

6. Potential Temperature

The stability of the atmosphere may seem counter-intuitive. We are aware from our everyday experience that hot air rises, so why is it that the troposphere is stable despite the temperature decreasing with height? In this and the next section we shall investigate why this is so. It will be shown that the stability of the troposphere depends on both the temperature and the decreasing pressure with height. The pressure and temperature can be combined into a single variable, the potential temperature, and it will be shown that the atmosphere is stable if the potential temperature increases with height. We shall define the potential temperature, θ , to be a quantity that will not change as air parcels are moved adiabatically (without heat exchange).

The first law of thermodynamics

Suppose that a small quantity of heat, dq , is added to a unit mass of gas. Conservation of energy requires that the heat added be balanced by an increase in the internal energy of the gas, du , and work done by the gas against its environment, dw . This may be expressed mathematically as

$$dq = du + dw, \quad (6.1)$$

and is known also as the *first law of thermodynamics*.

For a unit mass of gas, the volume V is simply the specific volume α . Then the work done *by* the gas when its specific volume increases by $d\alpha$ is

$$dw = p d\alpha .$$

Note that the work done = force \times distance. For example, for a unit mass of gas with cross-sectional area A , $d\alpha = A dx$, or $p d\alpha = (p A) dx = \text{force} \times \text{distance}$. (See Fig. 6.1.)

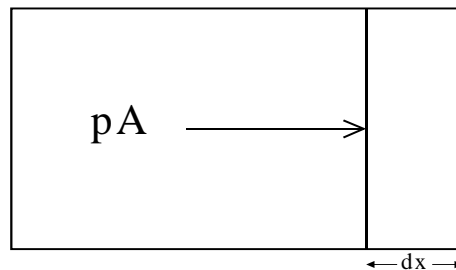


Figure 6.1. Work done in displacing a boundary of area A a distance dx .

Hence Eq. (6.1) becomes

$$dq = du + p d\alpha . \quad (6.2)$$

Suppose that the heat added to the gas increases its temperature from T to $T+dT$ without a change in phase. The ratio dq/dT is called the *specific heat*. The specific heat defined in this way can have any number of values depending on how the gas changes as the heat is added.

For example, if the volume of the gas remains constant, the ratio dq/dT , denoted by c_v , is called *the specific heat at constant volume*. In this situation, no work is done by the gas in expanding, whereupon Eq. (6.2) gives $dq = du$, i.e.,

$$c_v = \left(\frac{du}{dT} \right)_{\alpha=\text{constant}} . \quad (6.3)$$

However, the internal energy of an ideal gas is independent of volume since the molecules of an ideal gas do not exert any attractive or repulsive forces on each other. (In fact, it can be shown that the average kinetic energy of a gas molecule = $3kT/2$, where $k = R^*/N$ is Boltzman's constant.) Consequently, the internal energy depends only upon the temperature, in which case Eq. (6.3) gives

$$du = c_v dT.$$

Hence, Eq. (6.2) can be re-expressed as

$$dq = c_v dT + p d\alpha. \quad (6.4)$$

As a second example, consider a case in which the heat is added to the gas in such a way that the pressure remains constant. Then we may define a *specific heat at constant pressure* as

$$c_p = \left(\frac{dq}{dT} \right)_{p=\text{constant}} \quad (6.5)$$

In this case, some of the heat input dq will go into doing work as the gas expands and pushes against its environment. Therefore, a larger quantity of heat will need to be added to the gas to raise its temperature by the same dT as in the case $\alpha = \text{constant}$.

To show this we write the perfect gas equation in the differential form (using the product rule)

$$pd\alpha + \alpha dp = R_d dT. \quad (6.6)$$

Using Eq. (6.6) to eliminate $pd\alpha$ from Eq. (6.4) gives

$$dq = (c_v + R_d) dT - \alpha dp. \quad (6.7)$$

At constant pressure, $dp = 0$, whereupon Eq. (6.5) gives

$$c_p = c_v + R_d. \quad (6.8)$$

For dry air, $c_v = 717 \text{ J deg}^{-1} \text{ kg}^{-1}$.

Combining Eqs. (6.7) and (6.8) we obtain an alternative form of the first law of thermodynamics, i.e.,

$$dq = c_p dT - \alpha dp. \quad (6.9)$$

Note that αdp is *not* the work done, but for an isothermal process $p d\alpha + \alpha dp = 0$, and hence $dw = p d\alpha = -\alpha dp$. (An isothermal process is one for which the temperature remains constant, and hence $dT = 0$.)

Adiabatic changes

As an air parcel moves around in the atmosphere, it tends to receive little external heat input, at least on a time scale of a few hours. This is due to the relatively low thermal and radiative conductivity of air. Thus, air parcels carry their heat with them and suffer negligible heat loss or gain. Accordingly, it is a good approximation to assume that changes of state are adiabatic.

The mathematical statement that no heat is gained or lost is $dq = 0$. Thus, from Eq. (6.4)

$$0 = c_v dT + p d\alpha.$$

If the air parcel expands ($d\alpha > 0$), it must cool ($dT < 0$) as it does work ($p d\alpha$) against its environment.

Alternatively, from Eq. (6.9),

$$0 = c_p dT - \alpha dp.$$

Using the perfect gas equation, this becomes

$$0 = c_p dT - \frac{R_d T}{p} dp$$

or, upon dividing by T ,

$$\frac{dT}{T} - \kappa \frac{dp}{p} = 0,$$

where $\kappa = R/c_p$. Therefore, integrating from (T, p) to (θ, p_*) gives

$$\ln T - \kappa \ln p = \ln \theta - \kappa \ln p_*, \quad (6.10)$$

where θ and p_* are reference values of temperature and pressure. For *dry air*, $\kappa = 0.286$. Alternatively, Eq. (6.10) may be written as

$$T = \theta \left(\frac{p}{p_*} \right)^\kappa. \quad (6.11)$$

θ is called the *potential temperature* of the air parcel. Physically, the potential temperature of an air parcel is the temperature the parcel would have if it were brought adiabatically to the standard pressure p_* ; generally we take $p_* = 1000$ mb. Since the potential temperature is just the constant of integration, it is a conserved quantity for an air parcel in *adiabatic motion*, i.e., motion in which there are no heat sources or sinks.

In meteorology, it proves extremely useful to label air parcels with their potential temperature. In fact, the potential temperature is defined by

$$\theta = T \left(\frac{p_*}{p} \right)^\kappa. \quad (6.12)$$

We shall see that the vertical stability of a dry atmosphere can be characterized by the variation of θ with height. Meteorologists have developed *aerological diagrams* which allow the pressure and temperature to be plotted and the potential temperature to be read off.

Aerological Diagrams

There are many different types of aerological diagrams, but they all share some important characteristics. All of them allow points to be plotted according to their temperature and pressure, and the potential temperature to be read off. In addition, information about moist processes, such as the likelihood of cloud formation, can be obtained from the diagrams. However, in this course, we only consider dry processes.

The aerological diagram used in this course is the skewT-logP diagram used by the Australian Bureau of Meteorology. As the name suggests, the vertical axis is log(pressure) and the other axis is skew (diagonal) temperature. A simplified version of this diagram is shown in Figure 6.2. The horizontal lines are lines of equal pressure and are labelled in hPa, the dashed lines (positive slope) are lines of constant temperature and are labelled in $^{\circ}\text{C}$. The negatively sloping solid lines are *dry adiabats*, and indicate lines of equal potential temperature. Another way to think about dry adiabats is to realise that motion along these lines is adiabatic motion – no heat is exchanged.

Remembering the definition of potential temperature, it is easy to find the potential temperature given the pressure and temperature. Plot the (pressure, temperature) reading on the aerological diagram, then follow a dry adiabat to $p=1000\text{hPa}$ and read off the temperature at this point.

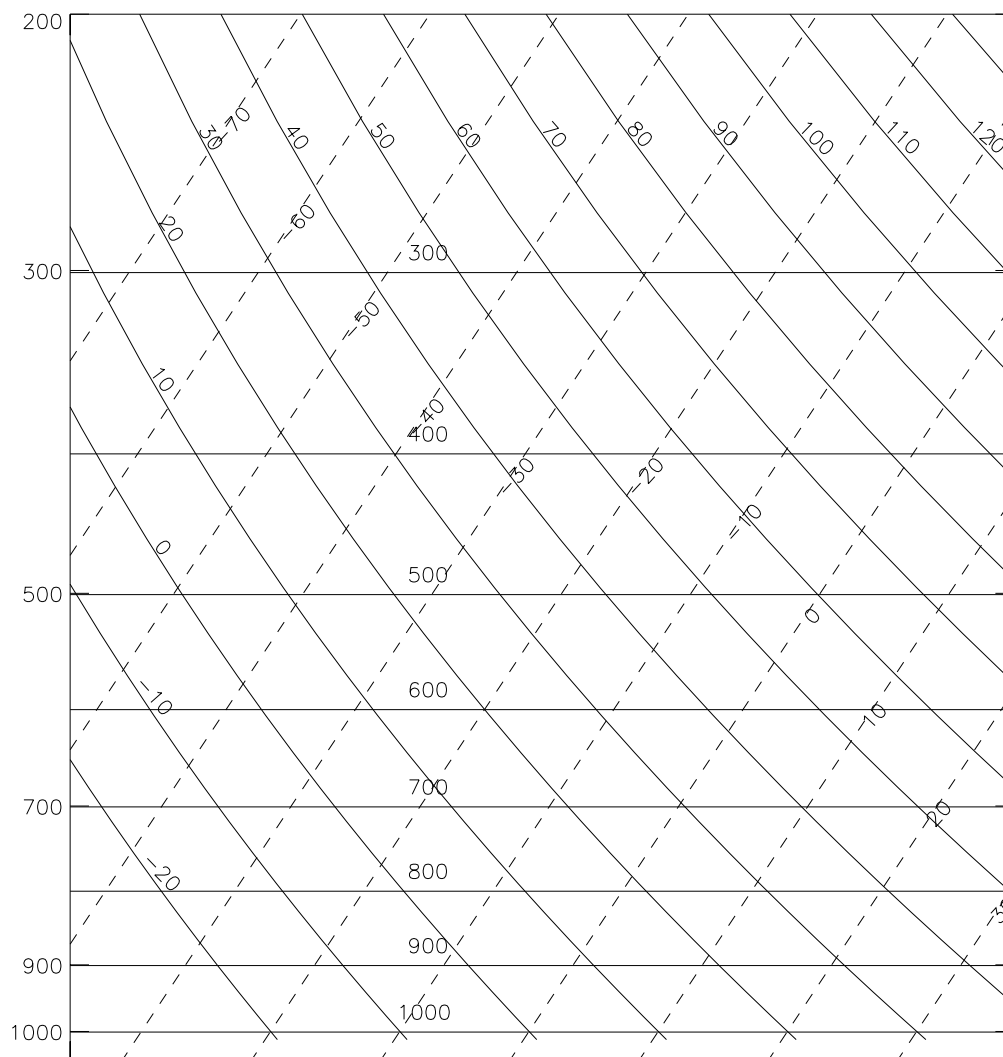


Figure 6.2 Simplified aerological diagram.