0.$$

This means that cumulus clouds must pump large amounts of AM up, out, and away from the storm center.



We can estimate the magnitude of this cloud friction using

$$F_{\theta c} \simeq \frac{1}{r}\left(\frac{dM}{dt} - F_{\theta s} r\right)$$

This requires that we estimate the surface drag to obtain a value of $F_{\theta s}$. Toward this end, start w/ the approx. momentum eq. (neglect pressure torques)

$$\frac{dM}{dt} = F_\theta r$$

where we let $F_\theta = -g\frac{\partial \tau_\theta}{\partial p} - g\frac{\partial \overline{u'\omega'}}{\partial p}$ (τ_θ is the horizontal stress; the second term is the vertical gradient of eddy angular momentum flux)

We expand

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + u\frac{\partial M}{r\partial \theta} + \frac{1}{r}\frac{\partial Mvr}{\partial r} + \frac{\partial M\omega}{\partial p} = \left(-g\frac{\partial \tau_\theta}{\partial p} - g\frac{\partial \overline{u'\omega'}}{\partial p}\right) r$$

($\frac{\partial M}{\partial t}$: assume steady state; $u\frac{\partial M}{r\partial\theta}$: symmetric; $\frac{\partial M\omega}{\partial p}$: small)

So

$$v\frac{\partial M}{\partial r} = \left(-g\frac{\partial \tau_\theta}{\partial p} - g\frac{\partial \overline{u'\omega'}}{\partial p}\right) r$$

→ we can obtain $\overline{u'\omega'}$ profile by integrating over p

We will approx. $\tau_\theta = C_D \rho u (u^2+v^2)^{1/2}$ (Bulk aerodynamic formulation)

Starting @ the sfc we integrate the above eq. upward into the atmosphere

- Step ①: compute $M = ur + \frac{f_0 r^2}{2}$
- Step ②: compute $v\frac{\partial M}{\partial r}$
- Step ③: compute wind stress τ_θ and its vertical gradient.
- Step ④: compute $\overline{u'\omega'}$ @ level k+1 assuming it is known at the lower level (at the sfc assume $\overline{u'\omega'} = 0$)

move to next level up and repeat

(c) Horizontal advection terms

Inside $\frac{dM}{dt}$ are advection terms in the horizontal plane, namely $-\underline{V}\cdot\nabla M$. These are generally small terms compared to friction torques. The positioning of a hurricane wrt the subtropical high can lead to high momentum parcels being advected into the storm. A storm passing south of the subtropical high is in a favorable position to benefit from the injection of high AM parcels.

As the flow in the tropics is easterly, which is opposite the sense of the earth's rotation, the tropical easterlies gain AM from the sfc. Though this effect is probably not very important for hurricanes, a storm moving into the mid-lats will move from an environment where the sfc supplies AM to the large scale flow to one in which the large scale flow is drivened of westerly AM by the sfc.

In light of this discussion we see that the following conditions will lead to a strong, intensifying hurricane (in terms of AM alone)

① symmetric (perfect circular) shape so that pressure torques are negligible.
② few outer convective rain bands to allow parcels to flow closer to storm center without being deflected into the vertical.
③ favorable position south of subtropical high.

* angular momentum principle in Hurricane.

5534
MET?: T. N. Krishnamurti (?)

- Describe the angular momentum principle and its control on the intensity of a hurricane. Describe:

a) the constant angular momentum profile for inflowing air at low levels.
b) the role of pressure torques
c) the role of frictional torques
d) how one can estimate the surface drag using the angular momentum budget.

look at my note p20-22

* angular momentum in Hurricane

5534

MET?(Tropical): T. N. Krishnamurti (1 hour) *

• Write down the equation for the time rate of change of angular momentum of a parcel for a local cylindrical coordinate. Discuss the various torques. Describe how you might interpret the maximum intensity of a hurricane taking into account the sources, sinks and the boundary values of a parcel's angular momentum.

look at my note p20-22

* angular momentum budget in Hurricane

MET5534: T. N. Krishnamurti (?)

• You are to make use of angular momentum budget using a local cylindrical coordinate (centered on a hurricane) as a frame of reference. Show how such a budget, based on real data, can be used to find the variation of surface drag (of a hurricane) as a function of radial distance for the storms center. Show how the same data sets can be used to find a relationship between the surface drag coefficient and the wind speed. Present a sequential use of data, identifying such, for the entire stream of above computations. What do the final results typically look like for a hurricane.

look at my note p20-22

MET5534 (Tropical (small scale) Meteorology): T. N. Krishnamurti (1 hour)

- Hurricane Opel of 1995 exhibited several interesting intensity changes as it moved towards Pensacola. You are to formulate a research problem which is to address possible investigations of these intensity changes. Elaborate on the following:

1) What kind of intensity change are you addressing?
2) What mathematical framework would you need?
3) What hypothesis would you be testing?
4) What data sets do you need?
5) What computations would you be doing?
6) What can you expect to learn from your approach?

Opal.

MWR, 128, 917–946

Bender & Ginis

* Potential vorticity

5534
MET?: T. N. Krishnamurti (?) , 1998 and before

• Question

a) Derive the adiabatic-inviscid potential vorticity equation,

$$\frac{d}{dt}\left[\zeta_a \frac{\partial\theta}{\partial p}\right] = 0$$

b) Describe the formation of a middle latitude lee trough (in the lee of the Rockies) using this conservation principle.

a) Eq of motion in isentropic coordinate is

$$\frac{\partial u}{\partial t} + V_\theta \cdot \nabla_\theta u + \frac{d\theta}{dt}\frac{\partial u}{\partial \theta} - fv = -\frac{\partial M}{\partial x} + F_x \qquad (1)$$

$$\frac{\partial v}{\partial t} + V_\theta \cdot \nabla_\theta v + \frac{d\theta}{dt}\frac{\partial v}{\partial \theta} + fu = -\frac{\partial M}{\partial y} + F_y \qquad (2)$$

where $V_\theta = u\hat{i} + v\hat{j}$ is horizontal velocity in θ-surface

$M = C_p T + gz$ the Montgomery streamfunction.

and F_x, F_y the x- and y-component of friction

For adiabatic inviscid eq

$$\frac{d\theta}{dt} = 0 = F_x = F_y$$

Taking $\frac{\partial (2)}{\partial x} - \frac{\partial (1)}{\partial y}$ noting that $\zeta_\theta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$$\frac{\partial \zeta_\theta}{\partial t} + V_\theta \cdot \nabla \zeta_\theta + (\zeta_\theta + f)D + \frac{\partial f}{\partial y} v = 0$$

$$\frac{d}{dt}(\zeta_\theta + f) = -(\zeta_\theta + f)D \qquad (3)$$

$$\frac{d}{dt}(\ln(\zeta_\theta + f)) = -D$$

Now, continuity eq. in θ system

$$\frac{d}{dt}\left(\ln\frac{1}{\sigma}\right) + D = 0 \qquad (4)$$

where $\frac{1}{\sigma} = \frac{1}{-\frac{\partial\theta}{\partial p}} = -\frac{\partial p}{\partial \theta}$

Combining (3) + (4) to eliminate D, we have

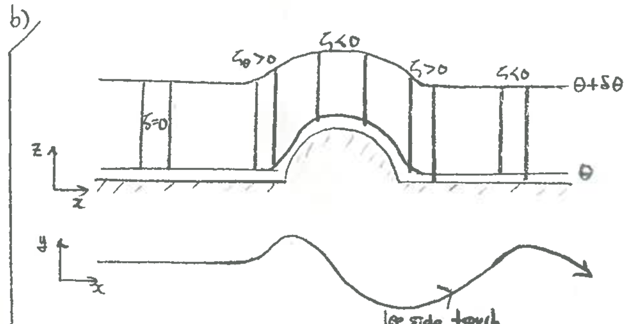
$$\frac{d}{dt}\ln(\zeta_\theta + f) = \frac{d}{dt}\ln\left(\frac{1}{\sigma}\right)$$

$$\frac{d}{dt}\left[\ln \sigma(\zeta_\theta + f)\right] = 0$$

$$\frac{d}{dt}\left[-\frac{\partial\theta}{\partial p}(\zeta_\theta + f)\right] = 0 \Longrightarrow \frac{d}{dt}\left(-\frac{\partial\theta}{\partial p}\zeta_a\right) = 0$$

Note : $\sigma = -\frac{\partial\theta}{\partial p} > 0$

b)



Suppose initially $\zeta_\theta = 0$ when the air moving toward the mountain, the column of air is stretched, that is, $\left|\frac{\partial\theta}{\partial p}\right|$ decreases to keep potential vorticity conserved. Thus, $\zeta_\theta > 0$, i.e. the air gains positive vorticity. When air approaching the top of the mountain air column is shrunk, the $\zeta_\theta < 0$.

Suppose that upstream of the mountain the flow is uniform zonal flow, so $\zeta =$

When the air is approaching the mountain barrier, there is a vertical stretching of air columns upstream of the topographic barrier. (the upstream streching is quite small). This stretching cause $-\frac{\partial\theta}{\partial p}$ to decrease, and hence ζ must bear positive in order to conserve potential vorticity.

As the column begins to cross the barrier its vertical extent decreases ; the relative vorticity must then become negative. Thus, the air column will acquire anticyclonic vorticity and move southward.

When the air column has passed over the mountain and returned to its original depth, it will be south of its original latitudes so that f will be smaller and the relative vorticity must be positive.

Thus the trajectory must have cyclonic curvature so that the column will be deflected poleward. When the parcel returns to its original latitude, it will still have a poleward velocity component and will continue poleward gradually acquiring anticyclonic curvature until its direction is again reversed. The parcel will then move downstream conserving potential vorticity by following a wavelike trajectory in the horizontal plane. Therefore, steady westerly flow over a large-scale mountain will result in a cyclonic flow to lee side trough followed by an alternating series of ridges and ridges downstream

★ Isentropic potential vorticity

5534

MET?(General): T. N. Krishnamurti (1 hour)

- Derive the isentropic potential vorticity equation. The conservation of potential vorticity implies changes among absolute vorticity and the static stability, how would you use that partioning to describe a lee cyclogenesis phenomena. Will it work equally well in the lee of mountains when the basic flows are westerly or easterly. Discuss.

look at the previous answer! + Holton ch4.3

Sol) We want to show

$$\frac{d}{dt}\left[\zeta_a \frac{\partial \theta}{\partial p}\right] = 0$$

where $\zeta_a = \zeta_\theta + f$ absolute vorticity on an isentropic sfc

$\zeta_a\left(-\frac{\partial \theta}{\partial p}\right)$ = Ertel's potential vorticity

(sometimes a factor of g is included, $\zeta_a\left(-g\frac{\partial \theta}{\partial p}\right)$)

We start from the isentropic u & v momentum eqs.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{d\theta}{dt}\frac{\partial u}{\partial \theta} - fv = -\frac{\partial M}{\partial x} + F_x \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{d\theta}{dt}\frac{\partial v}{\partial \theta} + fu = -\frac{\partial M}{\partial y} + F_y \quad (2)$$

where $\frac{d\theta}{dt}$ ≡ "vertical velocity" in θ-coordinates

$M = gz + C_pT$ ≡ Montgomery stream function.

(also, equivalent to dry static energy?)

For adiabatic motion $\frac{d\theta}{dt} = 0$, For frictionless motion $F_x = F_y = 0$

The above momentum eqs reduce to

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -\frac{\partial M}{\partial x} \quad (3)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -\frac{\partial M}{\partial y} \quad (4)$$

To derive a vorticity eq. we form $\frac{\partial}{\partial x}(4) - \frac{\partial}{\partial y}(3)$

$$\frac{\partial}{\partial x}(4) \Rightarrow \frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right) + \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + f\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{\partial M}{\partial y}\right)$$

$$-\frac{\partial}{\partial y}(3) \Rightarrow -\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) + u\frac{\partial}{\partial x}\left(-\frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right) - \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + f\frac{\partial v}{\partial y} + \frac{\partial f}{\partial y}v = +\frac{\partial}{\partial y}\left(\frac{\partial M}{\partial x}\right)$$

$$\frac{\partial}{\partial t}\zeta_\theta + u\frac{\partial}{\partial x}\zeta_\theta + v\frac{\partial}{\partial y}\zeta_\theta + \frac{\partial u}{\partial x}\zeta_\theta + \frac{\partial v}{\partial y}\zeta_\theta + f\nabla\cdot V + \beta v = 0$$

or

$$\frac{\partial}{\partial t}\zeta_\theta + V\cdot\nabla\zeta_\theta + (f+\zeta_\theta)\nabla\cdot V + \beta v = 0$$

$$\frac{\partial}{\partial t}(\zeta_\theta + f) + V\cdot\nabla(\zeta_\theta + f) = -(\zeta_\theta + f)\nabla\cdot V$$

or

$$\boxed{\frac{d}{dt}(\zeta_\theta + f) = -(\zeta_\theta + f)\nabla\cdot V} \quad (5) \quad \longleftarrow \text{here } \frac{d}{dt} = \frac{\partial}{\partial t} + V\cdot\nabla_\theta$$

↳ vorticity eq. in isentropic coord. for adiabatic-inviscid flow.

We want to replace the horizontal divergence, $\nabla\cdot V$, using the conti. eq.

Recall the mass flux form of the conti. eq. (in Cartesian coord.)

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho V + \frac{\partial}{\partial z}(\rho w) = 0 \quad \left(\text{note that } w = \frac{dz}{dt}\right)$$

This continuity eq. in (x, y, θ, t) coord. has similar form

$$\frac{\partial \sigma}{\partial t} + \nabla\cdot\sigma V + \frac{\partial}{\partial \theta}(\sigma\dot{\theta}) = 0 \quad \left(\text{where } \dot{\theta} = \frac{d\theta}{dt}\right)$$

The only difference is that we have replace density ρ by the parameter σ. The parameter σ plays the role of density in θ coordinates.

Consider an infinitesimal control volume with cross sectional area δA and vertical depth $\delta\theta$. Such a volume has mass

$$\delta M = \rho\,\delta A\,\delta z = \delta A\left(-\frac{\delta p}{g}\right) = \frac{\delta A}{g}\left(-\frac{\partial p}{\partial \theta}\right)\delta\theta = \sigma\,\delta A\,\delta\theta$$

↳ $\sigma = -\frac{1}{g}\frac{\partial p}{\partial \theta}$

The parameter σ is defined such that when it is multiplied by the "volume" $\delta A\delta\theta$ in θ coord. we obtain the mass element δM. As such we say σ plays the role of density in θ coord.

Using the total derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + V\cdot\nabla_\theta$ and assuming adiabatic flow ($\dot{\theta} = 0$), we rewrite the continuity eq. as

$$\frac{\partial \sigma}{\partial t} + V\cdot\nabla\sigma + \sigma\nabla\cdot V + \frac{\partial}{\partial \theta}(\sigma\dot{\theta}) = 0 \quad (\text{last term} \to 0)$$

$$\boxed{\frac{1}{\sigma}\frac{d\sigma}{dt} = -\nabla\cdot V} \quad (6)$$

We use this to replace $\nabla\cdot V$ in (5) with the result being

$$\frac{d}{dt}\ln(\zeta_\theta + f) = \frac{d}{dt}\ln\sigma \longrightarrow \frac{d}{dt}\left[\ln\left(\frac{\zeta_\theta + f}{\sigma}\right)\right] = 0$$

from which follows

$$\boxed{\frac{d}{dt}\left(\frac{\zeta_\theta + f}{\sigma}\right) = 0} \quad ; \ \sigma = -g\frac{\partial p}{\partial \theta}$$

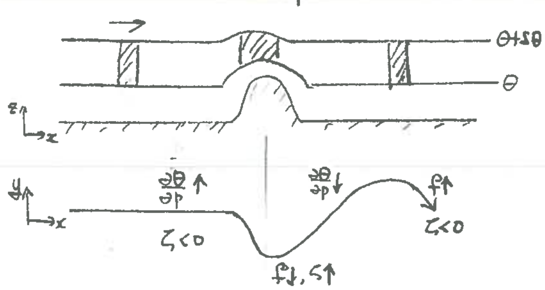
Replace $\sigma = -g\frac{\partial p}{\partial \theta}$ to get

$$\frac{d}{dt}\left[-\frac{1}{g}\frac{\partial \theta}{\partial p}(\zeta_\theta + f)\right] = 0$$

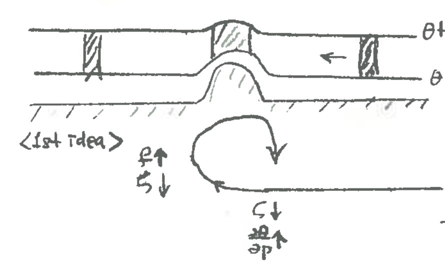
As $-\frac{1}{g}$ is constant

$$\boxed{\frac{d}{dt}\left[\frac{\partial \theta}{\partial p}\zeta_a\right] = 0}$$

i) For westerlies $(\zeta + f)\frac{\partial \theta}{\partial p}$ = const.

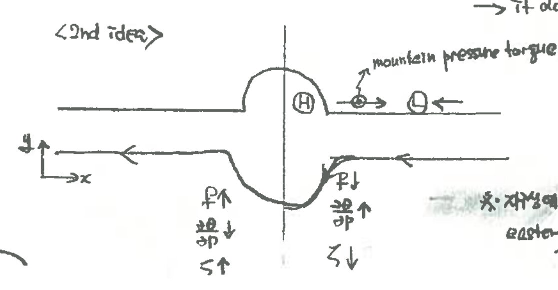


ii) For easterlies



→ no-steady state

→ it doesn't exist in nature



※ 자연에는 이렇게 조직적인 easterly가 없음.

* Potential vorticity

MET5534 (Tropical MET): T. N. Krishnamurti (?)

- Define potential vorticity
  - Write down an expression for potential vorticity in the isentropic frame of reference.
  - How do diabatic effects produce changes in the potential vorticity.
  - It is sometimes said that the arrival of an upper PV wave can influence surface cyclone development. Explain the process.

* look at my note p22 + 23 !

* Statistical methods for hurricane track prediction

MET? 5534 (Tropical): T. N. Krishnamurti (45 minutes) *

• Describe the operational statistical methods for hurricane track prediction. How do the HURRAN and the various NHCXX incorporate past and present information to issue forecasts? What are skill of performance of these methods; how can these methods be improved?

?

* PBL dynamics

5534
MET?: T. N. Krishnamurti (?)

- Describe the planetary boundary layer dynamics as a departure from the usual Ekman laws with the inclusion of advective acceleration. Describe the nature and transition in the planetary boundary layer following the Somali jet over the Arabian sea.

→ look at my note p24.

Sol) In the mid-lats, the main characteristics of BL wind profile can be best described as an Ekman spiral (i.e. the balance between Coriolis, friction, and the pressure gradient forces). In the tropics, a discussion of the BL must use a different approach.
(Consider that $f \to 0$ as $\phi \to 0°$)

Let's start w/ the governing eqs.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + K\frac{\partial^2 u}{\partial z^2} \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + K\frac{\partial^2 v}{\partial z^2} \quad (2)$$

T, A, V, C, P, F

We group the terms as follows

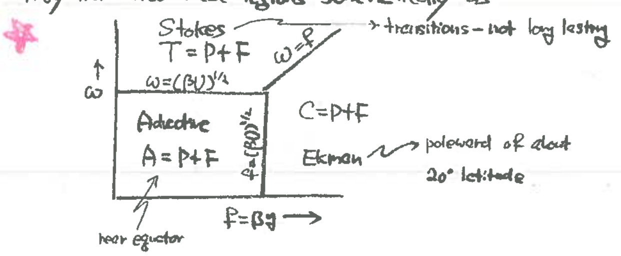
- T = local time tendencies
- A = horizontal adv.
- V = vertical adv.
- C = Coriolis terms
- P = horizontal pressure gradient
- F = frictional drag

Mahrt & Young (1972) showed that in different regions of the tropics we have different types of balance between the above terms in the BL.

They identified 3 types of tropical BL regions.

1. Ekman → C = P+F
2. Advective → A = P+F (also called drift BL)
3. Stokes → T = P+F

They illustrated these regions schematically as



In this scale analysis they set

$$\frac{\partial}{\partial t} = \omega\frac{\partial}{\partial t'} \ ; \ u = Uu' \ ; \ \frac{\partial}{\partial x} = (U/\beta)^{-1/2}\frac{\partial}{\partial x'} \ ; \ f = \beta y \quad (3)$$

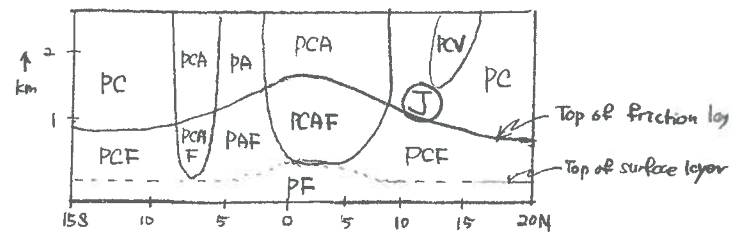
⇒ Now we use the concepts discussed above to describe the nature and transition in the PBL following the Somali jet over the Arabian sea. Krishnamurti & Wong (1979) used a simple model (steady state) described as follows

$$0 = -v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} + fv - \frac{1}{\rho_0}\frac{\partial P}{\partial x} + K\frac{\partial^2 u}{\partial z^2}$$

$$0 = -v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - fu - \frac{1}{\rho_0}\frac{\partial P}{\partial y} + K\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

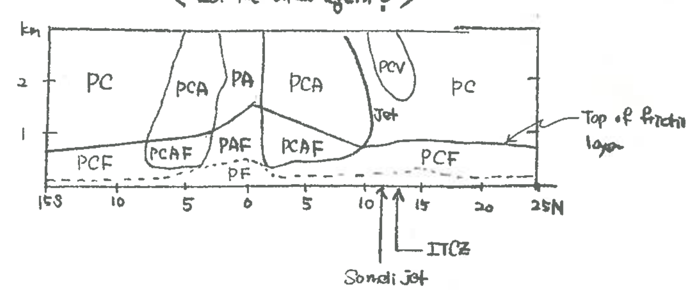
The pressure gradient was prescribed. They integrated this model over a meridional-vertical domain @ 60°E from 15°S to 25°N. They successfully simulated the Somali Jet @ 12°N and 1 km height. They examined the balance of forces over the domain and produced the following diagram.



They concluded that the balance of forces in the BL is found to settle to a set of values quite different from that of the initial Ekman layer. For example a surface layer near the ground everywhere w/ the balance between pressure gradient & frictional forces. Their calculations show that the Somali jet is located @ the poleward edge of the region where the horizontal advective term (A) becomes less important. The ITCZ is noted to form in the general region between the advective & Ekman layer. Their simulation shows a surface layer near the ground in which the pressure gradient & friction balance each other. Above the sfc layer is a layer where an Ekman balance (P+F = C) dominates away from the near equatorial regions. Closer to the equator C↓ and they found the horizontal advective terms become equally important. This gave a four way balance (P, C, A, F). In the immediate vicinity of the equator $C \to 0$ and the balance become that of an advective or drift BL ⇒ P, A, F balance. This pattern of the meridional variation in the importance of various forces is mirrored above the top of the friction layer with the exception that the friction force is no longer important.

It should be noted that in their study a steady state framework was adopted. Thus this approach is not suited for investigating the time evolution of the tropical BL.

<Let me draw again ☺>



- Just how did Mahrt & Young scale this eq?

Using (1) & (2) and substituting the scales (3) we obtain (drop vertical advection)

$$\omega U\frac{\partial u'}{\partial t'} + U^2\left(\frac{U}{\beta}\right)^{-1/2}\left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right) - \beta y U u' = P + F$$

↓

$$\omega\frac{\partial u'}{\partial t'} + (\beta U)^{1/2}\left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial v'}{\partial y'}\right) - \beta y u' = \frac{P}{U} + \frac{F}{U}$$

We identify 3 time scales

ω^{-1} , $(\beta U)^{-1/2}$, $(\beta y)^{-1}$

Now consider the following 3 cases

i) If $\omega < \beta y$ and $(\beta U)^{1/2} < \beta y$ then the balance is between C, P, F
→ an Ekman BL

ii) $f = \beta y < (\beta U)^{1/2}$ + $\omega < (\beta U)^{1/2}$, the balance is A, P, F
This is an advective or drift BL

iii) $(\beta U)^{1/2} < \omega$ + $\beta y < \omega$ ⇒ the balance is T, P, F
This is a Stokes BL.

• Summary of 2D results from Krishnamurti + Wong (1979)

Near the ocean surface, in the sfc layer, the essential balance is between the pressure gradient and frictional forces. In the overlying friction layer, as one proceeds northward from 15°S, Ekman balance is dominant until one reaches ~10°S. The equatorial region is characterized by a balance between the pressure gradient, horizontal advective, and frictional forces. Poleward in both directions from this equatorial region, Coriolis force becomes significant. As a result winds back with height to the south and veer with height to the north, producing diffluence and a region of descending motion in the equatorial friction layer.

* [The low level jet in the 2D simulation lies @ ~11°N. A slow veering of westerly winds occurs south of this latitude and a more rapid veering occurs to the North. This configuration results in a convergence of both mass and momentum flux @ the latitude of the jet. The region of the ITCZ near 12°N is dominated by strong vertical advection associated with ascending motion above a region of large Ekman convergence on the cyclonic shear side of the strong low level jet. Furthermore, the region is surrounded by an Ekman balance north of 15°N and by a horizontally advective BL south of 10°N.] *

* PBL dynamics (Somali jet)

MET5534: T. N. Krishnamurti (1 hour)

• Question

a) In the context of the planetary boundary dynamics show the scale analysis for the balance of forces for the Ekman, Advective and the Stokes types. Derive appropriate frequency equations.

b) Sketch the wind and pressure fields for the Somali jet over the Arabian Sea during summer at the 1 km level, and discuss the balance of forces from south to north following the jet.

look at the previous answer !

* PBL (Somali jet)

1/1

MET5534: T. N. Krishnamurti (1 hour)

• Using appropriate length and time scales, nondimensionalise the zonal equation of motion in the planetary boundary layer, and discuss under what conditions you expect Ekman, Advective and Stokes boundary layers. Give an example of the balance of forces for the advective boundary layer and explain its role in the cross equatorial flows of the Somali jet.

look at the previous answer

✓ Q : Origins/maintenance of ITCZ.

Sol) A common feature of the time mean tropics is the line of convection near the equator which spans the tropical oceans. (it is not really a solid line, rather over time convection occurs within this zone the locus of all convective events defines ITCZ). This feature is the so-called Inter Tropical Convergence Zone. Why should there be confluence in the winds and concentration of convective activity along such a zone? Below are some theories as to why we observe an ITCZ.

(a) Pike's theory

Pike prescribed, in a model, two belts of high SST's close to 10°N and 10°S with a low temperature belt near the equator. This distribution of SST's is roughly observed in the real oceans. Simulations with this boundary condition showed a preference for a single ITCZ. This showed that although the SST forcing was symmetric about the equator, the simulation evolved to an asymmetric pattern.

(b) Charney's CISK theory on ITCZ.

Charney presents a linear stability analysis for the growth rate of a line symmetric disturbance. He shows that the growth rate increases with the effect of the earth's rotation due to (i) frictional (Ekman) convergence, (ii) the lapse rate of θe. Charney showed that the combined effect of (i) and (ii) produces a maximum growth rate (i.e. an ITCZ) some 7° from the equator. This theory prefers a double ITCZ. However, it does not rely on high SSTs to obtain ITCZ-like features.

(c) Holton's theory

Holton pointed out that there is a critical latitude @ which the angular frequency of a wave matches the Coriolis frequency. From this observation he showed that the frequency spectrum of the westward propagating waves matches the Coriolis frequency @ a latitude close to 10°N. This is the latitude where convergence, rising motion, and the ITCZ would be expected to form via this theory.

* note for parcel method.

The parcel method assumes

① the parcel maintains its identity in thermodynamic processes.

② the parcel in no way disturbs or interacts with the environmental air.

③ the parcel has uniform properties throughout.

④ " " pressure instantaneously adjusts to the pressure of the surrounding.

What is a "parcel"?

↳ a buoyant element of air with size + shape unspecified.

Shortcomings of the parcel method include

① mixing of the parcel with the ambient air is neglected

② the weight of condensed water (some of which is carried along with the parcel) is neglected.

③ aerodynamic drag is neglected.

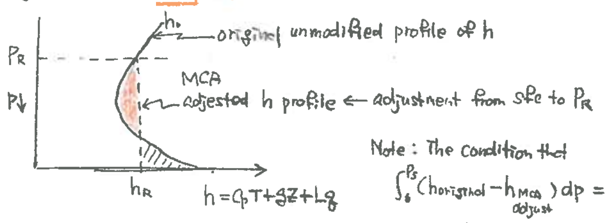
④ the effects of compensating downward motions of the surrounding air are neglected.

Q: moist convective adjustment, Dry convective adjustment, Shallow convection.

Sol)

Moist convective adjustment (MCA)

The idea is to modify soundings where large scale ascending motions occur in a conditionally unstable column. MCA adjusts the sounding in such a way that the final sounding is moist neutral subject to the condition that total moist static energy remains constant (invariant) during the adjustment. *



Note: The condition that $\int_{P_s}^{P_s}(h_{original} - h_{MCA\,adjust})\,dp = 0$ requires the area (shaded) = area (hatched)

Below a reference level P_R the adjustment via MCA satisfies the relation

$$\int_{P_R}^{P_s} h_o\,dp = h_R(P_R - P_s)$$ (← surface pressure)

Determining h_R is an iterative process. We apply the following algorithm @ any level p above the LCL.

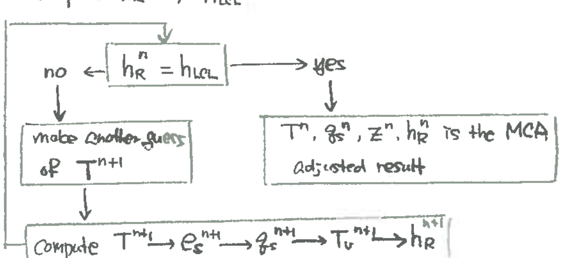
① Guess a value of T_s
② Get e_s using the Teten's formula
③ Get q_s : $q_s = \frac{0.622\,e_s}{p - 0.378 e_s}$
④ Get q : $\frac{q}{q_s} = 0.8$
⑤ Get T_v : $T_v = (1 + 0.61q)T_s$
⑥ Get z_s : $\frac{\partial g z_s}{\partial p} = -\frac{RT_v}{p}$
⑦ Compute h_R : $h_R = c_pT_s + gz_s + Lq_s$

If h_R is not same as graph, make another guess and repeat.

Let's draw a flowchart.
If we know T, q, z, p @ the LCL, the we systematically adjust layers above the P_{LCL} using the following iterative method

At any level $p < P_{LCL}$

- guess $T^o \to e_s^o \to q_s^o \to T_v^o \to gz^o \to h_R^o$
- compare h_R^o w/ h_{LCL}



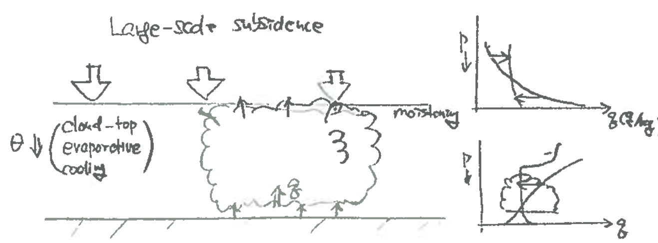
It has been found that the MCA adjusted sounding usually has a temperature difference of a few degrees (°c) and a moisture difference of a few g/kg from the original sounding. In real data applications MCA can indicate large ΔT, Δq adjustments which would shock a model. In GCMs, however, MCA is applied every time step to prevent large ΔT, Δq adjustment

Dry convective adjustment

This is a process by which superadiabatic lapse rates are removed from the model atmosphere. Typically dry adjustment is performed whenever a dry unstable column of air is found @ a model grid point. The procedure used consists of removing heat from the base of the superadiabatic layer and adding heat to the top subject to the constraint that dry static energy in the affected layer remains invariant. This adjustment is applied iteratively from top to bottom in the model's atmospheric column.

Shallow convection

This consists of removing moisture from the PBL and transferring it to the subcloud layer. This process is important since it compensates for the drying and warming due to large scale subsidence. Essentially it is a vertical diffusion scheme which parameterizes the turbulent transport of heat + moisture away from the PBL by shallow, non-precipitating cumulus clouds.



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* The cumulus parametrization problem : A review

William M. Frank, 1983, MWR, 111, 1859-1871

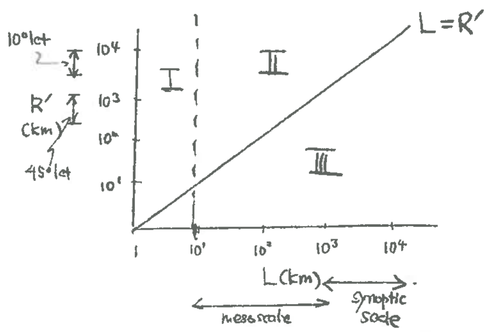
• Scale considerations

(1) Define Rossby radius of deformation

$$R' = \frac{NH}{(\zeta+f)^{1/2}(2\frac{V}{R}+f)^{1/2}}$$

$$= \frac{\text{relative resistence to vertical displacements resulting from static stability}}{\text{relative resistence to horizontal displacements resulting from inertial stability}}$$

N ≡ Brunt Vaisala frequency
H ≡ scale height of circulation
V ≡ rotational component of the wind
R ≡ radius of curvature



<physical dimensions of circulation>

I. Small scale turbulence and individual convective cells and clouds ⇒ probabilistic treatment in most NWP models.

II. Circulations with significantly unbalanced flow in which the divergent component can not be relegated to a secondary status relative to the rotational component. (?)

III. nearly balanced, quasi-horizontal flow regime, large scale circulation evolve slowly + secondary circulations (including vertical motions) are largely controlled by primary circulations

(2) Scales of convective processes.

(a) Generally, dynamically small (L<R') rotational circulations tend to decay rapidly due to the cascade of energy to smaller scales. Dynamically large (L ≳ R') tend to exhibit quasi-horizontal, nearly balanced flow + decay more slowly → lifetime of several days.
⇒ does not apply to convective regimes in which though they are dynamically small they may persist for days

(b) dynamically small circulations have very complex + sensitive interactions between the convection + spatially larger scale divergence fields
↓ divergence perturbations (gravity waves) spread out over a distance ~R'
↳ while convection may be controlled by small as well as large scale processes, the convection itself can influence the divergent flow (large scale processes) over spatially large areas.

(c) Interactions between heating on a dynamically small scale + the rotational flow on that scale are relatively small. Geostrophic adjustment shows that for dynamically small systems the mass field adjusts to the rotational wind field. ↳ heating effects.

(3) Implication of scale considerations to parameterization

• In region III cumulus parameterization seems somewhat simpler (compared to region II) ?

(1) Strong relationship between large scale flow + convection

(2) region III circulations are generally less sensitive to the exact rate or timing of latent heat release than are those of dynamically smaller circulations.

• Region II - complex parameterization required to account for
(1) Interaction between large scale divergence fields + convection ↳ rate of release of latent heat.
(2) additionally must be able to simulate evolution of intense mesoscale systems which are only weakly linked to large scale circulation.
(3) Possibly necessary to directly account for impact of convection + mesoscale circulations on momentum fields.

• Mesoscale Circulations

Latent heat release in convective regions is generally organized in a variety of mesoscale patterns ⇒ lines of cumulus, squall lines, supercells, MCCs, ...
The presence of these mesoscale circulations has reveal important effects

(1) Scale of mesoscale circulation can approach R' in which case latent heat release can have important feed back to the rotational flow, likely to occur in high latitudes + when ζ is large.

(2) rate of convective heating may be more determined by mesoscale circulatio than the large scale flow ⇒ failure to properly parameterize the mesoscale circulations will lead to incorrect specification of the timing + magnitude of the latent heat release.

(3) Importance of mesoscale up/down-drafts.

(4) Subgrid scale modifications to radiational heating due to mesoscale circulations + their resultant cloud patterns.

(5) unresolved mesoscale organization + circulations affect virtually all o the model assumptions concerning the effects of the large scale fields upon the clouds → question feasibility of parameterizing convection using cloud model in course resolution models.

(6) Observational studies suggest that @ least some of the apperent source/sink terms in the momentum + vorticity budgets result from the effects of mesoscale circulations (particularly in the upper troposphere)

(7) Dynamics of convective mesoscale circulations (scales between convection + R') are poorly understood.

Q: describe Ogura + Cho's cumulus model. → bimodality

Sol)

Ogura + Cho proposed a method whereby cumulus cloud populations are determined from large scale variables. This method combines large scale heat and moisture balance considerations w/ a simple steady state 1D model for an individual cumulus cloud. From this, the cloud base mass flux is determined. Essentially, they related Q_1, Q_2 and Q_R with $m_B(\lambda)$. This gave rise to an integral eq. which was numerically solved. Applying real data to this model, they found a bimodal distribution of vertical mass flux w/ a max @ the top of the BL (600 mb) and another in the upper troposphere (200mb, deep convection)
↳ shallow convection.

More details on the model which describes the interaction between the large scale + cumulus convective scale systems. Recall expressions for Q_1 + Q_2

(1) $Q_1 = \frac{\partial \bar{S}}{\partial t} + \nabla \cdot \bar{V}\bar{S} + \frac{\partial}{\partial p}\bar{\omega}\bar{S}$ $\quad$ $\overline{(\ )}$ denotes large scale mean

(2) $Q_2 = -L\left[\frac{\partial \bar{q}}{\partial t} + \nabla \cdot \bar{V}\bar{q} + \frac{\partial}{\partial p}\bar{\omega}\bar{q}\right]$

Q_1, the apparent heat source, is balanced by radiative heating/cooling Q_R, the net condensation $L(C-e)$ and cumulus convective processes $= -\frac{\partial}{\partial p}\overline{\omega' s'}$ (vertical eddy flux of eddy dry static energy)

Q_2, the apparent moisture sink, is balanced by net condensation $L(C-e)$ and cumulus convective processes
↳ vertical eddy flux of eddy moisture $= +\frac{\partial}{\partial p}\overline{\omega' q'}$

So

(3) $Q_1 = Q_R + L(C-e) - \frac{\partial}{\partial p}(\overline{\omega' s'})$

(4) $Q_2 = L(C-e) + L\frac{\partial}{\partial p}(\overline{\omega' q'})$

$\Big\}\ Q_1 - Q_2 - Q_R = -\frac{\partial}{\partial p}(\overline{\omega' h'})$ ← vertical eddy flux of moist static energy

Yanai defined an average mass flux across a unit area as

$$\bar{M} = -\bar{\omega}$$

which is divided into two parts

$$\bar{M} = M_c + \tilde{M}$$

$\tilde{M}$ → mass flux in the enviornment
M_c ↳ upward mass flux in cumulus clouds

We introduce the parameter σ which denotes the area occupied by cumulus clouds. It is easy to prove that the eddy fluxes of s, q, h can be expressed as

$$\overline{s'\omega'} = -M_c(S_c - \tilde{S})$$

$$\overline{q'\omega'} = -M_c(q_c - \tilde{q})$$

$$\overline{h'\omega'} = -M_c(h_c - \tilde{h})$$

we next define the entrainment rate λ as $\lambda = \frac{1}{m}\frac{dm}{dz}$. We use λ to identify the type of cloud. Using this spectral representation the eddy fluxes may be rewritten as

$$\overline{s'\omega'} = -\int_\lambda m(\lambda)(S_c - \tilde{S})\,d\lambda$$

$$\overline{q'\omega'} = -\int_\lambda m(\lambda)(q_c - \tilde{q})\,d\lambda$$

The mass + dry static energy eqs are, respectively

$$E(p) - \delta(p) + \frac{\partial M_c}{\partial p} = 0$$

↑ (entrainment) $\quad$ ↑ detrainment

$$E\tilde{S} + \delta\tilde{S} + \frac{\partial}{\partial p}\int m(\lambda) S_c\,d\lambda + LC = 0$$

we define the entrainment → $\delta \equiv$ detrainment

$$E(p) = \int_0^{\lambda_D} \frac{\lambda H}{P}\, m(\lambda)\,d\lambda$$

the detrainment

$$\delta(p) = m(\lambda_D)\frac{d\lambda_D}{dp} = m_B(\lambda)\eta(\lambda_D, p)\frac{\partial\lambda}{\partial p}$$

the nomalized mass flux

$$\eta(\lambda, p) = \exp\left[-\int_{P_B}^{P} \frac{\lambda H}{P}\,dp\right]$$

and

$$\frac{\partial}{\partial p}\overline{s'\omega'} = M_c\frac{\partial \tilde{S}}{\partial p} + LC$$

$$\frac{\partial}{\partial p}\overline{\omega' q'} = M_c\frac{\partial \tilde{q}}{\partial p} - C - \delta(\tilde{q}^* - \tilde{q})$$

Arakawa introduced the concept of a thin detrainment layer where the cloud loses its buoyancy. At this level (P_D) the cloud properties are the same as the saturation properties of the environment, i.e.

$$h_c(\lambda, P_D) = \tilde{h}^*(P_D)$$

$$q_c(\lambda, P_D) = \tilde{q}^*(P_D)$$

$$S_c(\lambda, P_D) = \tilde{S}^*(P_D)$$

Using all the above we obtain the following expressions for Q_1 + Q_2

$$Q_1 - Q_R = -L\bar{e} - \frac{\partial \tilde{S}}{\partial p}\int_0^{\lambda_D(p)} m_B(\lambda)\eta(\lambda, p)\,d\lambda$$

$$Q_2 = -L\delta(\tilde{q}^* - \tilde{q}) + L\frac{\partial \tilde{q}}{\partial p}\int_0^{\lambda_D(p)} m_B(\lambda)\eta(\lambda, p)\,d\lambda - L\bar{e}$$

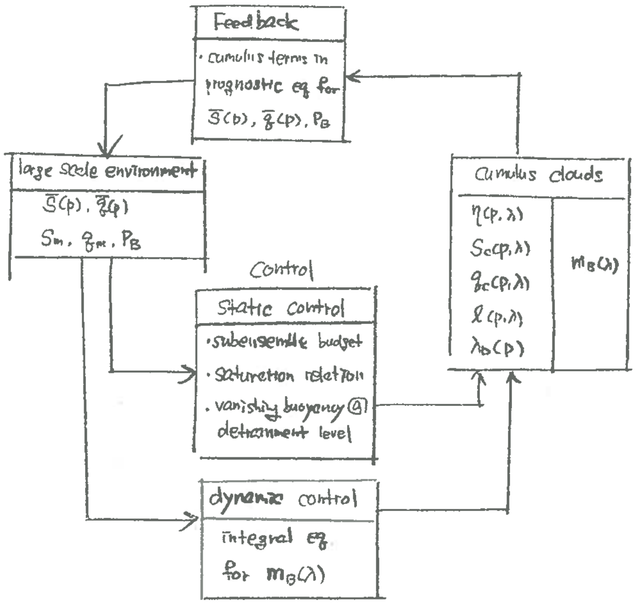
Or upon combining the two

$$\boxed{Q_1 - Q_2 - Q_R = L\delta(\tilde{q}^* - \tilde{q}) - \frac{\partial \tilde{h}}{\partial p}\int_0^{\lambda_D(p)} m_B(\lambda)\eta(\lambda, p)\,d\lambda}$$

This is the integral eq. they solved given large scale observations and a climatological Q_R profile. When the cloud base mass flux was plotted as a function of cloud height there was a maximum for cloud tops @ 600mb (shallow convection) + 200 mb (deep convection) with no other significant levels indicated between

Q: Arakawa–Schubert Scheme

Sol) Arakawa's theory describes how an idealized cumulus ensemble and a large scale field interact. The theory consists of three basic ingredients: feedback, static control, dynamic control. The feedback loop explains how the cumulus transport terms and source terms modify the large scale temperature and moisture fields. The control loops describe how the cloud ensemble properties is modulated by the large scale field.



The bases for this theory is the quasi equilibrium assumption (QEA) of the cloud work function $A(\lambda)$. The large scale field is divided into the subcloud mixed layer and the region above. Entrainment is assumed to take place @ all levels. Detrainment takes place in a thin layer @ cloud top (cloud top is assumed to be level of vanishing buoyancy, $h_c = \tilde{h}^*$). As a result of the above assumptions two important relations are deduced.

$$\rho\frac{\partial\bar{s}}{\partial t} = D(\hat{s} - \bar{s} - L\hat{l}) + M_c\frac{\partial\bar{s}}{\partial z} - \rho\bar{\underline{V}}\cdot\nabla\bar{s} - \rho\bar{w}\frac{\partial\bar{s}}{\partial z} + \bar{Q}_R$$

└→ dry static eq.

$$\rho\frac{\partial\bar{q}}{\partial t} = D(\tilde{q}^* + l - \bar{q}) + M_c\frac{\partial\bar{q}}{\partial z} - \rho\bar{\underline{V}}\cdot\nabla\bar{q} - \rho\bar{w}\frac{\partial\bar{q}}{\partial z}$$

└→ moisture eq.

For more details, look at my note p 18~20.

* Q_1 + Q_2 questions (AS + Kuo convection)

Sol) Consideration of the heat + moisture budgets of large scale tropical weather systems characterized by low level convergence + upper tropospheric divergence and thus general upward motion show that such budgets calculated from large scale motions alone reveal large magnitude residuals which have been termed

$Q_1 \equiv$ apparent heat source

$Q_2 \equiv$ apparent moisture sink.

Diagnostic budget studies typically define

$$Q_1 \equiv \frac{\partial \bar{S}}{\partial t} + \nabla\cdot\bar{S}\bar{V} + \frac{\partial}{\partial p}\bar{\omega}\bar{S} = Q_R + L(c-e) - \frac{\partial}{\partial p}\overline{(\omega' s')} \quad (1)$$

$$Q_2 \equiv -L\left[\frac{\partial \bar{q}}{\partial t} + \nabla\cdot\bar{q}\bar{V} + \frac{\partial}{\partial p}\bar{\omega}\bar{q}\right] = L(c-e) + L\frac{\partial}{\partial p}\overline{\omega' q'} \quad (2)$$

where

$\bar{A}$ denotes large scale value for A

Q_R is radiational cooling (<0) / heating (>0)

$\overline{\omega' s'}$ is the covariance of eddy vertical motion + eddy dry static energy.

$\overline{\omega' q'}$ is " of " " " + eddy moisture.

c is condensation, e is evaporation → c−e is net condensation
⇒ net gain in latent heat if > 0, net loss of heat if < 0.

$S = C_pT + gz$ (enthalpy; (geo)potential energy)

$h = S + Lq$ (latent heat energy)

- How did we get (1) + (2)?

Note that via the continuity eq

$$\nabla\cdot V + \frac{\partial \omega}{\partial p} = 0$$

We used this fact to rewrite

$$V\cdot\nabla S + \omega\frac{\partial S}{\partial p} \quad (\leftarrow \text{advection in dry static energy eq})$$

as $\nabla\cdot SV + \frac{\partial}{\partial p}(\omega S) = S(\nabla\cdot V + \frac{\partial \omega}{\partial p}) + V\cdot\nabla S + \omega\frac{\partial S}{\partial p}$ (first term = 0)

Thus the eq for the local time rate change in S,

$$\frac{\partial S}{\partial t} + V\cdot\nabla S + \omega\frac{\partial S}{\partial p} = Q_R + L(c-e) \leftarrow \text{This is a basic conservation eq } \left(\frac{dS}{dt} = \text{sources + sinks}\right)$$

was rewritten as

$$\frac{\partial S}{\partial t} + \nabla\cdot SV + \frac{\partial}{\partial p}(S\omega) = Q_R + L(c-e)$$

The next step was to average the above eq over an area containing many convective clouds but smaller than a synoptic scale system. Thus we decompose variables as

$S = \bar{S} + S'$ (S′: convective scale; $\bar{S}$: synoptic scale), $\omega = \bar{\omega} + \omega'$

where $\overline{S'} = 0 = \overline{\omega'}$

Then $\overline{\omega S} = \bar{\omega}\bar{S} + \overline{\omega' S'}$

$\overline{\omega' S'}$ → vertical flux of S associated w/ convection; $\bar{\omega}\bar{S}$ → vertical flux of dry static energy associated w/ large scale

We substitute this into our conservation eq for S.

$$\frac{\partial \bar{S}}{\partial t} + \nabla\cdot\bar{S}\bar{V} + \frac{\partial}{\partial p}(\bar{S}\bar{\omega}) = Q_R + L(c-e) - \frac{\partial}{\partial p}(\overline{s'\omega'})$$

where we have neglected all eddy terms except the vertical gradient of the vertical flux of dry static energy associated w/ convection (convection in our analysis is an eddy)

This is how we obtained (1). (2) was similarly derived.

- We can rewrite (1) + (2) as

$$Q_1 - Q_R = L(c-e) - \frac{\partial}{\partial p}(\overline{\omega' s'})$$

$$+) \quad -Q_2 = -L(c-e) - L\frac{\partial}{\partial p}(\overline{\omega' q'})$$

$$Q_1 - Q_R - Q_2 = -\frac{\partial}{\partial p}\left[\overline{\omega'(s' + Lq')}\right] \quad \because h = s + Lq \text{ = moist static energy}$$

or

$$\boxed{Q_1 - Q_R - Q_2 = -\frac{\partial}{\partial p}\overline{\omega' h'}}$$

The importance of these Q_1, Q_2 equations as they have been presented here is that they relate synoptic scale variables to eddy scale (convective) variables. It is necessary to relate (the vertical divergence of) the vertical eddy fluxes ($\overline{\omega' s'}$ or $\overline{\omega' h'}$) to properties of the clouds and their environment.

In p-coordinates, mass flux in the vertical is $M = \rho w = -\frac{\omega}{g}$. We will drop "g" and let $\bar{M} = -\bar{\omega}$ ⇒ mean mass flux. We separate the mass flux into two parts:

(a) mass flux by cloud updrafts, $-\omega_c\sigma$

(b) " " " environment vertical motion, $-\tilde{\omega}(1-\sigma)$

where σ is the fraction of total area occupied by ω_c.

Then

$$\bar{\omega} = \sigma\omega_c + (1-\sigma)\tilde{\omega}$$

(vertical motion in convection is much larger than the motion in the non-convec environment with the overall mean vertical motion comparable to $\tilde{\omega}$ because $\sigma \ll 1$;

We assume the realistic case where $|\omega_c| \gg |\tilde{\omega}| \sim |\bar{\omega}|$ and $\sigma \ll 1$.

Similar definitions for $\bar{S}$ and $\bar{q}$ are

$$\bar{S} = \sigma S_c + (1-\sigma)\tilde{S} = \sigma(S_c - \tilde{S}) + \tilde{S} \simeq \tilde{S}$$

$$\bar{q} = \sigma q_c + (1-\sigma)\tilde{q} = \sigma(q_c - \tilde{q}) + \tilde{q} \simeq \tilde{q}$$

With some algebra, the above expressions for $\bar{\omega}$, $\bar{S}$, $\bar{q}$, the ≈ assumptions in these expansions, and the assumption that $\sigma \ll 1$ we can obtain the following expressions for $\overline{\omega' s'}$, $\overline{\omega' q'}$, $\overline{\omega' h'}$ (also drop terms of order σ^2 or higher)

$$-\overline{s'\omega'} = -\sigma\omega_c(S_c - \tilde{S}) = M_c(S_c - \tilde{S})$$

$$-\overline{q'\omega'} = -\sigma\omega_c(q_c - \tilde{q}) = M_c(q_c - \tilde{q})$$

$$-\overline{h'\omega'} = -\sigma\omega_c(h_c - \tilde{h}) = M_c(h_c - \tilde{h})$$

↑ M_c is cloud mass flux

Since Q_1 + Q_2 may be computed from the mean, synoptic scale data alone, Q_R and $L(c-e)$ may be estimated, then the vertical distribution of eddy (cloud) energy flux may be determined from the synoptic scale data without direct knowledge of cloud properties. This energy-budget approach may be used in two ways.

① Diagnostic → attempt to determine the relative importance of cloud fluxes in determining large scale properties of atmospheric systems. Such studies may reveal systematic relationships between the cloud fluxes + the mean larger scale motion

② Prognostic mode → Given large scale forcing determine convective response and its feedback on the large scale. This is the problem of parameterization.

Successful parameterizations require

① identification of the subgrid scale process (to parameterize)

② determination of the importance of the process for the resolvable scales of motion.

③ (static control) intensive studies of individual cases in order to establish the fact that the relevant physics + dynamics are understood.

④ (dynamic control) formulation of quantitative rules for expressing the location, frequency of occurrence, and intensity of subgrid scale processes in terms of resolvable scales.

⑤ (feedback) formulation of quantitative rules for determining the grid scale average of the transports of mass, momentum, heat and moisture (by the parameterized processes) and verification of these rules by direct observation.

- Numerous observational studies indicate that synoptic/meso-scale motion exert major controls over the formation + maintenance of cumulus convection (particularly deep convection in the tropics). Upward motions + convergence of water vapor are two large scale parameters which are highly correlated w/ convective precip. These parameters are important in both tropical + extratropical convection. In the extratropics we additionally find that the large scale thermodynamic structure + vertical wind shear also strongly influence convection.

↓

Observations provide impetus to pressure parameterization

↓

Q_1, Q_2 budget equations provide mathematical framework upon which to build parameterizations.

<Laseur>

ⓐ Show Q_1 + $-Q_2$ are positive if ① vertical advection dominates ② we have low level large scale convergence + upper level large scale divergence

Take Q_1 first

$$Q_1 = \frac{\partial \bar{s}}{\partial t} + \nabla\cdot\bar{s}\bar{V} + \frac{\partial}{\partial p}(\bar{\omega}\bar{s}) = Q_R + L(c-e) - \frac{\partial}{\partial p}\overline{(\omega' s')}$$

via continuity

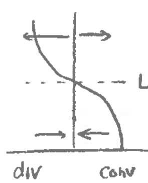
$$\nabla\cdot V + \frac{\partial\omega}{\partial p} = 0$$

Therefore

$$Q_1 = \frac{\partial \bar{s}}{\partial t} + \bar{V}\cdot\nabla\bar{s} + \bar{\omega}\frac{\partial \bar{s}}{\partial p}$$

we assume vertical advection dominates. Thus $Q_1 \simeq \bar{\omega}\frac{\partial\bar{s}}{\partial p}$

Under the condition of low level convergence + upper level divergence

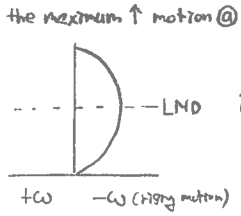


$$\nabla\cdot V = -\frac{\partial\omega}{\partial p}$$

so

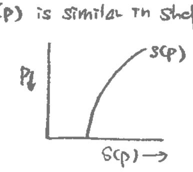
$$\omega(p_1) - \omega(p_0) = -\int_{p_0}^{p_1}(\nabla\cdot V)\,dp$$

We have generally rising motion through the depth of the troposphere w/ the maximum ↑ motion @ the LND



$\bar{\omega} < 0$ (rising motion)

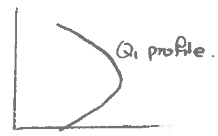
A typical profile of $\bar{s}$ in the tropics is shown below. Note that $\bar{s}(p)$ is similar in shape to $\bar{\theta}(p)$



The thing to notice here is that $\frac{\partial\bar{s}}{\partial p}$ is negative ⟶ $\bar{s}$ ↓ as p↑

Since both $\bar{\omega}$ and $\frac{\partial\bar{s}}{\partial p}$ are negative for the given synoptic conditions, then product is positive and

$$Q_1 \simeq \bar{\omega}\frac{\partial\bar{s}}{\partial p} > 0$$

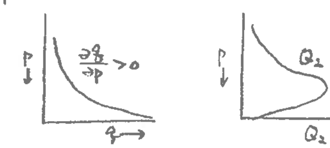


For $-Q_2$, we proceed as above. Let $-Q_2 \simeq L\bar{\omega}\frac{\partial\bar{q}}{\partial p}$

we know $\bar{q}$ ↓ as p↓ so $\frac{\partial\bar{q}}{\partial p} > 0$. In rising motion $\bar{\omega} < 0$. Thus

$$-Q_2 \simeq L\bar{\omega}\frac{\partial\bar{q}}{\partial p} < 0$$

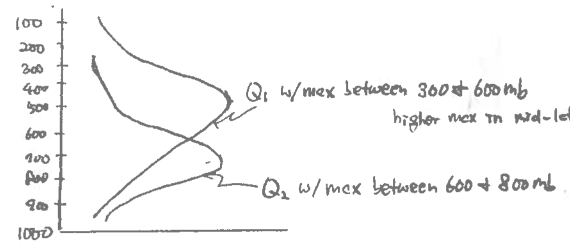
or $Q_2 > 0$.



- Q_1 + Q_2 profiles.

The vertical profiles of the apparent heat source Q_1 are generally similar in the tropical + extratropical studies. Most studies show a max in Q_1 between 600 + 300 mb. The Q_1 profiles diagnosed from time averaged data are smoother than those computed from instantaneous data or from short time averages. The Q_1 profile in mid-lats frequently show less apparent heating in the lower troposphere than do those in the tropics because of higher cloud bases and greater evaporating cooling.

The vertical profiles of Q_2, the apparent moisture sink, are also similar in the tropics and mid-lats, with a max sink present below 800 + 600 mb, somewhat lower than the maxima in the Q_1 profiles.



The overall effect of cumulus clouds is to warm + dry most of the troposphere. This warming + drying occurs mainly through adiabatic compression in the subsiding environment of the clouds. Detrainment of cloud water + the transport of heat + moisture in moist downdrafts are also important terms in the heat + water budgets of the lower troposphere.

Cumulus clouds are modify large scale vorticity + momentum structure. In the tropics, cumulus convection generally produces an apparent sink of vorticity in the lower troposphere and an apparent source in the upper troposphere. The same appears to be true in the mid-lats. Both tropical + extratropical convective clouds are affected strongly by vertical wind shear.

The heat + moisture budgets associated w/ extratropical convective systems differ from those in the tropics in several ways.

① Storage terms $(\frac{\partial\theta}{\partial t}, \frac{\partial q}{\partial t})$ are often as large as other terms in the midlats whereas they are small in the tropics.

② effect of evaporation can be larger in extratropical systems due to drier air + higher cloud base.

③ moist convection in mid-lats is positively correlated w/ static stability unlike tropical convection.

<Laseur>

ⓑ What additional processes play a role in Q_1, Q_2 budgets (from large-scale means)

Recall that we have

$$Q_1 \equiv \frac{\partial \bar{s}}{\partial t} + \nabla \cdot \bar{\chi}\bar{s} + \frac{\partial}{\partial p}(\bar{\omega}\bar{s}) = Q_R + L(c-e) - \frac{\partial}{\partial p}(\overline{\omega' s'})$$

└ definition of Q_1 ; ↳ from conservation eq $\frac{d\bar{s}}{dt}$ = sources + sinks plus expansion of s + ω into $\bar{\alpha} + \alpha'$

And

$$Q_2 \equiv -L\left[\frac{\partial \bar{q}}{\partial t} + \nabla \cdot \bar{\chi}\bar{q} + \frac{\partial}{\partial p}(\bar{\omega}\bar{q})\right] = L(c-e) + L\frac{\partial}{\partial p}(\overline{\omega' q'})$$

The RHS of the above eqs show us additional important terms in Q_1 + Q_2 budgets. These terms are

- Release of latent heat from physical processes in + around the cloud. This is the $L(c-e)$ term ⇒ net heating/cooling from water phase changes.
- Radiative effects represented by Q_R (the rate of radiative cooling/heating)
- Subgrid scale or smaller scale effects ($\frac{\partial \overline{\omega' s'}}{\partial p}$, $\frac{\partial \overline{\omega' q'}}{\partial p}$ terms) which include mixing (entrainment + detrainment) effects between the cloud and environment.

  In Arakawa's scheme there is a thin layer of detrainment @ the cloud top (the level where a rising parcel in the cloud, no buoyancy, $h_c = \tilde{h}^*$). His scheme has no downdraft. Observations suggest downdrafts @ cloud bottom have a great effect on the energy budgets.

<LaSeur>

◎ Outline the major assumptions in Arakawa–Schubert.

The basic idea of AS is that the cumulus ensemble alters the large scale temperature + moisture fields through cumulus updraft induced large scale subsidence and detrainment of cloud water to the environment. The environmental subsidence produces warming + drying and affect the depth of the boundary layer. Detrainment of cloud liquid water causes cooling + moistening. The major assumptions of AS can be summarized as follows:

① Quasi-equilibrium assumption (QEA) ⇒ the destabilization of the atmosphere by large scale processes (eg. low level moisture convergence, radiational cooling of upper troposphere, sfc fluxes of latent + sensible heat) is balanced by the stabilization of the atmosphere by convection. The QEA is used to close the AS scheme. It is not necessary nor used in diagnostic studies

② Common cloud base with cloud types in the ensemble identified by the entrainment rate $\lambda = \frac{1}{m}\frac{dm}{dz}$

③ One single value of q_c (specific humidity in cloud) + S_c and h_c is attributed to each cloud, distinguished by λ. (use of multiple cloud ensemble so we call this a spectral method)

④ Steady state plume model used for entrainment. Values of q_c, S_c, h_c assigned to various clouds as specified by entrainment. Entrainment occurs in all levels of the cloud except the topmost.

⑤ Clouds may have different cloud bases within the ensemble in the real atmosphere but not in the AS scheme.

⑥ A constant λ is assumed for simplicity

⑦ They do not consider the possibility of latent heat release and absorption of energy due to freezing and melting ⇒ crude to non-existent cloud microphysics

⑧ The conversion of cloud droplets to raindrop is parameterized as simply being proportional to the cloud liquid water content in the cloud ensemble.

As a result of these deficiencies, models such as those of Arakawa (1971), Yanai et al (1973), Ogura et al (1973)

① overestimated the cloud mass flux and therefore, also overestimated the compensating subsidence ⇒ too much warming + drying

② they ignore the fact that shallow clouds significantly contribute to the total cloud base flux.

③ detrainment @ a single level means the lateral detrainment over the life cycle of each cloud is not taken into account. What effect this has on computed cloud population properties is an unknown factor.

Improvements over these models were made by Johnson (1976) who included a downdraft in the parameterization. This lead to more realistic values of the cloud and environment fluxes. However, they introduced new problems in the model

① the downdraft entrained air only from the environment. entrainment from the updraft can also be important.

② the downdraft descended along a moist adiabat. This is a not always true. Since their downdrafts entrained only the dry, subsaturated environmental air, there must be adequate moisture in the cloud to saturate the entrained air if it is to descend moist adiabatically.

③ The magnitude of the downdraft was crudely approx. as an empirically derived fraction of the updraft. The downdraft origin was similarly based on a fraction of the pressure depth of the cloud.

↳ this fraction was specified with guidance from observations

Despite these weaknesses, the addition of downdrafts reduces M_c and therefore $\tilde{M}$ with the corresponding reduction in subsidence. The heating + drying profiles appear more realistic in the lower troposphere. It should be noted that Johnson's model had weakness similar to earlier models

① constant entrainment assumption

② latent heat of fusion was ignored

③ single detrainment layer assumed

④ mesoscale downdrafts associated w/ mesoscale organization of convection (eg squall lines) was not included.

Feedback interactions between the downdraft + subcloud boundary layer → the neglect of precip. downdraft in water vapor budget analysis of the subcloud layer during convectively disturbed conditions leads to a substantial overestimation of the between cloud or environmental transport of water vapor into this layer.

⑤ all rain is precipitated instantly (no evaporation)

⑥ all clouds detrain in a thin layer @ cloud top ⇒ $h_c = \tilde{h}^*$ ← no buoyancy.

⑦ detrained cloud air has values $S_c = \tilde{S}^*$, $T_c = \tilde{T}^*$, $h_c = \tilde{h}^*$, $q_c = \tilde{q}^*$

In the AS scheme, the apparent heat source + apparent moisture sink are large scale features. OR the overall effect of convection and radiation. These views are one and the same in terms of the Q_1 + Q_2 budgets. Convection affects the large scale environment by warming + drying it through induced subsidence in the environment. Detrainment of cloud liquid water @ cloud top cools + moistens the environment. The large scale processes affect the cloud ensemble by destabilizing the atmosphere.

ⓓ Defficiencies of AS

According to observational studies of actual cumulus ensembles, the important features which are not incorporated into the scheme are as follows;

① Only updrafts are included, downdraft are neglected. Downdrafts can make the BL cooler and more moist by detraining cloud air @ the cloud base. Downdrafts can affect the cloud base.

② In the model one single value of q, s, h is attributed to the cloud. In the real case these values have a horizontal distribution. Accurate specification of these values can impact microphysics

③ In the model, all clouds detrain at a single level ⇒ the cloud top. In the real case, cloud air may detrain @ all levels, especially from the cloud base into the BL due to downdrafts.

④ In the model all rain is precipitated immediately with taking into consideration evaporation of rainwater. In the real case, evaporation of cloud water is an issue.

* To arrive at a generalized framework for cumulus parameterization schemes, we adopt the terminology below which describes parameterization in terms of 3 aspects

① dynamic control — determines modulation of convection by the environment.
  → tells us where and how strong the convection will be.

② static control — determines the updraft/downdraft properties and such internal mechanisms such as detrainment, entrainment, and cloud microphysics

③ feedback — specifies the modification of the environment by convection
  → it distributes the total integrated heating & drying in the vertical.

	Kuo	AS
dynamic control	large scale convergence/ advection of moisture	rate of destabilization by large scale changes
Static Control	represent in cloud thermodynamics using a moist adiabat.	Constant entrainment rate w/ little to no microphysics
Feed back	heating & moistening are proportional to $T_c - T$ and $q_c - q$. physical processes which cause heating & moistening are not explicitly considered	Convective stabilization is just enough to maintain steady state w/ large scale destabilization rate. Convection influences the environment through environment subsidence and detrainment @ the top and/or bottom of the cloud. The latent heat release maintains the vertical mass flux of the clouds

MET5541: T. N. Krishnamurti (1 hour) * → Ch2.

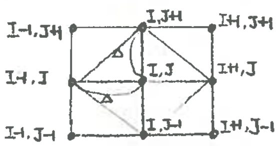
- Question

a) Derive an appropriate stencil for a 4th order accurate Laplacian or describe a procedure for implementing it.

b) Using the above lattice describe in principle how one constructs a fourth order accurate Jacobian which satisfies the quadratic invariance. ($\overline{\zeta}, \overline{K}$)

a) There are two kinds of stencils. $\nabla^2\psi$

– first 9-point stencil



Taylor's expansion (diamond)

$$\psi(I+1,J) = \psi(I,J) + \frac{\partial\psi}{\partial x}\Big|_{I,J}\Delta x + \frac{\partial^2\psi}{\partial x^2}\Big|_{I,J}\frac{(\Delta x)^2}{2!} + \frac{\partial^3\psi}{\partial x^3}\Big|_{I,J}\frac{(\Delta x)^3}{3!} + \frac{\partial^4\psi}{\partial x^4}\Big|_{I,J}\frac{(\Delta x)^4}{4!} + hot \quad \text{——} ①$$

$$\psi(I-1,J) = \psi(I,J) - \frac{\partial\psi}{\partial x}\Big|_{I,J}\Delta x + \frac{\partial^2\psi}{\partial x^2}\Big|_{I,J}\frac{(\Delta x)^2}{2!} - \frac{\partial^3\psi}{\partial x^3}\Big|_{I,J}\frac{(\Delta x)^3}{3!} + \frac{\partial^4\psi}{\partial x^4}\Big|_{I,J}\frac{(\Delta x)^4}{4!} + hot \quad \text{——} ②$$

$$\psi(I,J+1) = \psi(I,J) + \frac{\partial\psi}{\partial y}\Big|_{I,J}\Delta y + \frac{\partial^2\psi}{\partial y^2}\Big|_{I,J}\frac{(\Delta y)^2}{2!} + hot \quad \text{——} ③$$

$$\psi(I,J-1) = \psi(I,J) - \frac{\partial\psi}{\partial y}\Big|_{I,J}\Delta y + \frac{\partial^2\psi}{\partial y^2}\Big|_{I,J}\frac{(\Delta y)^2}{2!} - hot. \quad \text{——} ④$$

Let $\Delta x = \Delta y = \Delta$, $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \nabla^2\psi$

① + ② + ③ + ④ →

$$\star\ \psi(I+1,J) + \psi(I-1,J) + \psi(I,J+1) + \psi(I,J-1) - 4\psi(I,J)$$
$$= \nabla^2\psi\Big|_{I,J}\Delta^2 + 2\frac{(\Delta x)^4}{4!}\left(\frac{\partial^4\psi}{\partial x^4} + \frac{\partial^4\psi}{\partial y^4}\right) + 2\frac{(\Delta x)^6}{6!}\left(\frac{\partial^6\psi}{\partial x^6} + \frac{\partial^6\psi}{\partial y^6}\right) + hot \quad \text{(I)}$$

Taylor's expansion (square)

$$\psi(I+1,J+1) = \psi(I,J) + \Delta\left(\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y}\right)\Big|_{I,J} + \frac{\Delta^2}{2!}\left(\frac{\partial^2\psi}{\partial x^2} + 2\frac{\partial^2\psi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial y^2}\right)\Big|_{I,J}$$
$$+ \frac{\Delta^3}{3!}\left(\frac{\partial^3\psi}{\partial x^3} + 3\frac{\partial^3\psi}{\partial x^2\partial y} + 3\frac{\partial^3\psi}{\partial x\partial y^2} + \frac{\partial^3\psi}{\partial y^3}\right)\Big|_{I,J} + hot \quad ①'$$

$$\psi(I-1,J-1) = \psi(I,J) - \Delta\left(\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y}\right)\Big|_{I,J} + \frac{\Delta^2}{2!}\left(\frac{\partial^2\psi}{\partial x^2} + 2\frac{\partial^2\psi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial y^2}\right)\Big|_{I,J}$$
$$- \frac{\Delta^3}{3!}\left(\frac{\partial^3\psi}{\partial x^3} + 3\frac{\partial^3\psi}{\partial x^2\partial y} + 3\frac{\partial^3\psi}{\partial x\partial y^2} + \frac{\partial^3\psi}{\partial y^3}\right)\Big|_{I,J} + hot \quad ②'$$

$$\psi(I-1,J+1) = \psi(I,J) - \Delta\left(\frac{\partial\psi}{\partial x} - \frac{\partial\psi}{\partial y}\right)\Big|_{I,J} + \frac{\Delta^2}{2!}\left(\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial^2\psi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial y^2}\right)\Big|_{I,J}$$
$$- \frac{\Delta^3}{3!}\left(\frac{\partial^3\psi}{\partial x^3} - 3\frac{\partial^3\psi}{\partial x^2\partial y} + 3\frac{\partial^3\psi}{\partial x\partial y^2} - \frac{\partial^3\psi}{\partial y^3}\right)\Big|_{I,J} + hot \quad ③'$$

$$\psi(I+1,J-1) = \psi(I,J) + \Delta\left(\frac{\partial\psi}{\partial x} - \frac{\partial\psi}{\partial y}\right)\Big|_{I,J} + \frac{\Delta^2}{2!}\left(\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial^2\psi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial y^2}\right)\Big|_{I,J}$$
$$+ \frac{\Delta^3}{3!}\left(\frac{\partial^3\psi}{\partial x^3} - 3\frac{\partial^3\psi}{\partial x^2\partial y} + 3\frac{\partial^3\psi}{\partial x\partial y^2} - \frac{\partial^3\psi}{\partial y^3}\right)\Big|_{I,J} + hot \quad ④'$$

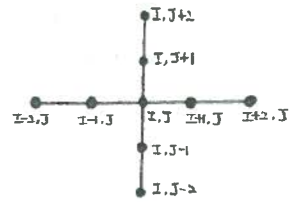
①' + ②' + ③' + ④' →

$$\star\ \psi(I+1,J+1) + \psi(I-1,J-1) + \psi(I-1,J+1) + \psi(I+1,J-1) - 4\psi(I,J)$$
$$= \nabla^2\psi\Big|_{I,J}(\sqrt{2}\Delta)^2 + 2\frac{(\sqrt{2}\Delta)^4}{4!}\left(\frac{\partial^4\psi}{\partial x^4} + \frac{\partial^4\psi}{\partial y^4}\right) + 2\frac{(\sqrt{2}\Delta)^6}{6!}\left(\frac{\partial^6\psi}{\partial x^6} + \frac{\partial^6\psi}{\partial y^6}\right) + hot \quad \text{(II)}$$

4 * (I) − (II) → (eliminate $O(\Delta^4)$)

$$\therefore\ \nabla^2\psi\Big|_{I,J} = \frac{1}{2\Delta^2}\Big\{4[\psi(I+1,J) + \psi(I-1,J) + \psi(I,J+1) + \psi(I,J-1)]$$
$$- [\psi(I+1,J+1) + \psi(I-1,J-1) + \psi(I-1,J+1) + \psi(I+1,J-1)]$$
$$- 12\psi(I,J)\Big\} + O(\Delta^4)$$

– another 9 point stecil



(I) →

$$\psi(I+1,J) + \psi(I-1,J) + \psi(I,J+1) + \psi(I,J-1) - 4\psi(I,J)$$
$$= \nabla^2\psi\Big|_{I,J}\Delta^2 + 2\frac{\Delta^4}{4!}\left(\frac{\partial^4\psi}{\partial x^4} + \frac{\partial^4\psi}{\partial y^4}\right)\Big|_{I,J} + 2\frac{\Delta^6}{6!}\left(\frac{\partial^6\psi}{\partial x^6} + \frac{\partial^6\psi}{\partial y^6}\right)\Big|_{I,J} + hot \quad \text{——} Ⓐ$$

Similarly,

$$\psi(I+2,J) + \psi(I-2,J) + \psi(I,J+2) + \psi(I,J-2) - 4\psi(I,J)$$
$$= \nabla^2\psi\Big|_{I,J}(2\Delta)^2 + 2\frac{(2\Delta)^4}{4!}\left(\frac{\partial^4\psi}{\partial x^4} + \frac{\partial^4\psi}{\partial y^4}\right)\Big|_{I,J} + 2\frac{(2\Delta)^6}{6!}\left(\frac{\partial^6\psi}{\partial x^6} + \frac{\partial^6\psi}{\partial y^6}\right)\Big|_{I,J} + hot \quad \text{——} Ⓑ$$

16 * Ⓐ − Ⓑ →

$$\therefore\ \nabla^2\psi\Big|_{I,J} = \frac{1}{12\Delta^2}\Big\{16[\psi(I+1,J) + \psi(I-1,J) + \psi(I,J+1) + \psi(I,J-1)]$$
$$- [\psi(I+2,J) + \psi(I-2,J) + \psi(I,J+2) + \psi(I,J-2)]$$
$$- 60\psi(I,J)\Big\} + O(\Delta^4)$$

b) There are three forms of Jacobian

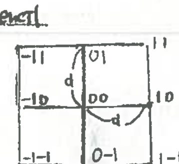
$$J(\zeta,\psi) = \frac{\partial\zeta}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\zeta}{\partial y}\frac{\partial\psi}{\partial x} \quad \text{——} ⓐ$$

$$J(\zeta,\psi) = \frac{\partial}{\partial x}\left(\zeta\frac{\partial\psi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\zeta\frac{\partial\psi}{\partial x}\right) \quad \text{——} ⓑ$$

$$J(\zeta,\psi) = \frac{\partial}{\partial y}\left(\psi\frac{\partial\zeta}{\partial x}\right) - \frac{\partial}{\partial x}\left(\psi\frac{\partial\zeta}{\partial y}\right) \quad \text{——} ⓒ$$

→ analytically the above are the same, but not in finite difference.

- 9 point stencil



$$J^{++}_{00}(\zeta,\psi) = \frac{1}{4d^2}[(\zeta_{10} - \zeta_{-10})(\psi_{01} - \psi_{0-1}) - (\zeta_{01} - \zeta_{0-1})(\psi_{10} - \psi_{-10})] \quad \text{——} ⓐ'$$

$$J^{+\times}_{00}(\zeta,\psi) = \frac{1}{4d^2}[\zeta_{10}(\psi_{11} - \psi_{1-1}) - \zeta_{-10}(\psi_{-11} - \psi_{-1-1})$$
$$- \zeta_{01}(\psi_{11} - \psi_{-11}) + \zeta_{0-1}(\psi_{1-1} - \psi_{-1-1})] \quad \text{——} ⓑ'$$

$$J^{\times+}_{00}(\zeta,\psi) = \frac{1}{4d^2}[\psi_{01}(\zeta_{11} - \zeta_{-11}) - \psi_{0-1}(\zeta_{1-1} - \zeta_{-1-1})$$
$$- \psi_{10}(\zeta_{11} - \zeta_{1-1}) + \psi_{-10}(\zeta_{-11} - \zeta_{-1-1})] \quad \text{——} ⓒ'$$

→ conservation of mean-square vorticity

$$\overline{\overline{\zeta J(\zeta,\psi)}} = 0$$

Using ⓐ', ⓑ', & ⓒ', we can prove that

$\overline{\overline{\zeta J^{++}(\zeta,\psi)}}$ & $\overline{\overline{\zeta J^{+\times}(\zeta,\psi)}}$ are not constant.

However,

$$\overline{\overline{\zeta J^{\times+}(\zeta,\psi)}} = 0, \quad \overline{\overline{\zeta J^{++}(\zeta,\psi)}} + \overline{\overline{\zeta J^{+\times}(\zeta,\psi)}} = 0$$

Similarly, kinetic energy.

$$\overline{\overline{\psi J^{++}(\zeta,\psi)}} + \overline{\overline{\psi J^{\times +}(\zeta,\psi)}} = 0$$

$$\overline{\overline{\psi J^{+\times}(\zeta,\psi)}} = 0$$

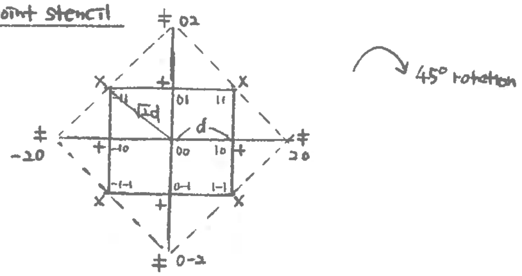
$$\therefore J_1(\zeta,\psi) = \frac{1}{3}[J^{++}(\zeta,\psi) + J^{+\times}(\zeta,\psi) + J^{\times +}(\zeta,\psi)] \quad \text{—— (*)}$$

→ conserve $\overline{\overline{\zeta^2}}$, $\overline{\overline{K}}$

After expand ζ + ψ in Taylor series and substituting into ①, ②, ③, we can find the accuracy of the J_1

→ second order accurate

• 13 point stencil



$$J_{00}^{\times\times}(\zeta,\psi) = \frac{1}{8d^2}[(\zeta_{11}-\zeta_{-1-1})(\psi_{-11}-\psi_{1-1}) - (\zeta_{-11}-\zeta_{1-1})(\psi_{11}-\psi_{-1-1})]$$

$$J_{00}^{\times\ddagger}(\zeta,\psi) = \frac{1}{8d^2}[\zeta_{11}(\psi_{02}-\psi_{20}) - \zeta_{-1-1}(\psi_{-20}-\psi_{0-2}) - \zeta_{-11}(\psi_{02}-\psi_{-20}) + \zeta_{1-1}(\psi_{20}-\psi_{0-2})]$$

$$J_{00}^{\ddagger\times}(\zeta,\psi) = \frac{1}{8d^2}[\psi_{-11}(\zeta_{02}-\zeta_{-20}) - \psi_{1-1}(\zeta_{20}-\zeta_{0-2}) - \psi_{11}(\zeta_{02}-\zeta_{20}) + \psi_{-1-1}(\zeta_{-20}-\zeta_{0-2})]$$

$$\therefore J_2(\zeta,\psi) = \frac{1}{3}[J_{00}^{\times\times}(\zeta,\psi) + J^{\times\ddagger}(\zeta,\psi) + J^{\ddagger\times}(\zeta,\psi)] \quad \text{—— (**)}$$

→ conserve $\overline{\overline{\zeta^2}}$, $\overline{\overline{K}}$

→ second order accurate

(*), (**) →

$$J_3(\zeta,\psi) = 2J_1(\zeta,\psi) - J_2(\zeta,\psi)$$
$$= J(\zeta,\psi) + O(d^4)$$

Therefore, we can construct a fourth order accurate Jacobian which satisfies the quadratic invariance.

5541

MET? : Krish? (?)

• Write down the names of major subroutines (with arguments) you would need in order to solve a grid point barotropic vorticity equation model as a prediction problem for a given streamfunction. Assume β plane limited domain, cyclic boundary conditions along x. Outline a main program that would call the above subroutines in order to produce a 24 hour forecast of the streamfunction and to map it.

barotropic vorticity eq.

$$\frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi) - \beta\frac{\partial\psi}{\partial x} \equiv B$$

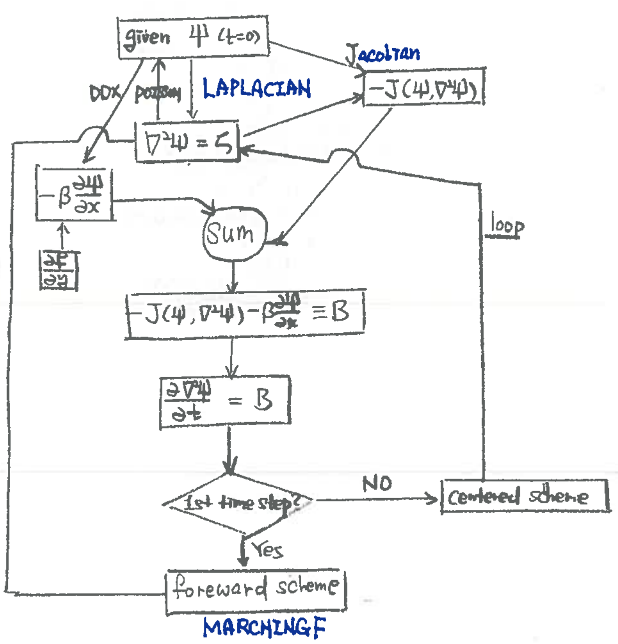
where ψ = streamfunction

$$J(\psi, \nabla^2\psi) = \frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x} \quad \text{(Jacobian)}$$

The names of major subroutines needed.

- DDX (ψ)
- LAPLACIAN (ψ, $\nabla^2\psi$)
- JACOBIAN (ψ, $\nabla^2\psi$, J)
- MARCHINGF (Δt, $\nabla^2\psi^n$, B, $\nabla^2\psi^{n+1}$) MARCHINGC (Δt, $\nabla^2\psi^n$, $\nabla^2\psi^{n-1}$, $\nabla^2\psi^{n+1}$, B)
- POISSON ($\nabla^2\psi$, ψ)

Flow chart



Outline of main program

```
PROGRAM VORT
declare dimension (ψ, ∇²ψ, J, B, .... )
ψ = given
CALL LAPLACIAN (ψ, ∇²ψ)        DO 10 I=1, (nΔt)  ← 24hr
CALL JACOBIAN
CALL DDX
SUM
IF ( I=1 ) then
 CALL MARCHINGF
 ELSE
 CALL MARCHINGC
 ENDIF
10 CONTINUE
STOP
END
```

→ Think more & make this organized!

MET5541: T. N. Krishnamurti (?) *

- Question

a. Discuss the computational stability of a semi-implicit time differencing scheme applied to a simple linear wave equation. Discuss its advantage over an explicit method.

b. In its application to shallow water show the separation of the terms for the fast (→ implicit) and the slow modes (→ explicit). Briefly indicate how a Helmholtz equation pops up in this formulation.

a. A simple linear wave eq.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \quad \text{——— (1)}$$

The implicit finite-difference analog of the linear eq.

$$\frac{U_m^{n+1} - U_m^n}{\Delta t} = -\frac{c}{2}\left(\frac{U_{m+1}^{n+1} - U_{m-1}^{n+1}}{2\Delta x} + \frac{U_{m+1}^n - U_{m-1}^n}{2\Delta x}\right) \quad \text{——— (2)}$$

where m is the space index and n is the time index

Eq (2) can be solved by inverting a matrix with proper B.C. (tridiagonal matrix)

Assuming $U_m^n = U^n e^{ikm\Delta x}$

$$\frac{U^{n+1} - U^n}{\Delta t} = -\frac{c}{2}\,\frac{U^{n+1}(e^{ik\Delta x} - e^{-ik\Delta x})}{2\Delta x} - \frac{c}{2}\,\frac{U^n(e^{ik\Delta x} - e^{-ik\Delta x})}{2\Delta x}$$

where $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

$$U^{n+1}\left(1 + ic\frac{\Delta t}{2\Delta x}\sin k\Delta x\right) = U^n\left(1 - ic\frac{\Delta t}{2\Delta x}\sin k\Delta x\right)$$

∴ $\lambda = \frac{U^{n+1}}{U^n} = \frac{1 - ic\frac{\Delta t}{2\Delta x}\sin k\Delta x}{1 + ic\frac{\Delta t}{2\Delta x}\sin k\Delta x}$ (amplification factor)

$|\lambda| = 1$ ⟶ always stable ! (unconditionally stable)

* in a linear eq. there are no non-linear terms.

→ advantage

In the explicit time-differencing schemes, the time-step should satisfy the CFL condition. The implicit time-integration scheme permits longer time steps than specified by the CFL condition and is therefore more economical than explicit time-difference scheme

b. The application of the semi-implicit time-differencing scheme to shallow-water model.

In the shallow water model, we have a layer of fluid with constant density

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$$

momentum eq.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + \frac{\partial \phi}{\partial x} = 0 \quad \text{——— (a)}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + \frac{\partial \phi}{\partial y} = 0 \quad \text{——— (b)}$$

continuity eq.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{——— (i)}$$

(i) →

$$W_{top} - W_{bottom} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)h = 0 \quad (\because u \,\&\, v \text{ are independent of height})$$

Let, H = mean depth of the fluid
h′ = perturbation height
→ $h = H + h'$

Assuming $W_{bottom} = 0$ with a flat surface

$$W_{top} = \frac{dh}{dt} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - h'\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

also

$$\frac{dh}{dt} = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - h'\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad \text{——— (c)}$$

$g \times$ (c) → ($\because \phi = gh$)

$$\therefore \frac{\partial \phi'}{\partial t} + \frac{\partial}{\partial x}(u\phi') + \frac{\partial}{\partial y}(v\phi') + \bar{\Phi}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad \text{——— (C)}$$

where $\bar{\Phi} = gH$, $\phi' = \phi - \bar{\Phi}$

→ A linearized shallow-water model

→ Two of the solutions are gravitational modes, and the third is a Rossby wave.

⇒ First, linearized form on a non-rotating frame ($f = 0$)

$$\frac{\partial u}{\partial t} + \frac{\partial \phi'}{\partial x} = 0 \quad \text{——— (I)}$$

$$\frac{\partial v}{\partial t} + \frac{\partial \phi'}{\partial y} = 0 \quad \text{——— (II)}$$

$$\frac{\partial \phi'}{\partial t} + \bar{\Phi}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad \text{——— (III)}$$

← no mean flow

$\frac{\partial}{\partial t}$(III) and making use of (I), (II) →

$$\frac{\partial^2 \phi'}{\partial t^2} - \bar{\Phi}\nabla^2\phi' = 0$$

Let's assume that the perturbation is only in x-direction

$$\frac{\partial^2 \phi'}{\partial t^2} - \bar{\Phi}\frac{\partial^2 \phi'}{\partial x^2} = 0$$

Assuming $\phi' = e^{ik(x-ct)}$

$c^2 = \bar{\Phi} = gH$ or $c = \pm\sqrt{gH}$ ⟶ 2 gravity waves.

↑ gravity wave phase speed

⇒ Second, linearized eq. including f and assuming non-divergent

$$\frac{\partial u}{\partial t} - fv = -\frac{\partial \phi'}{\partial x} \quad \text{——— (i)}$$

$$\frac{\partial v}{\partial t} + fu = -\frac{\partial \phi'}{\partial y} \quad \text{——— (ii)}$$

where $u = -\frac{\partial \psi}{\partial y}$, $v = \frac{\partial \psi}{\partial x}$, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$\frac{\partial}{\partial x}$(ii) $- \frac{\partial}{\partial y}$(i) ⟶ (Linearized form of the vorticity eq.)

$$\frac{\partial \zeta}{\partial t} = -\beta v \qquad \because \beta = \frac{df}{dy},\ \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla^2\psi$$

$$\frac{\partial \nabla^2\psi}{\partial t} = -\beta\frac{\partial \psi}{\partial x}$$

Assuming $\psi = \hat{\psi}e^{i(kx + ly - \nu t)}$

$\nu(k^2 + l^2) = -k\beta$ ⟶ $\nu = \frac{-k\beta}{k^2 + l^2}$

∴ $C_x = \frac{\nu}{k} = \frac{-\beta}{k^2 + l^2}$ ⟶ a Rossby wave.

∴ We observe that the shallow-water eqs contain both slow-moving Rossby waves and high-frequency gravity waves.

From ⓐ, ⓑ and ⓒ →

$$\frac{\partial u}{\partial t} + \frac{\partial \phi'}{\partial x} = -\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv\right) = N_u \quad \text{——— (A)}$$

$$\frac{\partial v}{\partial t} + \frac{\partial \phi'}{\partial y} = -\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu\right) = N_v \quad \text{——— (B)}$$

$$\frac{\partial \phi'}{\partial t} + \Phi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\left(\frac{\partial u\phi'}{\partial x} + \frac{\partial v\phi'}{\partial y}\right) = N_h \quad \text{——— (C)}$$

gravity wave terms ⟨linear⟩ implicit | Rossby wave terms ⟨non-linear⟩ explicit

Here, lets drop the ( )' in ϕ' term for simplicity.

Finite-difference analog of (A) →

$$\frac{u^{n+1} - u^{n-1}}{2\Delta t} + \frac{1}{2}\left(\frac{\partial \phi^{n+1}}{\partial x} + \frac{\partial \phi^{n-1}}{\partial x}\right) = (N_u)^n$$

$$u^{n+1} = u^{n-1} - \Delta t\left(\frac{\partial \phi^{n+1}}{\partial x} + \frac{\partial \phi^{n-1}}{\partial x}\right) + 2\Delta t (N_u)^n \quad \text{——— (A)'}$$

Similarly

(B) → $$v^{n+1} = v^{n-1} - \Delta t\left(\frac{\partial \phi^{n+1}}{\partial y} + \frac{\partial \phi^{n-1}}{\partial y}\right) + 2\Delta t (N_v)^n \quad \text{——— (B)'}$$

(C) → $$\phi^{n+1} = \phi^{n-1} - \bar{\Phi}\Delta t\left[\left(\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y}\right) + \left(\frac{\partial u^{n-1}}{\partial x} + \frac{\partial v^{n-1}}{\partial y}\right)\right] + 2\Delta t (N_h)^n \text{——— (C)'}$$

→ A single eq. for ϕ ($\frac{\partial}{\partial x}$(A)', $\frac{\partial}{\partial y}$(B)' —subst.→ (C)')

$$\phi^{n+1} = \phi^{n-1} + \bar{\Phi}(\Delta t)^2 \nabla^2 \phi^{n+1} + \bar{\Phi}(\Delta t)^2 \nabla^2 \phi^{n-1}$$
$$- \bar{\Phi}\Delta t(\nabla \cdot \underline{V}^{n-1}) - 2\bar{\Phi}(\Delta t)^2\left(\frac{\partial}{\partial x}(N_u)^n + \frac{\partial}{\partial y}(N_v)^n\right)$$
$$- \bar{\Phi}\Delta t(\nabla \cdot \underline{V}^{n-1}) + 2\Delta t (N_h)^n$$

$$\bar{\Phi}(\Delta t)^2 \nabla^2 \phi^{n+1} - \phi^{n+1} = \underbrace{-\left[\phi^{n-1} + \bar{\Phi}(\Delta t)^2 \nabla^2 \phi^{n-1} - 2\bar{\Phi}\Delta t(\nabla \cdot \underline{V})^{n-1}\right]}_{F^{n-1}}$$
$$\underbrace{+ 2\bar{\Phi}(\Delta t)^2\left[\frac{\partial}{\partial x}(N_u)^n + \frac{\partial}{\partial y}(N_v)^n\right] - 2\Delta t (N_h)^n}_{G^n}$$

$$\therefore \boxed{\nabla^2 \phi^{n+1} - \frac{\phi^{n+1}}{\bar{\Phi}(\Delta t)^2} = \frac{F^{n-1} + G^n}{\bar{\Phi}(\Delta t)^2}}$$

↳ Helmholtz eq. for the variable ϕ^{n+1}

a. Again!

Semi-implicit scheme

$$\frac{\partial F}{\partial t} = i\omega F$$

where $\omega = \alpha + \beta$, $\alpha < \beta$

α : low frequency mode — nonlinear term, explicitly

β : high frequency mode — linear term, implicitly

$$\frac{F_m^{n+1} - F_m^{n-1}}{2\Delta t} = i\alpha F_m^n + i\beta\frac{(F_m^{n+1} + F_m^{n-1})}{2}$$

or

$$F_m^{n+1} = F_m^{n-1} + 2\Delta t\left[i\alpha F_m^n + \frac{i\beta}{2}(F_m^{n+1} + F_m^{n-1})\right]$$

Let
$$F_m^n = F^n e^{ikm\Delta x}$$
$$F^{n+1} = G F^n$$

$$\left(G F_m^n = G^{-1} F_m^n + 2\Delta t\left[i\alpha F_m^n + \frac{i\beta}{2}(G F_m^n + G^{-1} F_m^n)\right]\right) \times \frac{G}{F_m^n}$$

$$G^2 = 1 + 2\Delta t\left[i\alpha G + \frac{i\beta}{2}(G^2 + 1)\right]$$

$$(1 - i\beta\Delta t)G^2 - 2i\alpha\Delta t G - (1 + i\beta\Delta t) = 0$$

$$G = \frac{i\alpha\Delta t \pm \sqrt{1 + \beta^2\Delta t^2 - \alpha^2\Delta t^2}}{1 - i\beta\Delta t}$$

If $1 + \beta^2\Delta t^2 - \alpha^2\Delta t^2 \geq 0$, stable

i.e. $1 + \beta^2\Delta t^2 > \alpha^2\Delta t^2$ always ← since $\beta > \alpha$

MET5541: T. N. Krishnamurti (1 hour) *

• Write down the shallow water equation in semi-implicit form and describe the Helmholtz equation for the free surface height. Discuss the invariants of the problem for a closed domain. What may be appropriate boundary conditions for this problem?

* look at my note p7 and 8 + previous question

$$\nabla^2\phi^{n+1} - \frac{\phi^{n+1}}{\bar{\phi}(\Delta t)^2} = \frac{F^{n-1}+G^{n}}{\bar{\phi}(\Delta t)^2}$$

• The invariants (a closed domain)

$\overline{K}$ = const.

$\overline{\zeta_a}$ = const.

$\overline{\zeta_a^m}$ = const.

• cyclic boundary condition

Think more ??

MET5541: T. N. Krishnamurti (?)

- Given a linearized system of shallow water equations (linearized about a constant basic flow), show that the frequency equation for wave motions is a cubic. Show that this cubic has two solutions which describe gravity waves moving in opposite directions and a unidirectional Rossby wave.✓ Show furthermore that the use of the geostrophic approximation retains only the Rossby wave and the gravity wave solution vanish.

Hint: Use as a start equations of the type:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)_M u'-fv'+g\frac{\partial h'}{\partial x}=0$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial v'}{\partial x}+\beta v'+f\frac{\partial u'}{\partial x}=0$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)h'+v'\frac{d\bar{h}}{dy}+\bar{h}\frac{\partial u'}{\partial x}=0$$

- look at the previous question and note p.7 + 8.
- Geostrophic approximation

$$\begin{cases} +fv_g = \frac{\partial \phi}{\partial x} \\ -fu_g = \frac{\partial \phi}{\partial y} \end{cases}$$

⋮

MET5541: T. N. Krishnamurti (?, 1997)

- In the spectral transform method over a sphere explain clearly what is:

a) a fourier-legendre transform

b) an inverse fourier-legendre transform

Show how you would use the above to calculate the fourier-legendre transform of the nonlinear term of the barotropic vorticity equation starting from a global field of $\Psi(\lambda,\Theta)$ where λ is the longitude and Θ is the latitude. ↳ streamfunction

✓ Define what is meant by the terms gaussian latitude and gaussian quadrature.

- A Fourier–Legendre transform

It transforms field variables (such as $A(\lambda,\mu)$) from grid to spectral space, in order to compute A_n^m (spectral coefficients)

The steps are

Step 1 Perform the Fourier transform along latitudes.

Fourier coeff → $A^m(\mu) = \frac{1}{2\pi}\int_0^{2\pi} A(\lambda,\mu)e^{-im\lambda}d\lambda$

Step 2 Perform the Legendre transform

$$A_n^m = \int_{-1}^{1} A^m(\mu)P_n^m(\mu)d\mu$$

↳ associate Legendre functions

- An inverse Fourier–Legendre transform

It transforms field variables from spectral to grid space, in order to get $A(\lambda,\mu)$

Step 1 Perform the reverse Legendre transform

$$A^m(\mu_k) = \sum_n A_n^m P_n^m(\mu_k)$$

Step 2 Perform the reverse Fourier transform

$$A(\lambda_j,\mu_k) = \sum_m A^m(\mu_k)e^{im\lambda_j}$$

Thus one proceeds from spherical harmonic components A_n^m to Fourier components A^m and then to grid-point values $A(\lambda,\mu)$

- barotropic vorticity eq.

$$\frac{\partial}{\partial t}(\nabla^2\psi + f) = -J(\psi, \nabla^2\psi + f)$$

where the J is the nonlinear term, $\nabla^2\psi = \zeta$

→ on a sphere, the above eq. can be written

$$\frac{\partial\zeta}{\partial t} = \frac{1}{a^2\cos\theta}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial\lambda}(\zeta+f) - \frac{\partial\psi}{\partial\lambda}\frac{\partial}{\partial\theta}(\zeta+f)\right)$$

$$= \frac{1}{a^2}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial\lambda}(\zeta+f) - \frac{\partial\psi}{\partial\lambda}\frac{\partial}{\partial\theta}(\zeta+f)\right) \quad \because \mu = \sin\theta$$

↳ barotropic nonlinear vorticity eq. on a sphere.

where $\frac{\partial f}{\partial\lambda} = 0$, $\frac{\partial f}{\partial\mu} = 2\Omega$ → there's no latitude dependence

$$\frac{\partial\zeta}{\partial t} = \frac{1}{a^2}\left(\frac{\partial\psi}{\partial\mu}\frac{\partial\zeta}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}\frac{\partial\zeta}{\partial\mu}\right) - \frac{2\Omega}{a^2}\frac{\partial\psi}{\partial\lambda}$$

|||

$F(\lambda,\mu)$: non-linear term

$$\frac{\partial\zeta}{\partial t} = F(\lambda,\mu) - \frac{2\Omega}{a^2}\frac{\partial\psi}{\partial\lambda} \quad (1)$$

→ This eq. can be written as in spectral form ∵ (look note P12)

$$\frac{d\psi_n^m}{dt} = \frac{2\Omega im}{n(n+1)}\psi_n^m - \frac{a^2}{n(n+1)}F_n^m$$

Now, let's consider the non-linear term F_n^m

In words, $\frac{\partial\psi}{\partial\lambda}$, $\frac{\partial\psi}{\partial\mu}$, $\frac{\partial\zeta}{\partial\lambda}$ and $\frac{\partial\zeta}{\partial\mu}$ are calculated on grid points by projecting the spectral coefficients onto the space domain. There are then multiplied to get values of the nonlinear terms $F(\lambda,\mu)$ on the grid points. Then perform the Fourier–Legendre transform of $F(\lambda,\mu)$ to get F_n^m

$$F(\lambda,\mu) = \frac{1}{a^2}\left(\frac{\partial\psi}{\partial\mu}\frac{\partial\zeta}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}\frac{\partial\zeta}{\partial\mu}\right)$$

$$= \frac{1}{a^2(1-\mu^2)}\left((1-\mu^2)\frac{\partial\psi}{\partial\mu}\frac{\partial\zeta}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}(1-\mu^2)\frac{\partial\zeta}{\partial\mu}\right)$$

Let $\psi(\lambda,\mu) = \sum_m\sum_n \psi_n^m(t)Y_n^m(\lambda,\mu)$ so that $\because Y_n^m = P_n^m e^{im\lambda}$

$$(1-\mu^2)\frac{\partial\psi}{\partial\mu} = \sum_m\sum_n \psi_n^m(t)e^{im\lambda}\left[(1-\mu^2)\frac{\partial}{\partial\lambda}P_n^m(\mu)\right]$$

∵ recurrence term

$$(1-\mu^2)\frac{d}{d\mu}P_n^m(\mu) = -n\epsilon_{n+1}^m P_{n+1}^m(\mu) + (n+1)\epsilon_n^m P_{n-1}^m(\mu)$$

$$\epsilon_{n+1}^m = \left(\frac{n^2-m^2}{4n^2-1}\right)^{\frac{1}{2}}$$

$$(1-\mu^2)\frac{\partial\psi}{\partial\mu} = \sum_m\sum_n \psi_n^m[-n\epsilon_{n+1}^m P_{n+1}^m(\mu) + (n+1)\epsilon_n^m P_{n-1}^m(\mu)]e^{im\lambda}$$

↑ inverse transform

$$\frac{\partial\psi}{\partial\lambda} = \sum_m\sum_n im\psi_n^m(t)P_n^m(\mu)e^{im\lambda}$$

Similarly for $(1-\mu^2)\frac{\partial\zeta}{\partial\mu}$ and $\frac{\partial\zeta}{\partial\lambda}$

Therefore, we have $F(\lambda,\mu)$ on the grid points.

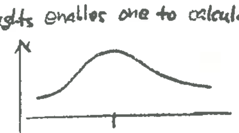
Then perform the Fourier–Legendre transform of F to get F_n^m

- Gaussian quadrature is used for the Legendre transform of data in the north–south direction.

The use of Gaussian latitudes and weights enables one to calculate the Legendre transform exactly.

✗ the eq. → latitude size big

the pole → " " small



Eg.

MET5541 (NWP): T. N. Krishnamurti (?) , 1998

- Using the barotropic non-divergent vorticity equation as a frame of reference:
  - MS Seeking Students: Describe a numerical procedure for its solution.
  - Ph.D. Seeking Students: Describe the spectral transform method for the solution procedure.

* look at note page 12

① derive vorticity eq.
② Write it down on a sphere
③ Arrange it and write it as spectral form
④ Explain the treatment of non-linear term well.
⑤ time differencing.

look at the previous two question !

5541

MET? : T. N. Krishnamurti (?)

- The spectral ~~force~~ form of the nondivergent barotropic model is given by the following equation

$$\frac{d\psi_{m,n}}{dt} = +\frac{2\Omega m}{n(n+1)} i\psi_{m,n} - \frac{a^2}{n(n+1)} F_{m,n}$$

Starting from the above equation, describe the transform method for solving the above equation. Include in your discussion how you handle the time differencing, the linear and the nonlinear terms and the eventual mapping of the solution.

look at page 12 of note

* Someone's answer

Given a map of wind (u,v) on a sphere, produce a one day forecast map of wind over the globe.

$$\frac{d\psi_n^m}{dt} = +\frac{2\Omega i m}{n(n+1)}\psi_n^m - \frac{a^2}{n(n+1)} F_n^m$$

↳ spectral form of the barotropic vorticity eq.

where $F = \frac{1}{a^2}\left(\frac{\partial\psi}{\partial\mu}\frac{\partial\zeta}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}\frac{\partial\zeta}{\partial\mu}\right)$

$$= \frac{1}{a^2(1-\mu^2)}\left[(1-\mu^2)\frac{\partial\psi}{\partial\mu}\frac{\partial\zeta}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}(1-\mu^2)\frac{\partial\zeta}{\partial\mu}\right] \quad \text{——— ①}$$

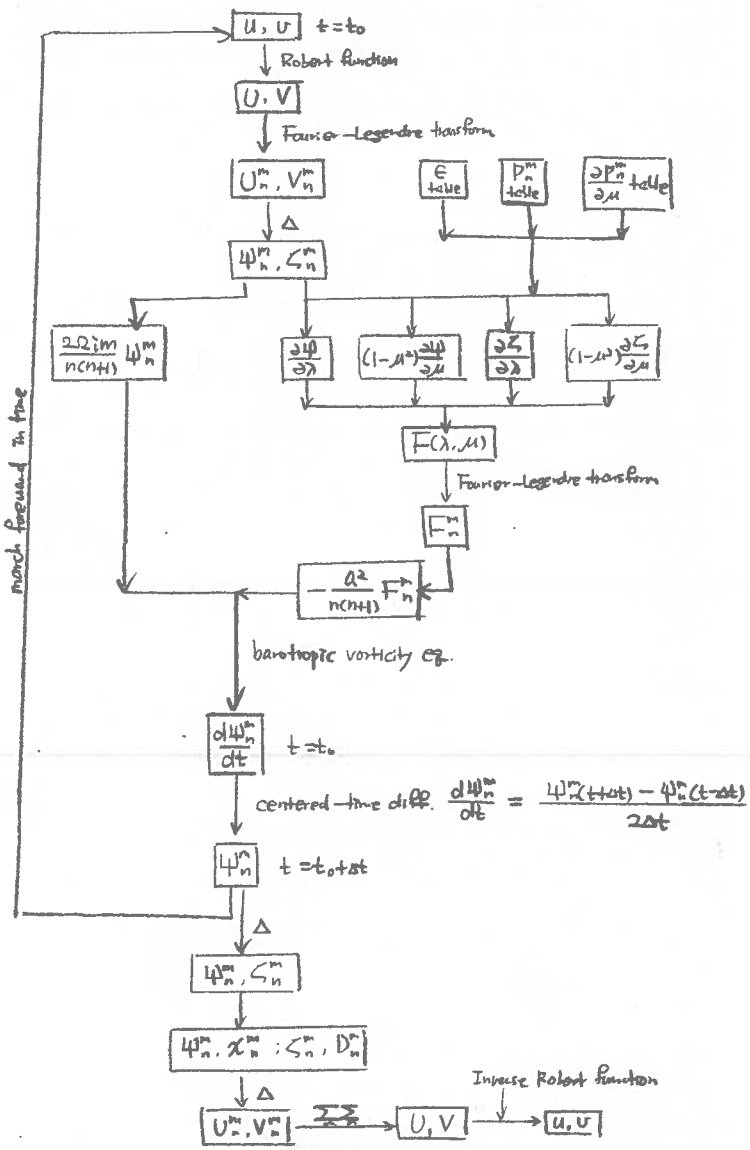
Let $\psi(\lambda,\mu) = \sum_m\sum_n \psi_n^m(t) Y_n^m(\lambda,\mu)$

then $\frac{\partial\psi}{\partial\lambda} = \sum_m\sum_n im\psi_n^m(t) P_n^m(\mu) e^{im\lambda}$

$$(1-\mu^2)\frac{\partial\psi}{\partial\mu} = \sum_m\sum_n \psi_n^m\left[-n\epsilon_{n+1}^m P_{n+1}^m(\mu) + (n+1)\epsilon_n^m P_{n-1}^m(\mu)\right] e^{im\lambda}$$

Similarly for ζ

Flow chart



MET5541(Dynamical Weather Prediction): T. N. Krishnamurti (1 hour, 1995)

• Question

✓ a) Write down the barotropic vorticity equation on a sphere.

✓ b) Using spherical harmonics as a basis function, derive the spectral form of the barotropic vorticity equation. Y_n^m

✓ c) Show the following for the solution procedure for the weather forecast model:

i) Use of the transform method to handle linear and the nonlinear terms.

ii) Provide a flow chart using a simple time differencing scheme for this problem.

↳ easy to answer

look at the note. page12.

MET5541: T. N. Krishnamurti (1 hour) *

• Derive the spectral form of the barotropic nondivergent vorticity equation. Discuss in detail how the transform method is used to handle a linear and the nonlinear terms. Finally show how you would make a one time step forecast starting from the wind components *u*, *v* over the globe at 500 mb.

refer to the note p12

MET5541: T. N. Krishnamurti (?) *

• Show clearly how the spectral transform method works in the formulation of the barotropic vorticity equation. Start from the vorticity equation and show the legendre-fourier transform of the spectral equation and clearly portray the one time step solution procedure for each of the terms. Use any standard time differencing scheme as your frame of reference.

• look at p12 of my note

MET5541: T. N. Krishnamurti (1 hour) *

• The spectral form of the semi-implicit shallow water equation are given by:

$$-n(n+1)\frac{\partial \psi_{m,n}}{\partial t} = \alpha_{m,n}(A,B) \tag{1}$$

$$-n(n+1)\frac{\partial \chi_{m,n}}{\partial t} = \alpha_{m,n}(B,-A) - \left[\nabla^2\left(\frac{U^2+V^2}{2\cos^2\theta} + \phi'\right)\right]_{m,n} \tag{2}$$

$$\frac{\partial \phi'_{m,n}}{\partial t} = \alpha_{m,n}(C,D) - \bar{\phi}F_{m,n} \tag{3}$$

where $A = U(\nabla^2\psi + f)$, $B = V(\nabla^2\psi + f)$

$C = U\phi'$, $D = V\phi'$, $C = U\phi'$, $F = \nabla \bullet \mathbf{V}$, $\mathbf{V} = U\mathbf{i} + V\mathbf{j}$

and the operator α is defined as follows:

$$\alpha = \frac{1}{\cos^2\theta}\left[\frac{\partial(\ )}{\partial \lambda} + \frac{\cos\theta\partial(\ )}{\partial \theta}\right] \tag{4}$$

Explain the following:

a) Explain clearly why we use the semi-implicit algorithm.

b) Starting from u, v and z at a single level as a function of latitude and longitude, draw a detailed flow chart, the spectral method for the solution of this problem.

c) One always encounters a Helmholtz equation in this problem; elaborate where and show how it is solved spectrally.

a) In order to increase time-step Δt,

linear term → implicitly
nonlinear term → explicitly

In shallow-water model, the linear terms are integrated implicitly, while the non-linear terms are integrated explicitly

vorticity eq. → explicitly
divergence and continuity eq → implicitly

b) look at my note p15 → flow chart.

c) " "

MET5541: T. N. Krishnamurti (1 hour) *

• Describe the semi implicit spectral shallow water model. Specifically outline the following.

(a) Spectral closed system.
(b) Treatment of the Helmholtz equation for the free surface height.
(c) Treatment of nonlinearity
(d) Treatment of the semi-implicit time differencing.

Too many details are not necessary, convince me that you know in principle how one can put these ideas together spectrally.

look at my note P14, p15 ?

MET? (NWP): T. N. Krishnamurti (?)

- Describe the closed system of equations (motion, hydrostatics, mass continuity and thermodynamics) on the x-y-σ plane where $\sigma = p/p_s$. Start from the x, y, p system on the Beta plane, p_s is the station level pressure.

look at p16, p17

The eqs for a global model in the p-coordinate (x, y, p)

- the horizontal eq.

$$\frac{d\mathbf{V}}{dt} = -f\mathbf{k}\times\mathbf{V} - \nabla\phi + (F) \quad \text{(friction)} \qquad (1)$$

- the hydrostatic eq.

$$\frac{\partial\phi}{\partial p} = -\alpha \qquad (2)$$

- the thermodynamic eq.

$$C_p\frac{dT}{dt} - \alpha\frac{dp}{dt} = \dot{Q} \rightarrow \frac{\partial T}{\partial t} = -\mathbf{V}\cdot\nabla T - \omega\left(\frac{\partial T}{\partial p} - \frac{RT}{C_p p}\right) + \frac{\dot{Q}}{C_p} \qquad (3)$$

- mass continuity eq.

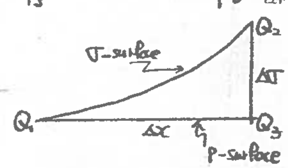
$$\nabla\cdot\mathbf{V} + \frac{\partial\omega}{\partial p} = 0 \qquad (4)$$

- eq. of state

$$p = \rho RT \qquad (5)$$

$\Rightarrow$ 6 eqs + 6 unknows $(u, v, \omega, \rho, T, z)$

$$\sigma = \frac{p}{p_s} \longrightarrow \sigma = \begin{cases} 1 & \text{at the earth's surface} \\ 0 & \text{at the TOA} \end{cases}$$



$$\frac{Q_2 - Q_1}{\Delta x} = \frac{Q_3 - Q_1}{\Delta x} + \frac{Q_3 - Q_2}{\Delta\sigma}\frac{\Delta\sigma}{\Delta x}$$

If $\Delta x \rightarrow 0$, $\Delta\sigma \rightarrow 0$

$$\left.\frac{\partial Q}{\partial x}\right|_p = \left.\frac{\partial Q}{\partial x}\right|_\sigma + \frac{\partial Q}{\partial\sigma}\left.\frac{\partial\sigma}{\partial x}\right|_p \qquad (a)$$

Similarly

$$\left.\frac{\partial Q}{\partial y}\right|_p = \left.\frac{\partial Q}{\partial y}\right|_\sigma + \frac{\partial Q}{\partial\sigma}\left.\frac{\partial\sigma}{\partial y}\right|_p \qquad (b)$$

$$\left.\frac{\partial Q}{\partial t}\right|_p = \left.\frac{\partial Q}{\partial t}\right|_\sigma + \frac{\partial Q}{\partial\sigma}\left.\frac{\partial\sigma}{\partial t}\right|_p \qquad (c)$$

$$\frac{\partial Q}{\partial p} = \frac{\partial Q}{\partial\sigma}\frac{\partial\sigma}{\partial p} \qquad (d)$$

First, let's consider substantial time derivative (d/dt) in σ-coord.

$$\frac{dQ}{dt} = \left.\frac{\partial Q}{\partial t}\right|_p + u\left.\frac{\partial Q}{\partial x}\right|_p + v\left.\frac{\partial Q}{\partial y}\right|_p + \omega\frac{\partial Q}{\partial p} \quad \text{in } (x, y, p)$$

using (a) ~ (d)

$$\frac{dQ}{dt} = \left.\frac{\partial Q}{\partial t}\right|_\sigma + u\left.\frac{\partial Q}{\partial x}\right|_\sigma + v\left.\frac{\partial Q}{\partial y}\right|_\sigma + \left(\left.\frac{\partial\sigma}{\partial t}\right|_p + u\left.\frac{\partial\sigma}{\partial x}\right|_p + v\left.\frac{\partial\sigma}{\partial y}\right|_p + \omega\frac{\partial\sigma}{\partial p}\right)\frac{\partial Q}{\partial\sigma}$$

$$= \left.\frac{\partial Q}{\partial t}\right|_\sigma + u\left.\frac{\partial Q}{\partial x}\right|_\sigma + v\left.\frac{\partial Q}{\partial y}\right|_\sigma + \frac{\partial Q}{\partial\sigma}\frac{d\sigma}{dt}$$

$$\therefore \frac{dQ}{dt} = \left.\frac{\partial Q}{\partial t}\right|_\sigma + u\left.\frac{\partial Q}{\partial x}\right|_\sigma + v\left.\frac{\partial Q}{\partial y}\right|_\sigma + \dot{\sigma}\frac{\partial Q}{\partial\sigma} \qquad (e)$$

where $\dot{\sigma} = \frac{d\sigma}{dt}$

Next, pressure gradient in σ-coord.

$$-g\left.\frac{\partial z}{\partial x}\right|_p = -g\left.\frac{\partial z}{\partial x}\right|_\sigma - g\left.\frac{\partial\sigma}{\partial x}\right|_p\frac{\partial z}{\partial\sigma}$$

$\because$ hydrostatic eq. $g\frac{\partial z}{\partial\sigma} = g\frac{\partial z}{\partial p}\frac{\partial p}{\partial\sigma} = -\frac{RT}{p}\frac{\partial p}{\partial\sigma}$

Let $Q = p$ in eq (a) and noting $\left.\frac{\partial p}{\partial x}\right|_p = 0$

$$0 = \left.\frac{\partial p}{\partial x}\right|_\sigma + \frac{\partial p}{\partial\sigma}\left.\frac{\partial\sigma}{\partial x}\right|_p$$

$$\therefore -g\left.\frac{\partial z}{\partial x}\right|_p = -g\left.\frac{\partial z}{\partial x}\right|_\sigma - \frac{RT}{p}\left.\frac{\partial p}{\partial x}\right|_\sigma$$

Similarly, $-g\left.\frac{\partial z}{\partial y}\right|_p = -g\left.\frac{\partial z}{\partial y}\right|_\sigma - \frac{RT}{p}\left.\frac{\partial p}{\partial y}\right|_\sigma \qquad \because \phi = gz$

In vector form $\Rightarrow$

$$-\nabla_p\phi = -\nabla_\sigma\phi - \frac{RT}{p}\nabla_\sigma p \qquad (f)$$

The Hydrostatic eq in σ-coord.

$$\frac{\partial\phi}{\partial p} = -\alpha = -\frac{1}{\rho} = -\frac{RT}{p}$$

$$\frac{\partial\phi}{\partial\sigma}\frac{\partial\sigma}{\partial p} = -\frac{RT}{p} \qquad \because \left(\frac{\partial\sigma}{\partial p} = \frac{\partial}{\partial p}\left(\frac{p}{p_s}\right) = \frac{1}{p_s}\right)$$

$$\frac{p}{p_s}\frac{\partial\phi}{\partial\sigma} = -RT$$

$$\therefore \sigma\frac{\partial\phi}{\partial\sigma} = -RT \qquad (g) \text{ — (II)}$$

mass continuity eq.

$$\left.\frac{\partial u}{\partial x}\right|_p + \left.\frac{\partial v}{\partial y}\right|_p + \frac{\partial\omega}{\partial p} = 0$$

$\because$ (a) $\rightarrow \left.\frac{\partial u}{\partial x}\right|_p = \left.\frac{\partial u}{\partial x}\right|_\sigma + \frac{\partial u}{\partial\sigma}\left.\frac{\partial\sigma}{\partial x}\right|_p$

(b) $\rightarrow \left.\frac{\partial v}{\partial y}\right|_p = \left.\frac{\partial v}{\partial y}\right|_\sigma + \frac{\partial v}{\partial\sigma}\left.\frac{\partial\sigma}{\partial y}\right|_p$

$$\frac{\partial\omega}{\partial p} = \frac{\partial}{\partial p}\left(\frac{dp}{dt}\right) = \frac{\partial}{\partial\sigma}\left(\frac{dp}{dt}\right)\frac{\partial\sigma}{\partial p} \quad \leftarrow (e)$$

$$= \frac{\partial}{\partial\sigma}\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \dot{\sigma}\frac{\partial p}{\partial\sigma}\right)\frac{\partial\sigma}{\partial p}$$

$$= \left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \dot{\sigma}\frac{\partial}{\partial\sigma}\right)\frac{\partial p}{\partial\sigma} + \frac{\partial u}{\partial\sigma}\frac{\partial p}{\partial x} + \frac{\partial v}{\partial\sigma}\frac{\partial p}{\partial y} + \frac{\partial\dot{\sigma}}{\partial\sigma}\frac{\partial p}{\partial\sigma}\right]\frac{\partial\sigma}{\partial p}$$

$$= \frac{\partial\sigma}{\partial p}\frac{d}{dt}\left(\frac{\partial p}{\partial\sigma}\right) + \left(\frac{\partial u}{\partial\sigma}\left.\frac{\partial p}{\partial x}\right|_\sigma + \frac{\partial v}{\partial\sigma}\left.\frac{\partial p}{\partial y}\right|_\sigma + \frac{\partial\dot{\sigma}}{\partial\sigma}\frac{\partial p}{\partial\sigma}\right)\frac{\partial\sigma}{\partial p}$$

$\because$ Letting $Q = p$ in (1) + (2)

$$0 = \left.\frac{\partial p}{\partial x}\right|_p = \left.\frac{\partial p}{\partial x}\right|_\sigma + \frac{\partial p}{\partial\sigma}\left.\frac{\partial\sigma}{\partial x}\right|_p \longrightarrow \left.\frac{\partial p}{\partial x}\right|_\sigma = -\frac{\partial p}{\partial\sigma}\left.\frac{\partial\sigma}{\partial x}\right|_p$$

$$0 = \left.\frac{\partial p}{\partial y}\right|_p = \left.\frac{\partial p}{\partial y}\right|_\sigma + \frac{\partial p}{\partial\sigma}\left.\frac{\partial\sigma}{\partial y}\right|_p \longrightarrow \left.\frac{\partial p}{\partial y}\right|_\sigma = -\frac{\partial p}{\partial\sigma}\left.\frac{\partial\sigma}{\partial y}\right|_p$$

therefore

$$\frac{\partial\omega}{\partial p} = \frac{\partial\sigma}{\partial p}\frac{d}{dt}\left(\frac{\partial p}{\partial\sigma}\right) - \frac{\partial p}{\partial\sigma}\left(\left.\frac{\partial\sigma}{\partial x}\right|_p\frac{\partial u}{\partial\sigma} + \left.\frac{\partial\sigma}{\partial y}\right|_p\frac{\partial v}{\partial\sigma} - \frac{\partial\dot{\sigma}}{\partial\sigma}\right)\frac{\partial\sigma}{\partial p}$$

$$\left.\frac{\partial u}{\partial x}\right|_\sigma + \frac{\partial u}{\partial\sigma}\left.\frac{\partial\sigma}{\partial x}\right|_p + \left.\frac{\partial v}{\partial y}\right|_\sigma + \frac{\partial v}{\partial\sigma}\left.\frac{\partial\sigma}{\partial y}\right|_p + \frac{\partial\sigma}{\partial p}\frac{d}{dt}\left(\frac{\partial p}{\partial\sigma}\right) - \frac{\partial p}{\partial\sigma}\left(\left.\frac{\partial\sigma}{\partial x}\right|_p\frac{\partial u}{\partial\sigma} + \left.\frac{\partial\sigma}{\partial y}\right|_p\frac{\partial v}{\partial\sigma} - \frac{\partial\dot{\sigma}}{\partial\sigma}\right)\frac{\partial\sigma}{\partial p} = 0$$

or

$$\left.\frac{\partial u}{\partial x}\right|_\sigma + \left.\frac{\partial v}{\partial y}\right|_\sigma + \frac{\partial\dot{\sigma}}{\partial\sigma} + \frac{\partial\sigma}{\partial p}\frac{d}{dt}\left(\frac{\partial p}{\partial\sigma}\right) = 0$$

$$\because \left(\sigma = \frac{p}{p_s}, \ \frac{\partial p}{\partial\sigma} = p_s, \ \frac{\partial\sigma}{\partial p} = \frac{1}{p_s}\right)$$

$$\frac{1}{p_s}\frac{dp_s}{dt} + \left.\frac{\partial u}{\partial x}\right|_\sigma + \left.\frac{\partial v}{\partial y}\right|_\sigma + \frac{\partial\dot{\sigma}}{\partial\sigma} = 0$$

$$\therefore \frac{d\ln p_s}{dt} + \left.\frac{\partial u}{\partial x}\right|_\sigma + \left.\frac{\partial v}{\partial y}\right|_\sigma + \frac{\partial\dot{\sigma}}{\partial\sigma} = 0 \qquad (h) \text{ — (III)}$$

From (f),

$$-\nabla_p\phi = -\nabla_\sigma\phi - \frac{RT}{p}\nabla_\sigma p = -\nabla_\sigma\phi - RT\nabla_\sigma\ln p$$

$$\because (p = \sigma p_s)$$

$$-\nabla_p\phi = -\nabla_\sigma\phi - RT\nabla_\sigma\ln p_s$$

Thus, the momentum eq. ① ⟶

$$\frac{d\mathbf{V}}{dt} = -f\mathbf{k}\times\mathbf{V} - \nabla\phi + \mathbf{F} \quad \text{(using ⓔ)}$$

$$\frac{\partial\mathbf{V}}{\partial t} = -(\mathbf{V}\cdot\nabla)\mathbf{V} - \dot{\sigma}\frac{\partial\mathbf{V}}{\partial\sigma} - f\mathbf{k}\times\mathbf{V} - (\nabla\phi + RT\nabla\ln p_s) + \mathbf{F}$$

$$\because \left((\mathbf{V}\cdot\nabla)\mathbf{V} = \nabla\left(\frac{\mathbf{V}\cdot\mathbf{V}}{2}\right) + \zeta\mathbf{k}\times\mathbf{V}\right)$$

$$\therefore \frac{\partial\mathbf{V}}{\partial t} = -(\zeta+f)\mathbf{k}\times\mathbf{V} - \dot{\sigma}\frac{\partial\mathbf{V}}{\partial\sigma} - \nabla\left(\phi + \frac{\mathbf{V}\cdot\mathbf{V}}{2}\right) - RT\nabla\ln p_s + \mathbf{F} \quad \text{— (I)}$$

The thermodynamic eq

$$C_p\frac{dT}{dt} - \alpha\frac{dp}{dt} = \dot{Q}$$

$$\because \frac{dp}{dt} = \frac{d}{dt}(\sigma p_s) = \dot{\sigma}p_s + \sigma\dot{p}_s$$

$$\frac{\partial T}{\partial t} = -\mathbf{V}\cdot\nabla T - \dot{\sigma}\frac{\partial T}{\partial\sigma} + \frac{RT}{C_p p}(\dot{\sigma}p_s + \sigma\dot{p}_s) + H_T \quad \left(H_T \sim \frac{\dot{Q}}{C_p}\right)$$

$$= -\mathbf{V}\cdot\nabla T - \dot{\sigma}\left(\frac{\partial T}{\partial\sigma} - \frac{RT}{C_p\sigma}\right) + \frac{RT}{C_p p_s}\frac{dp_s}{dt} + H_T$$

$$= -\mathbf{V}\cdot\nabla T - \dot{\sigma}\left(\frac{\partial T}{\partial\sigma} - \frac{RT}{C_p\sigma}\right) + \frac{RT}{C_p}\frac{d\ln p_s}{dt} + H_T$$

(← continuity eq.)

$$\therefore \frac{\partial T}{\partial t} = -\mathbf{V}\cdot\nabla T + \dot{\sigma}\gamma - \frac{RT}{C_p}\left(\nabla\cdot\mathbf{V} + \frac{\partial\dot{\sigma}}{\partial\sigma}\right) + H_T \quad \text{— (IV)}$$

where $\gamma = \frac{RT}{C_p\sigma} - \frac{\partial T}{\partial\sigma}$

There's no change in eq of state ⟶ $p = \rho RT$ — (V)

∴ (I), (II), (III), (IV) and (V) represent a closed system of eqs in (x, y, σ) coordinates.

5541

MET?: T. N. Krishnamurti (?)

• Describe how you would model large scale condensation (i.e. nonconvective precipitation process) for NWP. Your answer should include:

a) expressions for heating, moistening and rain rates,
b) conditions under which this is invoked, and
c) a discussion as to why such a process leads to the cooling of the environment.

look at my note p19 + p20 + one question

MET5541: T. N. Krishnamurti (1 hour)

• Derive a simple quasigeostrophic isentropic coordinate adiabatic forecast model. What are the dependent variables, and the closed system of equations. Describe the procedure for solution of time marching and boundary value issues.

MET?: T. N. Krishnamurti (?)

• You have no doubt attended a recent departmental seminar; be scientific in answering the following:

a) Write down the approximate title of this seminar and the speaker's name
b) Present a statement of the problem posed by the speaker.
c) A summary of the talk.
d) Present a critique of this seminar (pros and cons as you saw it).
e) Conclusions of this talk.

MET?: T. N. Krishnamurti (40 minutes)

• As a graduate student you were required to attend departmental seminars. For one of the seminars you attended (not your own seminar) discuss the following:

a) Approximate title of seminar.
b) Approach to the problem.
c) Main results
d) Limitation of the work.

Ensemble forecast of a typhoon flood event.

Brian mackey

ABSTRACT

A high resolution nested regional spectral model and an ensemble perturbation system are combined to forecast the track, intensity, and precipitation arising from Super Typhoon Winnie of August, 1997. In just 24 to 36 hours, northern Taiwan and portions of eastern China were inundated with heavy rainfall amounts greater than 200 mm, which led to landslides and major flooding of crops. The prediction of floods is challenging since rainfall distributions can have a high degree of spatial and temporal variability. Thus, an ensemble of forecasts is generated to evaluate this event. Regional model resolutions are set at 0.5° and 0.25°. The results indicate very accurate track and intensity forecasts. For both the control and ensemble mean forecasts, the track position errors remain below 150 km through 72 hours while intensity errors are approximately 5 m s^{-1} at the time of landfall. Qualitatively, the overall 5-day precipitation patterns look realistic and compare favorably with the observed data. Upon examining individual stations with high accumulated rainfall values, most forecasts underestimate those station totals by 50 to 100 mm. A transform technique is then applied to each ensemble member's track so that the precipitation from each member accumulates along the ensemble mean track. A new relative ensemble mean is then created. Using a 0.25° resolution, both the control forecast and the relative ensemble mean, in general, provide the best rainfall prediction. This is especially the case for the areas near and to the north of where Winnie makes landfall in eastern China. Over these locations errors are generally between 10 and 60 mm.

1 INTRODUCTION

1.1 Overview

Tropical cyclones and floods are among the world's most serious threats to life and property. According to a World Meteorological Organization (WMO) study (De and Joshi, 1998), tropical cyclones rank second among all natural disasters in loss of life behind drought. As a result of landfalling tropical cyclones, nearly 900,000 deaths occurred around the world from 1967–1991. Each year about 80 tropical cyclones form in the tropical oceans with the northern and eastern hemispheres being preferred regions. Typhoons in the western North Pacific Ocean basin annually cause more than $4 billion in total losses based on estimates by the Asian and Pacific Typhoon Committee of the WMO (Chen, 1995).

The dangers of tropical cyclones include storm surges, strong winds, and flooding due to heavy rainfall. This last aspect can pose grave risks if certain conditions are met. The process begins while the tropical cyclone is still over the ocean. Here, the hydrological cycle is intensified due to the cyclone's massive energy exchanges. Because of warm sea surface temperatures, sustained high winds, and sea spray associated with large swells, the evaporation and transport of water vapor is significantly enhanced (Raj, 1998). Once the storm approaches land, several factors can cause severe flooding. First, a persistent supply of moist air may be drawn into a cyclone's circulation. This can be depicted on satellite imagery as cumulonimbus cloud lines usually on the equatorward side of a storm. Water vapor transport is maximized along these inflow channels, and intense rains may soon develop (Chen, 1995).

Then, as the tropical cyclone moves ashore, heavy rains may occur in four basic regions. One is within the core of the cyclone, which extends 15 to 50 km from the center of circulation. Within this area powerful upward vertical velocities may lead to torrential precipitation. If the ground has already become nearly saturated before this core reaches shore, then surface runoff would only be aggravated. In addition, persistent rainfall can result from spiral rainbands, which are usually found in the right semicircle and are 5–50 km wide and 100–300 km long. These spiral bands are often maintained via convergence. For instance, in the western north Pacific, an easterly low level jet may combine with monsoon southwesterlies and an equatorward wind component on the typhoon's west side. Even after landfall when the circulation is much weaker, spiral bands may still be sustained. Furthermore, heavy rains can also be observed in a third location north of landfalling tropical cyclones. For example, in China it is not uncommon for inverted troughs to form to the north of a cyclone within an area of low level horizontal wind shear and positive vorticity advection. The most intense rains are often found where the southeasterly low level jet intersects the inverted trough. Finally, it is also possible for heavy rains to evolve as much as 1000 km away from a tropical cyclone's center. For instance, over China a portion of a cyclone's large supply of water vapor may be transported into the vicinity of an approaching westerly trough via a strong southeasterly low level jet. Hence, the rainfall can be amplified along the approaching front in an area of strong moisture convergence (Chen, 1995).

Once a tropical cyclone's circulation is completely inland, it will begin to weaken, and the strong winds and deadly storm surge will no longer be a threat. Nevertheless, high winds may have already defoliated vegetation, which can seriously curtail rainfall interception and lead to increased runoff. High winds can also oppose river flows and, together with high tides, can decrease discharge at a river's mouth (Raj, 1998). Even as the winds finally begin to subside, heavy rainfall may become a serious issue, especially if the vortex is sustained through strong upper level divergence and low level convergence. Flooding is likely if the circulation can be maintained for about three days after landfall, but floods may still occur over time periods of 24 to 36 hours. A more critical situation arises if the cyclone vortex becomes nearly stationary. In this case, the cyclone environment is usually characterized by weak or offsetting steering currents. More specifically, the underlying causes may include (1) high pressure regions surrounding the cyclone; (2) tropical cyclone interaction with another low pressure system leading to looping or stagnation; (3) a strong, blocking high pressure region to the northwest of the cyclone; and (4) different layers of the atmosphere possessing opposite steering currents (Chen, 1995).

Moreover, coastal boundaries and mountainous terrain provide another important means for producing torrential rainfall. Increased convergence along the coastline to the right of a storm (in the northern hemisphere) can occur because of blocking and frictional effects. A land mass will often act as a block to the onshore air flow while also serving to increase frictional convergence as the winds move from the sea to the land. The convergence and consequent heavy rains are only exacerbated by steep topographic regions along the coast. In addition, mountains also may help cause tremendous floods as air currents are quickly forced upward. This is especially true if the ranges are oriented perpendicular to the low level flow. Heavy rain bands may then form over the same locations. Precipitation amounts may also be magnified by unique terrain features, such as V-shaped mountain ranges and long, narrow river valleys. Rainfall accumulations may be especially significant on valley floors (Chen, 1995).

The consequences of a flood event are often far-reaching. On steep hills and mountains, especially those which contain fragile soil or volcanic ash, landslides can easily occur. Dwellings, bridges and roads can be uprooted or destroyed and rivers diverted due to the excess debris. On the flood plain, the ponding of stagnant, dirty water can pose serious health risks. In addition, it is probable that major flooding will kill livestock and devastate crops and vegetation. The effects can often be long-lasting (Raj, 1998).

1.2 Objectives and Organization of Thesis

This study focuses on one particular typhoon flood event — Super Typhoon Winnie of August, 1997, which seriously affected portions of Taiwan and eastern China. Even though the typhoon did not become stationary, very heavy rains occurred near the core and spiral bands. In addition, the blocking and frictional effects of the Chinese land mass likely served to increase the convergence and heavy precipitation to the north of the typhoon's center. Moreover, steep hills and heavy rainfall in Taipei combined to induce landslides.

In order to investigate this event, the Florida State University Nested Regional Spectral Model (FSUNRSM) is employed, along with an empirical orthogonal function (EOF) perturbation method, to generate ensemble forecasts of the track, intensity, and precipitation. The regional model used here is based upon the Florida State University Global Spectral Model (FSUGSM). Both employ the same physics and both use the sigma (σ) coordinate in the vertical, where there are

14 levels. The horizontal resolution is set to approximately 0.9375° (T-126) for the global model and either 0.5° or 0.25° for the regional model. Further details about the FSUNRSM are available in Appendix B.

A successful prediction of a typhoon's heavy rain areas is predicated upon an accurate forecast of the typhoon's path, strength, and structure. Furthermore, it is crucial to correctly parameterize the cumulus convection and related heating. In addition, air-sea interactions and moisture, temperature, and wind fields surrounding the typhoon must be accurately represented as well as boundary parameters, such as sea surface temperatures and topography. Misconstructions in the physical parameterizations can seriously degrade a typhoon forecast.

By using a high resolution spectral model and an ensemble prediction system, the main goals of this thesis are to

- research different ensemble forecast strategies and explain why an empirical orthogonal function (EOF)-based perturbation technique is suitable for tropical weather prediction;

- perform a detailed case study of Super Typhoon Winnie and compare the observed track and intensities with those from the control and ensemble members;

- demonstrate the useful products of ensemble forecasting, which include multi-panel charts, probability distribution maps, cluster mean charts, and station time series;

- focus on the rainfall and subsequent flooding due to Winnie by comparing station and satellite-derived precipitation data to the model ensemble;

- investigate what effect, if any, initialization procedures have on perturbation growth and related ensemble spread;

- examine what effect, if any, a higher resolution regional forecast would have on the precipitation patterns and amounts.

In Chapter 2, the FSUGSM is explained. The dynamical and physical aspects of the model are detailed, as well as the two main initialization procedures. Ensemble forecast strategies is the topic of Chapter 3. Two techniques currently in use at operational centers — the singular vector and breeding methods — are described. Then, the EOF-based perturbation method used in this thesis is presented along with a brief discussion about ensemble forecasts of floods. In Chapter 4, the experiment methodology is set forth in addition to observational aspects of Super Typhoon Winnie and related flooding issues. In this study, both a lower resolution global model and a high resolution regional model are utilized. Sensitivity tests are then performed to determine the impacts of a very high resolution regional model forecast. The consequences of performing a physical initialization either before or after generating perturbations is also explored. In Chapter 5, the computational results of the ensemble forecasts are presented, and finally, Chapter 6 contains the conclusions and future work.

SUMMARY AND CONCLUSIONS

1.3 Discussion and Summary

Through the use of a high resolution regional spectral model and an EOF-based perturbation technique, this study aims to assess the effectiveness of an ensemble forecast of a typhoon flood event. In Chapter 1, an overview is given of how tropical-cyclone induced floods can occur. In Typhoon Winnie's case, these floods are likely a result of heavy rains associated with the typhoon's core and spiral bands. In addition, the land mass of China may have contributed to the flooding via frictional convergence and blocking effects. The steep topographic features of northern Taiwan definitely helped in forcing heavy rains and subsequent landslides there.

The FSUGSM is described in Chapter 2 and the physical initialization technique is examined in detail. This 24-hour pre-integration procedure helps synthesize the rainfall and moisture fields with the other atmospheric variables, thus creating a physically realistic initial state and hopefully reducing the spin-up time of the model.

Ensemble forecasting and the EOF-based perturbation method are both detailed in Chapter 3. It is shown that this ensemble prediction system is suitable for tropical forecasting since it uses a nonlinear model with a complete physics package in order to obtain a realistic perturbation field. The primary eigenvector represents the optimal perturbation as it is projected onto fast-growing, dynamically unstable modes. This method is somewhat similar to the breeding method employed at NCEP since the initial perturbations are selected by the model itself (Zhang, 1997). The singular vector method used at the ECMWF is also a powerful ensemble technique in which fast-growing modes are chosen as the eigenvectors of the product of an operator by its adjoint. The most dynamically unstable regions – those that are most sensitive to different initial conditions – are efficiently sampled.

The ensemble experiments begin approximately 48 hours prior to landfall. In one experiment, the physical initialization technique is performed after the perturbations have been added. For this case, the initial wind components have been 're-laxed' and the perturbation growth rate suppressed. Thus, the ensemble forecast is rendered ineffective due to the fact that very few possible forecast directions have been sampled. In the second experiment, the perturbation generation is performed after the initialization, and the spread is much more robust, but not excessive. For this case study, the track and intensity of Typhoon Winnie is well-predicted by both the control and ensemble mean. This case is similar to one performed by Zhang (1997) during his simulation of 5 different

Atlantic-basin hurricanes. In that case, both the control and ensemble mean predicted the track of Hurricane Hugo equally well. Since the control (unperturbed) forecast is derived from the best guess initial conditions, one might expect this member to be among the best. Indeed, the control forecast is often given extra weight within an ensemble system. The ensemble mean track, however, should be superior if the forecast uncertainty is properly sampled. In general, with this EOF method, the overall track forecast skill over the control has been improved in several cases, including those performed by Zhang (1997). This degree of success is not guaranteed, however, for each individual case study. Many more cases need to be performed involving a variety of different synoptic situations.

In addition to the track and intensity charts, an assortment of useful products can be created from an ensemble forecast. Among these are 'stamp' maps, cluster mean charts, probability distribution maps, and station time series. The focus of the thesis is on the precipitation products from the ensemble members. As mentioned, precipitation is a very difficult variable to forecast even when other variables are well-predicted. For the 0.5° simulation, the control forecast produces realistic precipitation patterns, including mesoscale bands of enhanced precipitation. The ensemble mean is a little smoother, but the rainfall amounts are still adequate. A relative ensemble mean is defined by transforming each ensemble member's track and associated precipitation to the ensemble mean track. Then a new ensemble mean is taken. With this method, the goal is to reduce the amount of smearing, which can often occur when the ensemble members are far apart from the mean track. For the 0.5° run, however, the difference between the two means is not too noticeable. By examining station data, one can further test the ensemble forecast. In general, at the nine station locations, most of the forecasts did not produce sufficient precipitation. The best skill occurred near and to the north of where Typhoon Winnie made landfall. Furthermore, in most cases, the relative ensemble mean improved the final precipitation amount.

An intermediate 0.5° experiment is run to test the sensitivity of perturbation magnitudes, which are reduced by half in this test. The corresponding track spread at the time of landfall is also cut in half. It appears, therefore, that there is some sensitivity to tropical cyclone perturbation magnitudes, but further experiments will need to be performed to verify this.

A 0.25° resolution run is the final experiment. The track forecasts exhibit more spread, but the ensemble mean and control continue to show a high degree of skill. For the precipitation output, even finer detail is evident. The control forecast from this experiment is the most skillful, and the need for a relative ensemble mean in this situation is apparent, particularly after day 3. Overall, the highest rainfall prediction skill is near and just north of where Winnie makes landfall in China. Some noticeable improvements are made from the

0.5° run to the 0.25° simulation, but it may not be enough to justify tripling the computing time.

Numerical weather prediction of rainfall, especially rare event flooding cases, is very challenging, even when a storm's track and intensity are correctly modeled. A discontinuous, derived quantity like precipitation often possesses much variability in space and time. Thus, it is best to examine each individual ensemble member carefully in order to determine the maximum amount possible at any one location.

1.4 Future Work

This study evaluates an ensemble forecast of floods arising from a tropical cyclone landfall. Since this is still a relatively new area of study, further research is needed. This includes investigating modeling issues such as:

- the effectiveness of the EOF-based ensemble prediction system. The limited number of case studies performed thus far limits the skill assessment. Many additional simulations are needed involving weak, moderate, and strong tropical cyclones embedded within a variety of synoptic flows.

- the number of ensemble members. In this study, each model simulation includes seven ensemble members. The statistics generated from this sample are presented, but little validation of these statistics is given. For rare events, it is best to have a large number of members so that all possibilities may be realized.

- model resolution and computing time. Certainly, a high resolution model must be run in order to try to predict mesoscale precipitation distributions and spiral rainbands. Additional vertical resolution should also be explored. The most effective resolution, however, is one which gives the most value while using as few resources as possible. It is difficult to determine from this case study whether the 0.25° simulation is more effective than the 0.5° experiment.

- weighting the control (unperturbed) forecast. Not all forecasts have an equal probability of occurring. If the fastest growing modes dominate a forecast, then this forecast will be less likely to occur when compared to one that is normally evolving. Thus, after performing many simulations, some proper weight may be estimated for the control forecast.

In addition to the above considerations, further hydrological work is necessary. This includes, but is not limited to

- generating a flood potential map. For a particular region, this map would include information about the climatology, terrain, soil moisture, and of course, the precipitation forecast. Ideally, weighting factors would be used to redistribute rainfall for a more meaningful forecast.

- estimating (via Thiessen polygons, for example) how much precipitation has fallen over a given land area. This can be implemented for both station and model data. By performing statistical analysis on this spatial rainfall distribution, the forecast skill can be improved over that from a point precipitation forecast.