

On the theory of CISK

By ROGER K. SMITH*
University of Munich, Germany

(Received 16 November 1995; revised 31 May 1996)

SUMMARY

Theories for the cooperative interaction between cumulus convection and a developing cyclonic vortex proposed independently by Ooyama and by Charney and Eliassen in the early 1960s are re-examined in the light of recent studies and recent criticisms. It is suggested that the criticisms have sidestepped the concept of cooperative intensification as envisaged by Ooyama. However, a later, but much neglected, contention by Ooyama is supported: this is that the linear theories worked out by Charney and Eliassen and others are, in themselves, physically ill-founded. The relationship between Ooyama's cooperative intensification theory and the recent theory of Emanuel based on Wind-Induced Surface Heat Exchange (WISHE) is also discussed. The main difference between these two theories appears to be the degree of emphasis placed by the respective authors on the relative importance of convection and surface fluxes in the intensification of hurricanes.

KEYWORDS: Convection Cyclone intensification Tropical meteorology

1. INTRODUCTION

In a highly influential paper, Charney and Eliassen (1964, henceforth CE64) proposed a theory for the cooperative interaction between a field of deep cumulus clouds and an incipient cyclonic vortex. The underlying idea is that the gross buoyancy-force associated with a localized field of cumulus clouds near the centre of a vortical flow could act as a driving force and cause the vortex to intensify. CE64 envisaged a mechanism of interaction in which the rate of latent-heat release by the cumulus convection would be proportional to the convergence of moisture in the boundary layer beneath the vortex, while an increase in vortex strength would lead to increased moisture-convergence and therefore latent-heat release. This would represent a self-exciting process. CE64 constructed a linear model to illustrate the mechanism and found that the model had unstable modes which were assumed to be a manifestation in the model of the envisaged interaction. CE64 named the instability Conditional Instability of the Second Kind, or CISK, to distinguish it from the conventional conditional instability (of the first kind), which leads to the initiation of individual cumulus clouds.

A cooperative intensification theory was proposed independently by Ooyama (1964 and his important unpublished 1963 paper with the same title), and illustrated in simple (nonlinear) numerical-model calculations pertaining to hurricane intensification by Ooyama (1969). The theory is described succinctly by Ooyama (1982—section 4); see also section 6 of the present paper.

The so-called CISK-theory has enjoyed wide support from tropical meteorologists and has become firmly entrenched in the teaching of tropical meteorology and in modern textbooks (e.g. Holton 1992—section 9.7.2; James 1994—pp. 279–281). Moreover, the early papers stimulated much subsequent research. A list of references can be found in papers by McBride and Fraedrich (1995) and Fraedrich and McBride (1995) who considered extensions to the linear theory of CISK. An important and very relevant paper to the debate over CISK that these authors[†] and others appear to have overlooked is that by Ooyama (1982). Here 'others' includes Emanuel (1994), Emanuel *et al.* (1994) and Raymond (1994) who have been sharply critical of CISK theory.

* Corresponding author: Meteorological Institute, University of Munich, Theresienstrasse 37, 80333 Munich, Germany.

[†] While Fraedrich and McBride (1995) cite Ooyama (1982), they do not comment on his critique of the linear theory of cooperative instability.

This paper examines the issues once more, focusing on recent criticisms of CISK theory and on recent extensions of it. In doing so the paper expands on Ooyama's own penetrating critique of the linear theory of CISK and points out that recent criticisms of the theory do not apply to the original concept of the cooperative interaction between a field of deep cumulus clouds and a moderately strong cyclonic vortex as envisaged by Ooyama. Section 2 reviews briefly the derivation of the basic equations which, although well-known, enables us to refer to specific aspects which may often be overlooked. Section 3 discusses the cumulus parametrization problem, and section 4 Lilly's linear theory for convective instability. This enables us to set the scene for a discussion of Charney and Eliassen's linear CISK theory in section 5. Ooyama's cooperative intensification theory is the subject of section 6 and an appraisal of CISK follows in section 7. Finally, section 8 considers the relationship between Ooyama's cooperative intensification theory and the theory proposed by Emanuel (1986), which focuses on Wind-Induced Surface Heat Exchange (WISHE).

2. BASIC FORMULATION

The essence of the CISK problem, as posed by Charney and Eliassen, is contained in a two-level model formulated in rectangular Cartesian coordinates (x, y, z) with the physical height z as vertical coordinate (e.g. Charney 1973). Assuming that $\partial(\text{any quantity except pressure})/\partial y \equiv 0$, the *quasi-linearized*, hydrostatic, Boussinesq equations, represented in the vertical on the staggered grid shown in Fig. 1, take the form

$$\frac{\partial u_n}{\partial t} - f v_n = -\frac{1}{\bar{\rho}} \frac{\partial p_n}{\partial x} \quad (n = 1, 3) \tag{1}$$

$$\frac{\partial v_n}{\partial t} + f u_n = 0 \quad (n = 1, 3) \tag{2}$$

$$\frac{\partial \sigma_2}{\partial t} + \left(N^2 + \left(\frac{\partial \sigma}{\partial z} \right)_2 \right) w_2 = \frac{g}{c_p T_{02}} \dot{Q}_2 \tag{3}$$

$$0 = -\frac{1}{\bar{\rho}} \left(\frac{\partial p}{\partial z} \right)_2 + \sigma_2 \tag{4}$$

$$\frac{\partial u_n}{\partial x} + \left(\frac{\partial w}{\partial z} \right)_n = 0 \quad (n = 1, 3) \tag{5}$$

where (u_n, v_n) and p_n are the horizontal velocity components and pressure, evaluated at levels 1 and 3; $\sigma_2 = \frac{1}{2}H(p_3 - p_1)/\bar{\rho}$ and w_2 are the buoyancy force and vertical velocity evaluated at level 2; H is the total depth of air; \dot{Q}_2 is the diabatic heating rate at level 2 (e.g. that due to latent-heat release); N^2 is the square of the Brunt-Väisälä frequency, evaluated here at level 2; f is the Coriolis parameter; T_{02} is the basic-state temperature at

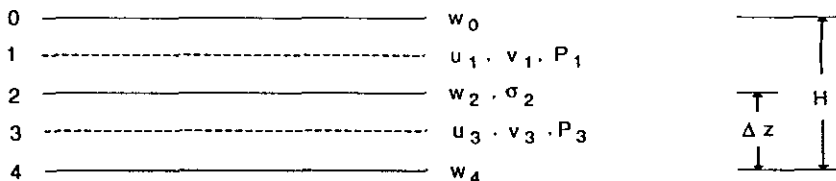


Figure 1. Staggered system of variables for the two-layer model formulation in Section 2. The quantity $\Delta z (= \frac{1}{2} H)$ is the discretization interval for vertical derivatives.

level 2; and $\bar{\rho}$ is a constant density. The equations are linear, except for the term $(\partial\sigma/\partial z)_2$ in (3). Omission of this term precludes the stabilizing effects of vertical heat-transport, for example by convection, from changing the effective static stability of the flow.

The term $(\partial w/\partial z)_n$ in (5) can be expressed in finite-difference form as $(w_0 - w_2)/(\frac{1}{2}H)$ when $n = 1$ and $(w_2 - w_4)/(\frac{1}{2}H)$ when $n = 3$. Omitting for the present the term $(\partial\sigma/\partial z)_2$ in (3), we obtain a closed problem for the eight unknown quantities u_n, v_n, p_n ($n = 1$ and 3), σ_2 and w_2 when boundary conditions are provided for w_0 and w_4 and when \dot{Q}_2 is prescribed. Possible boundary conditions are that $w_0 = w_4 = 0$.

3. THE CLOSURE PROBLEM

The representation of \dot{Q}_2 in (3) constitutes the closure problem for moist processes and, in general, necessitates the introduction of a moisture variable, say the water-vapour mixing ratio r , and a suitable equation for the water budget. Let us suppose that the equations are solved using finite differences in the horizontal. For the sake of argument, we may ignore the coarse vertical resolution in the present model. Then we consider two types of closure, depending on the horizontal resolution, denoted by I explicit closure, and II implicit closure.

In closure I, the assumption would be that the model has sufficient horizontal resolution to resolve individual clouds, but not the turbulent eddies which comprise these clouds (of course, the vertical resolution of the present model is inadequate to resolve the vertical structure of clouds). Then, the closure in the model is required only to relate turbulent fluxes to the mean-flow variables; the moisture effects enter in the specification of \dot{Q}_2 .

As a simple example, one might assume that, where there is upward motion and the air is saturated, the local diabatic heating rate \dot{Q} is

$$\dot{Q} = -L \frac{Dr_s}{Dt}, \quad (6)$$

where r_s is the saturated mixing ratio of air and L is the latent heat. If the motion were steady and r_s were horizontally uniform, this expression would reduce to $-Lw(\partial r_s/\partial z)$, or in flux form $-L\partial(wr_s)/\partial z$. Then the linearized form of (3) becomes

$$\frac{\partial\sigma_2}{\partial t} + w_2 \left(\frac{g}{\theta_0} \frac{d\theta_0}{dz} + \frac{L}{c_p T_0} \frac{\partial r_s}{\partial z} \right) = 0, \quad (7)$$

where T_0 and θ_0 are the basic-state temperature and potential temperature respectively. With the usual approximations that L and T_0 vary much more slowly with z than r_s , the term in brackets equals $(1/\theta_{e0})(d\theta_{e0}/dz)$, where $\theta_{e0} = \theta_0 \exp(Lr_s/c_p T_0)$ is an approximate formula for the pseudo-equivalent potential temperature of the cloud environment. Then (7) becomes

$$\frac{\partial\sigma}{\partial t} + N_m^2 w = 0, \quad (8)$$

where

$$N_m^2 = (g/\theta_{e0})(d\theta_{e0}/dz).$$

Equation (8) is the differential form of the equation $\sigma = -N_m^2 \delta z$, which determines the buoyancy of a parcel of *saturated* air when it is displaced a small vertical distance δz from a level where its buoyancy is zero. A parcel displacement is unstable if $N_m^2 < 0$, i.e. if $d\theta_e/dz < 0$. Therefore, a *saturated atmosphere* is unstable to small parcel displacements if $N_m^2 < 0$.

More sophisticated explicit formulations of convective heating might include some representation of microphysical processes as well as the effects of condensate on buoyancy (e.g. Rotunno and Emanuel 1987).

In closure II, the assumption would be that there is a scale separation between the horizontal resolution of the model and the horizontal scale of cumulus clouds, so that the latter are subgrid-scale phenomena. Then the collective effect of many clouds must be parametrized in terms of the large-scale (i.e. resolvable-scale) variables. In order for such a parametrization to be possible, the clouds must be in a state of statistical equilibrium with the large-scale flow. In particular, *in any initial value problem, a cloud field will be present if required by the initial large-scale conditions* (see, for example, Emanuel (1994)—p. 543). Moreover, the parametrization will be required to specify the amount of subgrid-scale heating associated with this cloud field at this instant so that the time integration of (1) to (5) may proceed.

Over the years, a variety of cumulus parametrization schemes have been proposed and these have been reviewed by Frank (1983), Molinari and Dudek (1992) and Emanuel (1994); see also Emanuel and Raymond (1993). An important point stressed by Ooyama (1982) was that, in general, the implementation of a particular cumulus-parametrization scheme does not eliminate the possibility that resolved-scale convection can occur, even though the latter may be unrealistic (for example, if the resolution of the model is too coarse).

As a lead in to the Charney–Eliassen problem, consider briefly a problem formulated by Lilly (1960) (who used a similar representation of \bar{Q}_2) which helps us see the limitations of the Charney–Eliassen formulation.

4. LILLY'S PROBLEM

Lilly considered the evolution of the flow in which a region of the atmosphere, $|x| < a$, was initially saturated and moist unstable with N_m^2 uniform and negative, while, outside this region, the atmosphere was stable with uniform static stability N . There was no basic flow and no background rotation ($f = 0$). The flow configuration is sketched in Fig. 2(a). In essence*, the linearized form of (1) to (5) is solved with a Type-I closure in which \bar{Q}_2 is assumed to be proportional to w_2 when $w_2 > 0$; otherwise, it is taken to be zero. Lilly ignored frictional and diffusive effects as he was interested in the growth of disturbances which were rapid compared with frictional or radiative damping timescales. Starting from a flow at rest, he found exponentially-unstable modes with growth rates increasing with decreasing a . He interpreted this result as support for the earlier studies of Bjerknes (1938) and Höiland (1939), which showed that, in the absence of frictional and diffusive effects as well as entrainment, convection occurs preferentially at zero horizontal scale. Strictly, the theory is one for potential instability† rather than conditional instability, since the atmosphere is assumed to be saturated from the outset for $|x| < a$. Moreover, the theory applies to small-amplitude air-parcel displacements in this region. Conditional instability, of its very nature, refers to instability arising from finite-amplitude displacements of air parcels. The model formulation is limited by the fact that the treatment of moist processes effectively assumes that the terms $\partial r_s / \partial t$ and $u_2 \partial r_s / \partial x$ in (6) can be neglected in comparison with $w_2 \partial r_s / \partial z$ in the specification of \bar{Q}_2 . Justification of the neglect of $\partial r_s / \partial t$ in (6) in the presence of an exponentially-growing disturbance is certainly questionable, but a further

* Lilly solved the continuous form of the linearized equations in pressure coordinates, but the essence of his problem is captured in the present simplified version.

† The distinction between potential instability and conditional instability is discussed, for example, by Emanuel (1994—p. 185).

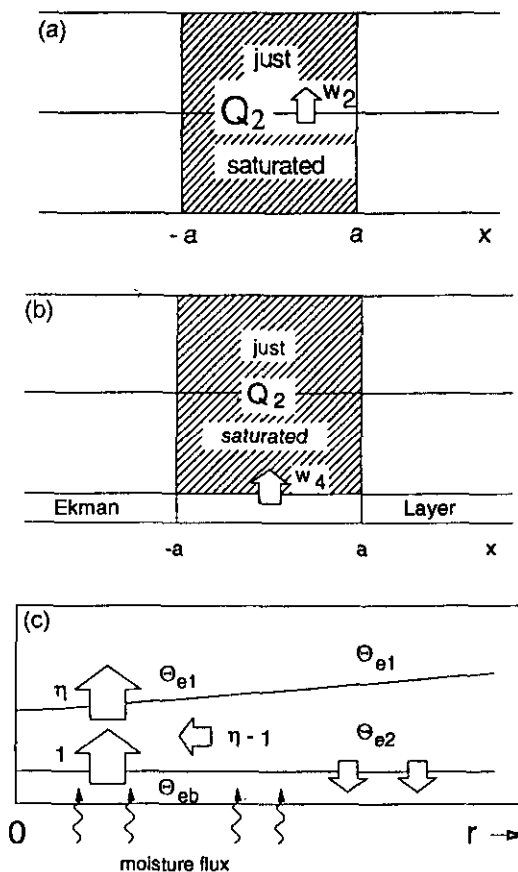


Figure 2. Flow configurations of the three problems discussed in the text: (a) Lilly's; (b) Charney and Eliassen's; (c) Ooyama's. Lilly's problem considers the stability of a region of air $|x| < a$ that is just saturated with a negative buoyancy frequency squared N_m^2 and surrounded by a stably stratified region. The diabatic heating rate Q_2 is proportional to w_2 . In Charney and Eliassen's problem, Q_2 is proportional to w_4 , which, in turn is proportional to the relative vorticity ζ_3 . In Ooyama's problem, Q_2 is proportional to ηw_4 , where η is a measure of the degree of convective instability (or convective available potential energy). Surface-moisture fluxes are present also.

problem is the linearization of (3) by removing the term $(g/\theta_0)(\partial\theta'/\partial z)$. This prevents the reduction of the effective static stability by heat being transported upwards with the 'evolving cloud'. Finally, the negative buoyancy effects of precipitation are excluded. Evidently, the model is, at best, one for the incipient stages of a developing cloud. In no way may the representation of heating be regarded logically as a Type-II closure for convection.

5. THE CHARNEY-ELIASSEN CISK PROBLEM

In essence, the problem formulated by CE64 is similar to Lilly's problem, except that the initial state is not one of rest, but one in which there is cyclonic vortical flow in gradient-wind balance with the horizontal (i.e. radial) temperature-gradient, and the effects of a surface boundary-layer are included through an Ekman-pumping term. The boundary layer thereby forces a secondary meridional circulation. The flow configuration is sketched in Fig. 2(b). The original problem was formulated in cylindrical coordinates,

but later versions used Cartesian coordinates in which the vortical flow was replaced by a localized region of enhanced horizontal shear, and geostrophic balance (as opposed to gradient-wind balance) was assumed (e.g. Bates 1973; Charney 1973). The surface boundary-condition on the vertical velocity, which enters the formulation through w_4 in (5), is that $w_4 = \gamma \zeta_3$, where $\zeta_3 = \partial v_3 / \partial x - \partial u_3 / \partial y$ is the vertical component of relative vorticity at level 3 and γ is a constant. Now, \dot{Q}_2 can be expressed as a linear combination of w_2 and w_4 , depending on how one constructs the finite-difference form of $\partial(wr_s)/\partial z$ in the approximation to the flux form of (6). In the present work, it is argued that the linear relationship between \dot{Q}_2 and the vertical velocity is a Type-I closure, irrespective of which linear combination of w_2 and w_4 is chosen. This is because, in the two-level model, w_2 will have the same sign as Q_2 , and hence the same sign as w_4 . Thus, qualitatively, the same dynamics are implied even if \dot{Q}_2 is related linearly to w_4 only*, provided, of course, that† $w_2 > w_4$. However, the linear relationship between w_4 and ζ_3 and the assumption of quasi-geostrophic balance in the vortex may be expected to distort the evolution of the convective mode. Also, as in Lilly's problem, it is hard to justify the neglect of $\partial r_s / \partial t$ in (6) in the presence of an exponentially growing disturbance. A key point is, however, that, as in Lilly's problem, there are no clouds present in the initial state; there is only a deep region of potentially unstable air in the inner region. While this was undoubtedly not the intention of CE64, it is all that is included in the mathematical formulation.

Like Lilly (1960), CE64 found unstable modes to their equations, but the growth rates were rather uniform over a broad range of horizontal scales. The feedback which gives rise to instability in the model is clear from the relationship that Q_2 is proportional to ζ_3 since middle-level heating will cause ascent throughout the troposphere. This ascent, in turn, will generate relative vorticity in the lower troposphere by vortex-line stretching. However, in reality, the heating aloft would ultimately stabilize the atmosphere to further convection, a feedback which is precluded in the CE64 model as a result, *inter alia*, of the neglect of the nonlinear term in (3). This feedback is permitted in the nonlinear formulation of Ooyama (1969) which is described below.

6. Ooyama's PROBLEM

Ooyama's formulation of a theory of cooperative intensification between cumulus clouds and an incipient vortex differs in certain respects to that of CE64, partly in the closure of moist processes and partly in concept. Ooyama (1964, 1969) considered a flow configuration with two layers of homogeneous fluid overlying a shallow boundary-layer of uniform thickness. He represented the heating effects of deep cumulus clouds in terms of a mass flux from the boundary layer to the upper layer wherever there is resolved-scale boundary-layer convergence; (see Fig. 2(c)). The representation is based on the idea that the deep cumulus clouds that form in such a region will entrain ambient air from the middle layer as they rise through it and detrain in the upper layer. The entrainment rate is determined as a function of time so as to satisfy energy conservation, with the assumption that the air detrained into the upper layer has the saturation equivalent potential temperature of ambient air in that layer. Thus, for each unit of mass transferred from the boundary layer into the upper layer, $\eta - 1$ units of mass are entrained from the middle layer and transferred to the upper layer also. In essence, the net heating of the upper layer is proportional to

* Strictly, of course, the coefficient of proportionality should be chosen to be a consistent approximation to (6).

† Otherwise there will be a contraction of vertical vorticity at level 3 and spin down will occur in spite of the latent-heat release. This would happen, for example, if the prescribed vortical flow is too strong to be supported by the latent-heat release implied by the model. Then frictional effects will reduce the shear, i.e. spin down will occur.

ηw_4 , using the notation of section 2. The entrainment parameter η is determined through an energy budget and can be expressed as

$$\eta = 1 + \frac{\theta_{eb} - \theta_{e1}^*}{\theta_{e1}^* - \theta_{e2}} \quad (9)$$

where θ_{eb} , θ_{e2} and θ_{e1}^* are respectively the equivalent potential temperatures of the air in the boundary layer and the middle layer (layer 2), and the saturated equivalent potential temperature in the upper layer (layer 1). Note that the parameter η depends on the degree of instability of the flow to deep convection, that is on the convective available potential energy (CAPE), which is proportional to $\theta_{eb} - \theta_{e1}^*$. In his nonlinear calculations, Ooyama (1969) took θ_{e1}^* to be a dependent variable of the problem, allowing η to vary with time. In fact, θ_{eb} was allowed to vary also, accounting for the heat fluxes from the ocean. He showed that, as the vortex develops a warm core structure aloft, the inner region becomes more neutral to convection whereupon η tends to unity and the entrainment declines to zero. Ooyama's representation of deep convective heating is distinctly different from those used by Lilly and CE64 and cannot be considered purely as a Type-I closure, although as in the CE64 scheme, the heating rate is proportional, *inter alia*, to the resolvable-scale boundary-layer convergence and hence to w_4 . The additional factors are the dependence of the heating rate on the degree of convective instability through the variable quantity η , and its relationship to the radial entrainment in the middle layer; these features are not present in the cumulus representations of Lilly or CE64. This entrainment is a crucial factor in Ooyama's model as it leads to convergence in the middle layer which is essential for vortex spin-up (Ooyama 1982).

Although Ooyama (1969) carried out a linear stability-analysis of his equations, he was at pains to point out the limitations of this analysis and that, like the previous calculations of CE64, it could not account for the horizontal scale of a tropical cyclone. He argued that the process of cooperative interaction between cumulus convection and a vortex must be intrinsically nonlinear. The processes represented in the nonlinear model form the basis of Ooyama's concept of cooperative interaction, or CISK, which he viewed as a theory for vortex intensification from a state in which organization of the convection by rotation was already present. The numerical integrations of the nonlinear equations were able to produce hurricane-like vortices with a considerable degree of realism, including their growth rate, radial scale and mature strength. Ooyama was also at pains to point out that his model calculations demonstrated that latent- and sensible-heat transfer from a warm ocean were crucial to vortex intensification.

An important aspect of the cooperative intensification theory of hurricanes proposed by Ooyama (1982) is the progressive reduction of the local Rossby radius-of-deformation as the inertial stability of the vortex increases in the inner-core region. This intrinsically nonlinear effect, he argues, progressively reduces the scale separation between the deep cumulus clouds and the balanced tangential circulation of the vortex in that region, so that the individual clouds become more and more under the control of the balanced dynamics. Presumably, as this occurs, a representation of convective heating based on resolvable-scale mass-flux convergence becomes more appropriate for the inner-core region.

7. AN APPRAISAL OF CISK

Ooyama (1982) was strongly critical of the so-called linear theories of CISK, pointing out their intrinsic inability in capturing the feedback effects of convection on the large-scale flow. This limitation applies also to the linear version of his own model (Ooyama 1969). In

addition, he noted that the term CISK had been abused to the degree that 'the acronym has become a useless term in any sensible communication'. Section 4 of his paper described the concept of a cooperative-intensification theory for hurricanes and Ooyama stressed that this concept was independent of any particular cumulus parametrization scheme. Furthermore, he criticized authors who refer to a particular cumulus parametrization scheme as a 'CISK parametrization'. He wrote: 'The present author views CISK in terms of the conceptual content that has grown and matured with advances in modelling work. Then, the spirit of CISK as the cooperative intensification theory is valid and alive.' He went on: 'There are those who continue on criticisms of CISK as the linear theory, ignoring all the later contributions that have cast a better light on the theory.' Ooyama also noted that CISK (in his sense of a cooperative-intensification theory) was *not* a theory for the growth of incipient disturbances, which the mathematical formulation of CE64 clearly was.

Recently, the theory of CISK has come under heavy criticism from other quarters and for apparently different reasons (e.g. Emanuel 1994; Emanuel *et al.* 1994; Raymond 1994). For example, Emanuel (1994—p. 527) challenged CE64's suppositions that cumulus convection could not be maintained in the tropical atmosphere in the absence of some large-scale circulation to replenish the moisture consumed by the convection, and that low-level moisture-convergence was necessary to maintain a sufficient level of conditional instability. He argued: 'The CISK idea overlooks the simple fact that convection is a response to the generation of instability, not to the supply of water *per se*.' He continued: 'The possibility of an instability due to the feedback involving vertical velocity was first raised by Ooyama (1964) and CE64. Models that produce CISK do not use statistical equilibrium of energy as a basis for incorporating the effects of cumulus convection, but instead use ill-posed schemes in which the heating is related to the total column moisture convergence.' In the same vein, Emanuel *et al.* (1994) argued: 'In essence it is proposed that a purely advective process could maintain or increase the APE of a disturbance, in clear violation of the laws governing energy conservation.' But is this true? Ooyama's scheme does not violate energy conservation! We note that these are all criticisms of the particular cumulus parametrization schemes used by CE64 and their followers; they do not apply to the cooperative-intensification theory proposed by Ooyama (1982—section 4). Moreover, while CE64's closure may be criticized for not using statistical equilibrium of energy, this is not true of the scheme used by Ooyama (1969). Indeed, as noted above, there are significant differences between Ooyama's parametrization scheme and those of CE64 and followers including McBride and Fraedrich (1995) and Fraedrich and McBride (1995). Raymond's critique of CISK was mainly directed at the concept known as wave CISK, envisaged as a cooperative instability between convection and some larger-scale atmospheric wave disturbance.

Ooyama was fully aware of the importance of surface heat-fluxes, especially the surface latent-heat flux, without which intensification in his (nonlinear) model did not occur. Moreover, Ooyama's cumulus parametrization scheme did not overlook the fact that convection is a response to the realization of instability. In fact, the scheme generates heating only at radii where CAPE exists. This is not to say that Ooyama's scheme itself does not have shortcomings, only that it does not suffer from many of the shortcomings of other parametrizations, such as the CE64 scheme, that have been used to study the cooperative intensification, or CISK, process.

McBride and Fraedrich (1995) criticized Emanuel (1989) for his statement that CISK models* made an incorrect assumption of the existence of a reservoir of CAPE to drive large-scale motions. They said: 'Strictly this is not correct as CAPE does not appear

* Meaning the CISK formulations of CE64 and followers.

explicitly in the basic equations used by CE64'. But if it is not correct, there is no other energy source to drive an instability!

It seems right that a proper appraisal of CISK should consider Ooyama's concept as summarized in his 1982 paper as well as that of CE64 and followers. It must take into account also the differences between the two formulations, including the important differences between the two representations of cumulus convection used by CE64 and Ooyama (1964, 1969). One might challenge the degree of realism of Ooyama's representation of convection, but, so far, an appraisal of this has not been provided by those who criticize CISK. Unlike the representations of Lilly and CE64, it is not simply a Type-I closure; its use in Ooyama's cooperative-intensification theory *can* be justified when the scale separation between the cumulus scale and balanced-vortex scale has become sufficiently reduced. Moreover, the closure, as implemented in the nonlinear model calculations of Ooyama (1969), includes a representation of nonlinear feedbacks not present in the CE64 model and its extensions. It is notable that, in these calculations, a vortex of tropical-depression scale and strength intensified to one of hurricane intensity and scale during a realistic period.

8. Ooyama's Theory and Wishe

Emanuel (1986) constructed a steady state theory for an axisymmetric hurricane, using a Type-II closure on convection in which the effect of clouds was postulated to bring the tropospheric temperature to a state in which the reversibly-calculated equivalent potential temperature θ_{er} , was uniform along an angular momentum surface*. This closure was based on radiosonde observations which showed that, on average, in convective regions in the tropics, soundings lie close to a reversibly-defined moist adiabat whose θ_{er} is approximately equal to the value at the top of the sub-cloud layer (Betts 1982; Xu and Emanuel 1989). The presumption is that convection brings the atmosphere to a state that is locally neutral (in the vertical) to moist convection, but the details of how this happens in the presence of precipitating convection are not well understood; (see, for example, Cohen and Frank 1989). Such an atmosphere would have precisely zero CAPE, if the latter is defined in terms of reversible parcel-ascent. The consequence of the parametrization is that the thermodynamic structure of the entire troposphere is determined by the θ_{er} in the surface boundary-layer; in particular, the large-scale horizontal gradient of θ_{er} throughout the troposphere is determined entirely by that in the boundary layer. In turn, the boundary-layer θ_{er} is influenced greatly by the flux of latent heat, and to a lesser extent sensible heat, from the ocean. The latent-heat flux, in particular, increases dramatically with increasing surface wind speed. Emanuel (1986-p. 586) argued that this is the crucial factor in the intensification of an incipient tropical vortex to one of hurricane strength and that, if it were not for this effect, a hurricane would not form. Thus, hurricane intensification is regarded essentially as a finite-amplitude instability associated with wind-induced surface-heat exchange, for which Emanuel coined the acronym 'WISHE'. The idea is that, as the strength of the vortex, and in particular that of the surface winds, increases, surface fluxes act to increase the broad-scale radial entropy-gradient in the boundary layer and that, through the action of convection, this gradient is communicated to the troposphere. This would account for the large radial gradient of θ_e observed across the inner-core region of a hurricane (e.g. Hawkins and Imbembo 1976-Fig. 16). The associated increasing large-scale buoyancy-gradient would lead to convergence above the boundary layer and

* In a rapidly-rotating vortex context, local neutrality has to be expressed along angular-momentum surfaces.

thereby to vortex spin-up. The evolution of a hurricane as envisaged above was demonstrated in numerical-model experiments by Rotunno and Emanuel (1987). The surface fluxes cannot increase the boundary-layer entropy above that corresponding to the local saturation entropy of the ocean surface temperature, but the reduction of surface pressure increases the levels of boundary-layer entropy that can be achieved. Factors limiting the maximum intensity that can be achieved by a particular storm include the effects of precipitation-driven downdraughts and the mixing effects of shallow convection as discussed by Emanuel (1989, 1991, 1995), who described more sophisticated representations of cumulus clouds.

The question is: how do these conceptual ideas compare with Ooyama's cooperative intensification theory? Emanuel (1986) noted that while his view departed in some ways from other conceptual views of tropical cyclones, the physics implied in his view were contained in virtually all contemporary models which allowed for convection and air-sea heat-transfer. In particular, this applies to Ooyama's nonlinear model described in section 6. The main difference is that Emanuel regarded the role of cumulus convection simply as a local means of distributing sensible and latent heat in the vertical. The implications for the three-dimensional (non-axisymmetric) version of Emanuel's model would be that patterns of convection are 'organized' by the patterns of boundary-layer entropy that develop, not directly by the patterns of boundary-layer convergence. There is no environmental CAPE, nevertheless CAPE develops as a result of surface-heat and moisture fluxes, but is instantaneously consumed by the convection as it redistributes the added heat through the troposphere. In the three-dimensional version of Ooyama's model (Shapiro 1992; Dengler and Reeder 1997), the pattern of convective mass-flux depends on the pattern of boundary-layer convergence, but again, where there is deep convection, the atmosphere is brought to a state of moist neutrality (in the vertical and with respect to a pseudo-adiabat). As noted, in section 6, this is accomplished by entraining ambient mid-tropospheric air into the 'clouds' at a rate necessary for air at the upper model-level to detrain at the saturation θ_e of that level. This requires a supply of CAPE which, as the initial CAPE is exhausted, is provided locally by sea-surface heat fluxes, as in the Emanuel model. Although, as pointed out by Emanuel (1986-p. 585), Ooyama's original calculations began from an initial state with a relatively large amount of CAPE, Dengler and Reeder (1997) have shown that this is not a necessary requirement of the model. The cloud-induced radial entrainment in the middle layer in Ooyama's model is wholly responsible for the vortex spin-up and, as the vortex approaches its mature stage, the entrainment declines and the convection becomes passive, with updraughts tending towards moist adiabats with the saturated (pseudo-) θ_e of the boundary layer, much as is assumed in Emanuel's steady-state model. This being the case, we may ask whether it is possible to distinguish between Ooyama's cooperative-intensification theory and Emanuel's WISHE-instability theory. The distinction appears to rest as much on semantics as on substance; at most, the emphasis on the role of convection is different, as explained below.

It remains, then, to appraise Ooyama's statement (Ooyama 1982-p. 377) that 'the spirit of CISK as a cooperative-intensification theory is valid and alive'. Much would appear to hinge on how one interprets 'the spirit of CISK' and the word 'cooperative'; the latter implies cooperation between convection and the large-scale flow. Few would deny that, as a tropical cyclone intensifies, the convection becomes organized, generally into a relatively symmetric ring of cloud which forms the eye wall of the storm, together with one or more spiral bands of deep convection extending outwards from the eye wall. The question is whether this organization is the cause of, or merely a consequence of the intensification. Emanuel (personal communication) would argue that while the convection is crucial, it is not causal, and that vortex amplification results not from a cooperation be-

tween convection and the flow, but as a result of feedback with surface fluxes, a feedback which he notes Ooyama's model goes far to demonstrate. Emanuel sees the role of convection as simply communicating the boundary-layer entropy-gradient to the troposphere. As explained above, this is precisely what the convection does in Ooyama's model.

9. CONCLUSIONS

Ooyama's criticisms of the linear CISK theories are supported; indeed, these criticisms are arguably more fundamental than those which have been levelled at these theories in recent papers. The latter criticisms attack only a particular representation of cumulus convection, which, it is argued, is in essence a somewhat limited, explicit, cloud-model closure rather than a parametrization of cumulus clouds. These criticisms apply also to the studies of McBride and Fraedrich (1995) and Fraedrich and McBride (1995).

Ooyama's cooperative intensification theory of hurricanes has been compared with Charney and Eliassen's theory of CISK and with Emanuel's WISHE-instability theory. Considerable differences have been shown between Charney and Eliassen's concept of CISK and Ooyama's cooperative-intensification theory of hurricanes. In particular, there are important differences in the way convection is represented in the two theories. Moreover, Ooyama's theory is an intrinsically nonlinear one, in which surface fluxes are of crucial importance.

It would appear that the differences between Emanuel's WISHE-instability theory and Ooyama's cooperative-intensification theory, as represented in his 1969 modelling study and as further elucidated in his 1982 paper, are largely restricted to technical differences in the formulation of convective heating and to the degree of emphasis placed on the role of convection in relation to that of surface fluxes by Emanuel. However, both authors agree that surface fluxes *and* convection are necessary for hurricane intensification.

ACKNOWLEDGEMENTS

I am very grateful to a number of people for their penetrative comments on an earlier draft of this manuscript; they include Kerry Emanuel, Brian Mapes, Vic Ooyama, Dave Raymond, Michael Reeder and Lloyd Shapiro. Some of the ideas crystallized during a visit to the Hurricane Research Division of NOAA in August and September 1995 where I had stimulating discussions on the subject of CISK with Drs Lloyd Shapiro and Vic Ooyama. Discussions about the Ooyama scheme with Klaus Dengler and Sarah Jones have also been most helpful. The paper was presented at a Workshop on Convective Parametrization held at the Centre for Dynamical Meteorology and Oceanography, Monash University, Australia in October 1995.

This research was supported by the United States Office of Naval Research through Grant Number N00014-95-1-0394.

REFERENCES

- | | | |
|---------------------------------|------|--|
| Bates, J. R. | 1973 | A generalization of the CISK theory. <i>J. Atmos. Sci.</i> , 30 , 1509–1519 |
| Betts, A. K. | 1982 | Saturation point analysis of moist convective overturning. <i>J. Atmos. Sci.</i> , 39 , 1484–1505 |
| Bjerknes, J. | 1938 | Saturated-adiabatic ascent of air through dry-adiabatically descending environment. <i>Q. J. R. Meteorol. Soc.</i> , 64 , 325–330 |
| Charney, J. G. | 1973 | Planetary Fluid Dynamics, in <i>Dynamical Meteorology</i> , P. Morel, Ed. Reidel |
| Charney, J. G. and Eliassen, A. | 1964 | On the growth of the hurricane depression. <i>J. Atmos. Sci.</i> , 21 , 68–75 |

- Cohen, C. and Frank, W. M. 1989 A numerical study of lapse-rate adjustment in the tropical atmosphere. *Mon. Weather Rev.*, **117**, 1891–1905
- Dengler, K. and Reeder, M. J. 1997 The effects of convection and baroclinicity on the motion of tropical-cyclone-like vortices. *Q. J. R. Meteorol. Soc.*, **123**, (In press)
- Emanuel, K. A. 1986 An air-sea interaction theory for tropical cyclones. Part I: Steady-state maintenance. *J. Atmos. Sci.*, **43**, 585–604
- 1989 The finite-amplitude nature of tropical cyclogenesis. *J. Atmos. Sci.*, **46**, 3431–3456
- 1991 The theory of hurricanes. *Ann. Rev. Fluid Mech.*, **23**, 179–196
- 1994 *Atmospheric Convection*. Oxford University Press
- 1995 The behaviour of a simple hurricane model using a convective scheme based on subcloud-layer entropy equilibrium. *J. Atmos. Sci.*, **52**, 3960–3968
- Emanuel, K. A. and Raymond, D. J. 1993 *The representation of cumulus convection in numerical models*. Eds. Meteorol. Monogr., 24, American Meteorological Society, Boston
- Emanuel, K. A., Neelin, J. D. and Bretherton, C. S. 1994 On large-scale circulations in convecting atmospheres. *Q. J. R. Meteorol. Soc.*, **120**, 1111–1143
- Fraedrich, K. and McBride, J. L. 1995 Large-scale convective instability revisited. *J. Atmos. Sci.*, **52**, 1914–1923
- Frank, W. M. 1983 The cumulus parametrization problem. *Mon. Weather Rev.*, **111**, 1859–1871
- Höiland, E. 1939 On the interpretation and application of the circulation theorems of V. Bjerknes. *Arch. Math. og Naturv.*, **42**, No. 5
- Hawkins, H. F. and Imbembo, S. M. 1976 The structure of a small, intense hurricane—Inez 1966. *Mon. Weather Rev.*, **104**, 418–442
- Holton, J. R. 1992 *An introduction to dynamic meteorology*, Academic Press
- James, I. N. 1994 *Introduction to circulating atmospheres*, Cambridge University Press
- Lilly, D. K. 1960 On the theory of disturbances in a conditionally-unstable atmosphere. *Mon. Weather Rev.*, **88**, 1–17
- McBride, J. L. and Fraedrich, K. 1995 CISK: A theory for the response of tropical convective complexes to variations in sea surface temperature. *Q. J. R. Meteorol. Soc.*, **121**, 783–796
- Molinari, J. and Dudek, M. 1992 Parametrization of convective precipitation in mesoscale numerical models: A critical review. *Mon. Weather Rev.*, **120**, 326–344
- Ooyama, K. 1964 A dynamical model for the study of tropical cyclone development. *Geophys. Int.*, **4**, 187–198
- 1969 Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci.*, **26**, 3–40
- 1982 Conceptual evolution of the theory and modelling of the tropical cyclone. *J. Meteorol. Soc. Japan*, **60**, 369–379
- Raymond, D. 1994 Cumulus convection and the Madden-Julian oscillation of the tropical troposphere. *Physica D*, **77**, 1–22
- Rotunno, R. and Emanuel, K. A. 1987 An air-sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model. *J. Atmos. Sci.*, **44**, 542–561
- Shapiro, L. J. 1992 Hurricane vortex motion and evolution in a three-layer model. *J. Atmos. Sci.*, **49**, 140–153
- Xu, K. and Emanuel, K. A. 1989 Is the tropical atmosphere conditionally-unstable? *Mon. Weather Rev.*, **117**, 1471–1479