MET5533: Krishnamurti

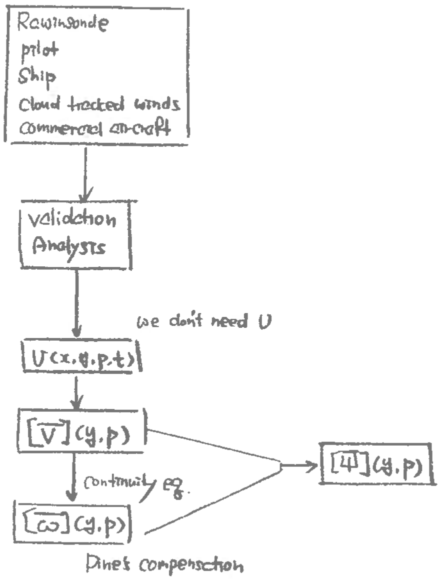
- Using the enclosed diagram as a frame of reference discuss the role of planetary scales and the zonally averaged scales on the maintenance of the eddy kinetic energy of a tropical disturbance.

look at note p5 ?

MET5533: Krishnamurti

- Given atmospheric wind data sets on a global grid, how would you construct a picture of the Hadley cell? List all step starting from global winds to the final calculation of the Hadley cell stream function.

<picture of the Hadley cell>



MET5533: Krishnamurti

- This question is on the dry and moist static stability

a) Discuss how the large scale divergence has opposite effects on the generation of dry and moist static stability over the lower troposphere of the tropics.

b) How would shallow stratocumulus cloud top cooling affect the dry and moist static stability.

c) How would surface evaporation be important for this problem.

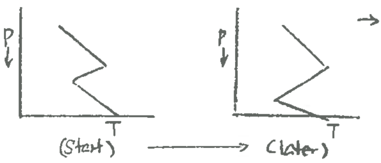
a) For the dry static stability, the divergence term is most important

If we just consider the divergence term in the dry static stability eg.,

$$\frac{\partial \overline{\Gamma_d}}{\partial t} = -\overline{\Gamma_d}\frac{\partial \overline{\omega}}{\partial p}$$

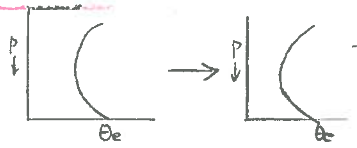
$\overline{\Gamma_d}$ is almost always positive and downward motion (low-level divergence) will cause $-\frac{\partial \overline{\omega}}{\partial p}$ to be positive

Therefore, dry static stability will increased. Large scale divergence has a stabilizing effect.



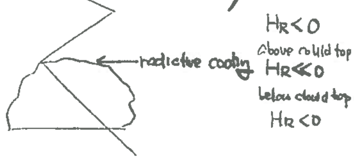
→ The trade wind inversion will be stabilized

For the moist static stability, the divergence term does not play an important role.



→ destalilize slightly

b) For the dry static stability



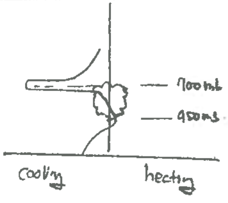
$H_R < 0$

above could top $H_R \ll 0$ $\Big\}\ -\frac{\partial}{\partial p}H_R > 0$ → producing inversion

below cloud top $H_R < 0$ $\Big\}\ -\frac{\partial}{\partial p}H_R < 0$ → destroying inversion

Cloud top radiative cooling destabilizes below inversion base (i.e. $\frac{\partial \overline{\Gamma_d}}{\partial t} < 0$), while stabilizes above inversion base (i.e. $\frac{\partial \overline{\Gamma_d}}{\partial t} > 0$)

For the moist static stability, $\frac{\partial \overline{\Gamma_m}}{\partial t} = -\frac{\partial}{\partial p}\overline{H_R}$

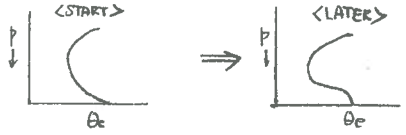


within the cloud layer

$-\frac{\partial \overline{H_R}}{\partial p} < 0$ hence $\frac{\partial \overline{\Gamma_m}}{\partial t} < 0$ → destabilizing

below the cloud, $-\frac{\partial \overline{H_R}}{\partial p} > 0$, hence $\frac{\partial \overline{\Gamma_m}}{\partial t} = 0$

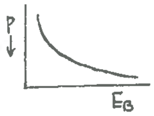
→ stabilizing



c) Surface evaporation term

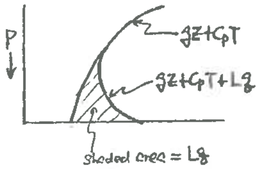
$$\frac{\partial \overline{\Gamma_m}}{\partial t} = \underbrace{-L\frac{\partial \overline{E_B}}{\partial p}}$$

→ surface flux of latent heat evaporation.



$\frac{\partial E_B}{\partial p} > 0$ hence $\frac{\partial \overline{\Gamma_m}}{\partial t} < 0$ → destabilizing

Conditional instability arises due to the presence of moisture (i.e. contribution from Lq)



If surface evaporation is very large ⇒ the slope will be greater ⇒ conditional instability will increase.

Therefore, surface evaporation is important in restoring conditional instability.

MET5533: Krishnamurti

- This question is on monsoon. Please provide short answer for each part.

a) What level and in what season is the Somali jet frequently observed.
b) Is the African easterly jet found south of the Tibetan High.
c) Is the Tropical easterly jet found south of the Tibetan High.
✓ d) What are typical values of evaporative flux (in energy units) over the Arabian Sea during the northern summer.
e) What is a PNA pattern?
f) How is the intraseasonal oscillation on the time scale of 30 to 50 days related to the dry and wet spells of the monsoon.
✓ ? g) What is downstream amplification within the monsoon environment, sketch and describe.
h) Show a very simple sketch on the workings of a differentially heated monsoon using the three familiar covariances.

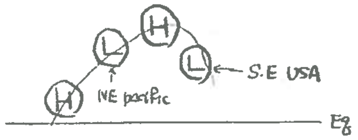
a) The Somali jet has maximum winds near 1.5 km level.
This jet is observed during summer season.
This is a well-known northern summer low-level jet.

b) No, it is located at eastern Africa.
(latitude : 13°N, maximum wind speed height : 600mb, longitude : around 0°)

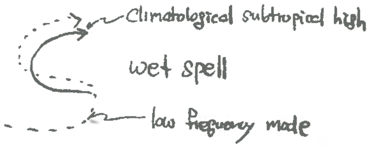
c) Yes.

d) around 400 W/m²

e) The PNA stands for Pacific-North-America.
This pattern is one of the tropical-mid-latitude teleconnections.
This is connected with ENSO events.
above 500 mb (North winter season)

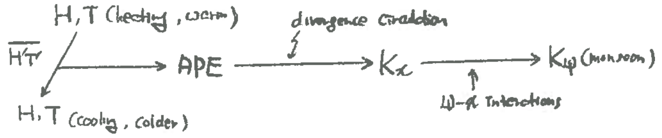


f) Intraseasonal oscillation propagate northward, if this wave interact with climatological wave, there will be a dry or wet spells.
example, China monsoon



g)
?

h)



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MET5533: Krishnamurti

- On the issue of the initial value approach to the assessment of combined instability of an easterly wave:

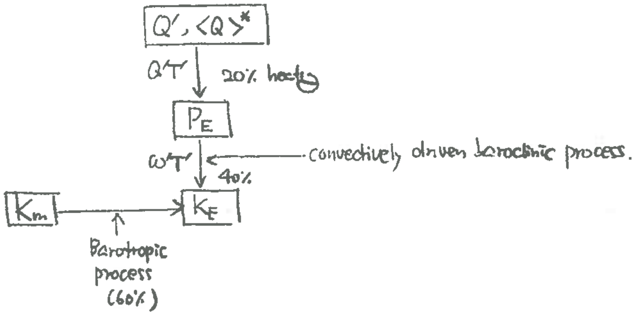
a) Describe how in principle the 10 spectral equations are obtained. (Do not derive, just state in words).

b) Given a solution to this problem how does one assess the mechanisms for the growth of an easterly wave.

a) From the linearized equations governing the perturbation motions, to obtain a numerical solution, the assumption is made that the x-dependence of each perturbation variable may be expressed as $Q' = Q_1 \sin kx + Q_2 \cos kx$, where Q_1 and Q_2 the amplitude, are functions of y, p, and t. One substitutes the above expression for u', v', w', T', ϕ' and the resulting five eqs are multiplied by either $\sin kx$ or $\cos kx$ and integrated over a wavelength ($2\pi/k$). Then a set of ten equations in the coefficients Q_1 and Q_2 is obtained.

b) We have to find out which, among $-\frac{\partial \bar{u}}{\partial y}$ (horizontal shear), $-\frac{\partial \bar{u}}{\partial p}$ (vertical shear) or Q (cumulus heating), are the most important for the growth of African waves

To do this, we have to examine the energetics



In case of combined instability, making three dimensional surfaces of αC_i (growth rate) is impossible. So we must use initial value approach

MET5534: Krishnamurti

- (40 minutes) You are given the solution of the Murrays cloud model. You are to make diagrams of vertical eddy flux of heat $\overline{W'T'}$ as a function Z and time, and of momentum $\overline{U'W'}$ as a function Z and time. Where the over bar is a zonal average, prime is a departure from zonal mean. You are then to make a diagram of the convergence of fluxes $-\frac{\partial}{\partial Z}\overline{W'T'}$ and $-\frac{\partial}{\partial Z}\overline{U'W'}$ as a function of Z and time. Write down systematically how you would use the model output and construct these diagrams. What do such diagrams mean to you?

Sol) After running the Murreys cloud model (two-dimensional), we can have outputs such as u, w and T at every grid points for different time steps.

For certain time step output, first we have to compute zonal mean of u, w and T at each model level. Using these $\overline{U}(z)$, $\overline{W}(z)$ and $\overline{T}(z)$, calculate the departure from zonal mean at every model grid points, i.e, $U'(x,z)$, $W'(x,z)$, $T'(x,z)$. And then multiply W' by U' and multiply W' by T', after that, take zonal mean.

Therefore we can calculate $\overline{U'W'}$ and $\overline{W'T'}$ at each model level, i.e., function of z. For other time step outputs, same procedure can be applied to calculate the vertical eddy flux of heat ($\overline{w'T'}$) and momentum ($\overline{u'w'}$)

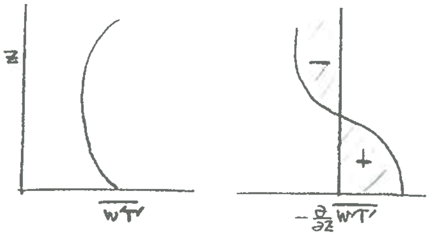
Also, using the above results, we can calculate the convergence of fluxes $-\frac{\partial}{\partial z}\overline{w'T'}$ and $-\frac{\partial}{\partial z}\overline{u'w'}$

With above results, we can easily make diagrams as a function of z and time.

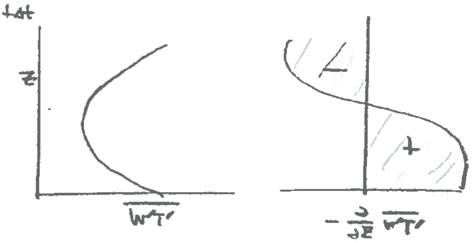
From these diagram, we can see the evolution (or changes) of flux convergence to develop cloud system environment.

For example

at t



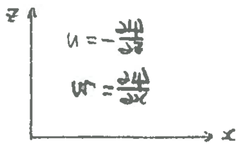
at t+Δt

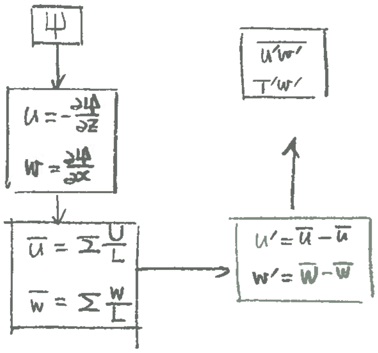


$\psi \rightarrow u, w$?

Murray used vorticity eq.: $\frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi) + \frac{g}{T_M}\frac{\partial T'}{\partial x} + \nu_M \nabla^4\psi$

$\mathbb{V} = -\frac{\partial\psi}{\partial z}\mathbb{i} + \frac{\partial\psi}{\partial x}\mathbb{k} = -\mathbb{j}\times\nabla\psi$





MET5534: Krishnamurti

- Describe the procedure for calculating the surface drag coefficient from the angular momentum budget of a hurricane. Answer the following:

a) Identify the data sets needed.
b) Basic equations needed.
c) Integration procedures needed.
d) How the drag coefficient is estimated.
e) What is the role of surface drag on the hurricane i.e. on the vorticity and on the kinetic energy (of the hurricane)?

Sol) The procedure for calculating the surface drag coefficient from the angular momentum budget of a hurricane.

The angular momentum per unit mass can be written as

$$M = V_\theta r + f_0 \frac{r^2}{2}$$

where V_θ is tangential velocity (azimuthally averaged)

Using the momentum eq, we can have, in cylindrical coordinate,

$$\frac{dM}{dt} = -g\frac{\partial z}{\partial \theta} + F_\theta r \text{ ——— ①}$$

where $\frac{dM}{dt}$ = 3-D change of angular momentum following a parcel

$-g\frac{\partial z}{\partial \theta}$ = pressure torques, ~0 in symmetric condition

$F_\theta r$ = Frictional torques (surface and cloud friction)

Eq ① can be written as flux form

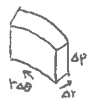
$$\frac{\partial M}{\partial t} = -\frac{\partial M V_\theta}{r\partial\theta} - \frac{1}{r}\frac{\partial}{\partial r}(M V_r r) - \frac{\partial M\omega}{\partial p} - g\frac{\partial z}{\partial\theta} + F_\theta r \text{ ——— ②}$$

which uses continuity eq.

$$\frac{\partial V_\theta}{r\partial\theta} + \frac{1}{r}\frac{\partial V_r r}{\partial r} + \frac{\partial\omega}{\partial p} = 0 \text{ ——— ③}$$

Integrating eq ② term by term, we have

$$\frac{\partial \overline{\overline{\overline{M}}}}{\partial t} = -\overline{\overline{(M_2 V_{r_2} r_2 - M_1 V_{r_1} r_1)}}^{\theta} + \overline{\overline{\overline{F_\theta}}}\,\pi(r_2^2 - r_1^2) \text{ ——— ④}$$

∵ 

$$-\frac{1}{g}\int_{p_0}^{p}\int_{r_1}^{r_2}\oint_{\theta}(\ )\, r\,d\theta\,dr\,dp$$

Also, we assumed a symmetric storm $(\frac{\partial}{\partial\theta} = 0)$

a) In equation ④, the data set needed are, (using aircraft)

i) day 1 and day 2 observations of $\frac{\partial M}{\partial t}$

ii) V_r (radial velocity), V_θ (tangential velocity) in cylindrical coord

So $\overline{\overline{\overline{F_\theta}}}$ can be deduced as a residual in eq ④

The following expression is valid for frictional torque

$$F_\theta = \underbrace{-g\frac{\partial \tau_\theta}{\partial p}}_{\text{surface friction}} \underbrace{\cancel{-gr\frac{\partial}{\partial p}\overline{u'\omega'}}}_{\text{Cloud(Atmosphere) friction}} \text{ ——— ⑤}$$

→ ignore

↳ vertical eddy flux of relative momentum by subgrid scale

where τ_θ = Surface frictional stress

$$= C_D \rho_0 V_\theta \sqrt{V_\theta^2 + V_r^2} \text{ ——— ⑥}$$

Let's ignore the second term in eq ⑤ in this procedure.

b) Hence, basic eqs needed are eqs ④ (or ①), ⑤ and ⑥

c) Integration procedures are mentioned before

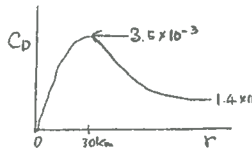
d) Eq. ⑤ can be written as finite difference form

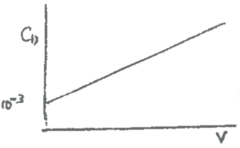
$$\overline{\overline{\overline{F_\theta}}} = -g\frac{\overline{\overline{\overline{\tau_\theta}}} - 0}{\Delta p} = -g\frac{\overline{\overline{\overline{\tau_\theta}}}}{\Delta p}$$ where Δp is the entire atmosphere.

$\overline{\overline{\overline{F_\theta}}}$ was obtained as a residual above, hence now we know $\overline{\overline{\overline{\tau_\theta}}}$

Therefore, using eq ⑥, we can solve for C_D (surface drag coefficient).

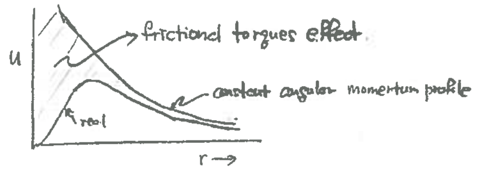
Then, we can plot C_D as functions of r or V





e) Hurricane Intensity is controled by this surface drag

This effect can be seen from below figure.



From the Ertel potential vorticity eq., the frictional torques terms are expressed,

$$\frac{d}{dt}\Big(\underbrace{-\zeta_{a\theta}\, g\frac{\partial\theta}{\partial p}}_{\text{potential vorticity}}\Big) = \cdots - \{\nabla\cdot(\underset{\sim}{F}\times k)\}\, g\frac{\partial\theta}{\partial p}.$$

where $\underset{\sim}{F} = -g\frac{\partial \tau}{\partial p}$. So, surface drag makes PV change

There is the relationship between PV and AM,

$$PV = -\frac{1}{r}\frac{\partial M}{\partial r}\, g\frac{\partial\theta}{\partial p}$$

using this, we know how Hurricane intesifies

Also, studying energetics of the Hurricane, we can find the role of surface drag on the K.E.

✓

MET5534: Krishnamurti

- Describe the heat budget of a tropical storm following the study of Kurihara and Tuleya. Answer the following in this context:

a) The heat budget equations and exploration of each term in this equation.
b) Salient terms within this framework during the formative stage of a tropical storm.
c) Mechanisms for the formation of a warm core.

Sol) Kurihara and Tuleya analyzed the growth of a warm core by the budgets of heat as follows.

a) The heat budget eq. is expressed in terms of temperature change on the moving system (a quasi-Lagrangian manner)

$$\frac{dT}{dt} = TA + TB + TC + TD + TE$$

where TA = relative horizontal thermal advection term (the ventilation effect of the relative wind)
TB = the effect of vertical advection and the dry adiabatic change
TC = the effect of condensation - convection.
TD = radiational term
TE = the effect of diffusion

b) Upper level warming is largely due to the excess of the condensation-convection heating over the cooling effect associated with the upward motion. Therefore TC is the most important term for generation of warm core. Although the diabatic heating effect of radiation (TD) plays an important role, the heating due to the condensation of water vapor is essential for the formation of a tropical storm.

c) Therefore, mechanisms for the formation of a warm core can be expressed; the warming results mainly from a delicate difference between the warming effect due to condensation-convection and the opposing cooling effect of upward motion.
In summary, the formation of a warm core is mainly due to the condensation-convection heating.

→ look at NWP note ?

MET5534: Krishnamurti

- Given the four equations for the surface similarity theory (for the unstable case) write an outline of a simple iterative fortran do loop and show how you would solve for
  - a) The Monin-Obukhov length.
  - b) Surface sensible heat flux.
  - c) Surface latent heat flux. and
  - d) The surface momentum flux.

What does one do with such products?

Sol) Four equations for the surface similarity theory for the unstable case are

$$L = u^{*2}/k\beta\theta^* \quad \text{———} ①$$

$$\frac{kz}{u^*}\frac{\partial\bar{u}}{\partial z} = \left(1-15\frac{z}{L}\right)^{-1/4} \quad \text{———} ②$$

$$\frac{kz}{\theta^*}\frac{\partial\bar{\theta}}{\partial z} = 0.74\left(1-9\frac{z}{L}\right)^{-1/2} \quad \text{———} ③$$

$$\frac{kz}{q^*}\frac{\partial\bar{q}}{\partial z} = 0.74\left(1-9\frac{z}{L}\right)^{-1/4} \quad \text{———} ④$$

where $\beta = g/\theta_0$, u^* = the friction velocity

L = Monin-Obukhov length and so on

For the unstable surface layer, $L<0$ or $Ri_B<0$

Let's write an outline of a simple iterative fortran do loop.

In other words, we have to solve for u^*, θ^*, q^* and L in below do loop.

```
   define several constants and large scale variables.
   $\zeta = 2.5$ ← From the non-dimensional flux vs $\zeta (= z/L)$
   $\Delta\zeta = 0.1$
   $L = -z/\zeta$
   DO 20 ITR =1, 1000
    eq ②
    eq ③
    eq ④
    $YL = u^{*2}/k\beta\theta^*$ ←——— eq ①
    $\zeta = \zeta - \Delta\zeta$
    $L = -z/\zeta$
   Compare L with YL, if they are close to each other,
   go out of do loop
 20 continue.
```

a) Therefore, using iterative method above, we obtained L, u^*, θ^* and q^*

b) Surface sensible heat flux (F_H) can be calculated by

$$F_H = -u^*\theta^*$$

c) Here, instead of surface latent heat flux, surface moisture flux (F_q) can be computed by

$$F_q = -u^* q^*$$

Surely, surface moisture flux can be converted to surface latent heat flux.

d) The surface momentum flux (F_M) can be solved by

$$F_M = u^{*2}$$

Therefore, we obtained Monin-Obukhov length and all fluxes.

Monin-Obukhov length can be used as stability check parameter

($L>0$: stable , $L<0$: unstable)

These fluxes are very important for studies of tropical convection and tropical disturbances, especially hurricane, the ITCZ, waves and low-level jets.

Also, for large-scale numerical weather prediction models, the exchange of energy (flux) between PBL and the atmosphere above may be more important then details within the PBL

TM I Krish

1. Zonally averaged tropical circulation
2. Zonally asymmetric features of the tropics
3. Gill's model
4. EL niño
5. Monsoon (Ψ-χ interaction)
6. Trade-wind inversion
7. African waves
8. Heat lows

TM II Krish

1. Sea Breeze and diurnal change
2. Convective related Issues
3. Planetary boundary layer fluxes
4. Simple radiation modeling, low clouds and θe
5. Hurricanes
6. Tropical squall lines in Africa
7. Orographics rainfall and convection, off shore convection

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① Zonally averaged tropical circulation

- long term averaging (over a season or month)

↳ to portray the zonally symmetric distributions of atmos. variables

$$[Q] = \frac{\oint Q\,dx}{\oint dx} \quad : \text{Zonal average}$$

$$\overline{Q} = \frac{\int_0^T Q\,dt}{T} \quad : \text{time average}$$

$[\overline{Q}] = f(y, p)$: meridional-vertical plane

- Zonal velocity $[\overline{u}]$

Over the equator → easterlies prevail from the surface up to 100 mb

Climatological westerly jet ; Strongest near 200 mb during winter

The latitude of strongest westerlies shifts from roughly 30°N during winter to roughly 45°N during Summer

vertical shear distribution

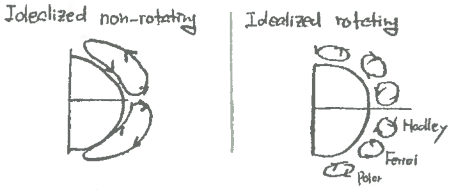
During Summer over the belt 0 to 20°N, easterlies increase with height

In this belt, during winter, easterlies increase in intensity between the surface and 850mb and decrease with altitude above that level.

Strongest tropical easterlies (fall ~ 300mb / spring ~ 700 mb

- Mean meridional circulation $[\overline{\psi}]$

⊏ Streamfunction


Idealized non-rotating | Idealized rotating

→ mass continuity eq

$$\frac{1}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{a}\frac{\partial v}{\partial \phi} - \frac{v\tan\phi}{a} + \frac{\partial \Omega}{\partial p} = 0$$

$$\left(\because \oint \left(\frac{\partial}{\partial \lambda}u\right)d\lambda = u\Big|_0^{2\pi} = 0\right)$$

$$\frac{1}{a}\frac{\partial}{\partial \phi}[\overline{v}] - \frac{[\overline{v}]\tan\phi}{a} + \frac{\partial[\overline{\Omega}]}{\partial p} = 0$$

$$\because \frac{\cos\phi}{a}\frac{\partial}{\partial \phi}[\overline{v}] - \frac{[\overline{v}]\sin\phi}{a} + \frac{\partial}{\partial p}[\overline{\Omega}]\cos\phi = 0$$

$$\frac{1}{a}\frac{\partial}{\partial \phi}\{[\overline{v}]\cos\phi\} + \frac{\partial}{\partial p}[\overline{\Omega}]\cos\phi = 0$$

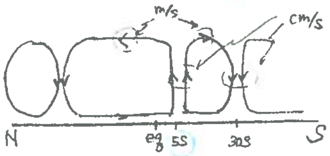
✓ $\frac{1}{a}\frac{\partial}{\partial \phi}\{[\overline{v}]\cos\phi\} + \frac{\partial[\overline{\Omega}]}{\partial p}\cos\phi = 0$

define streamfunction $[\overline{\Psi}]$ ⟶ unit kg/sec

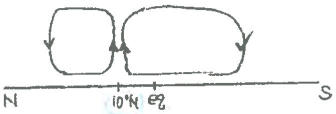
$$\frac{\partial[\overline{\Psi}]}{a\,\partial \phi} = -[\overline{\Omega}]\,\frac{2\pi a\cos\phi}{g}$$

$$\frac{\partial[\overline{\Psi}]}{\partial p} = [\overline{v}]\,\frac{2\pi a\cos\phi}{g}$$

$\langle\overline{\Psi}\rangle$

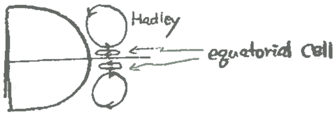


< Northern winter >



< Northern Summer >

※ 4 cell model



3 1/2 cell model

Hadley cell → thermally direct.

↳ This has important implications for the generation of zonal kinetic energy from the zonally available potential energy.

- Temperature field $[\overline{T}]$

mid-lat. : Strong meridional gradient.
tropics : lack of " "

the annual cold tropical tropopause

warm subtropical lower tropospheric temp. during the northern summer

The zonal average smooths out the land-ocean contrasts.

- Moisture field $[\overline{q}]$ ※ Virtual temperature correction

meridional gradient of moisture in the tropics is large and gives rise to a considerable meridional gradient of the virtual temperature

☆ Meridional transports by the zonally symmetric circulations

$$Q = [Q] + Q^*$$

$$Q = \overline{Q} + Q'$$

$$\because \begin{cases} [Q^*] = 0 = \overline{Q'} \\ [[A]] = [A] \\ [A^*[B]] = [A^*][B] = 0 \\ [A^*B^*] \neq 0 \end{cases}$$

mean meridional transport of Q

$$[\overline{Qv}] = [\overline{([Q]+Q^*)([v]+v^*)}]$$

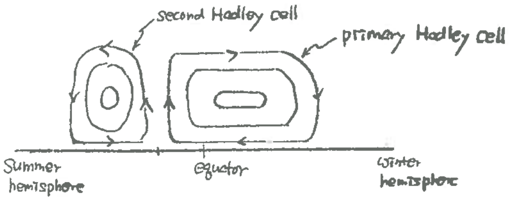
$$= [\overline{[Q][v]} + \cancel{\overline{Q^*[v]}} + \cancel{\overline{[Q]v^*}} + \overline{Q^*v^*}]$$

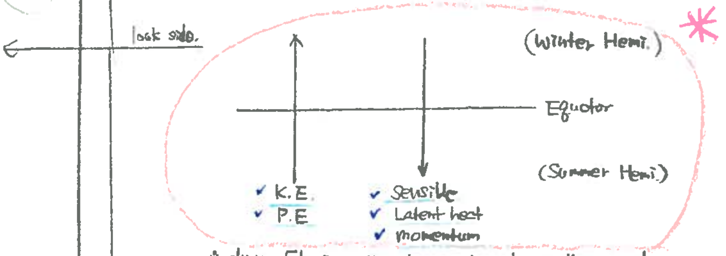
$$= \overline{[Q][v]} + [\overline{Q^*v^*}]$$

$$= \overline{([\overline{Q}]+[Q'])([\overline{v}]+[v'])} + [\overline{Q^*v^*}]$$

$$= [\overline{Q}][\overline{v}] + \overline{[Q'][v']} + [\overline{Q^*v^*}]$$

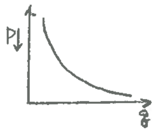
< Total transport = Transport by mean meridional circulation + Transport by transient eddies + Transport by Standing eddies >





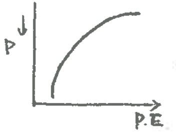
※ during Elniño : this transport is larger then usual

· moisture



: from winter to summer

· potential energy



: from summer to winter

· During the northern winter, the mean meridional circulation is fairly intense between the equator and 30°N.

✓· At the equator and 15°N, a large proportion of the transport is carried out by the mean meridional circulations

→ This is large for the sensible heat, P.E. and latent heat

✓· momentum, K.E. : transports by transient eddies begin to become large at 15°N and polewards

① Towards a theory of a Hadley cell

Study zonally symmetric motion

Governing eqs in Cartecian coord. are

$$\begin{cases} \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=0 \\ \frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial u\omega}{\partial p}-fv+\frac{\partial \Phi}{\partial x}+F_x=0 \\ \frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{\partial v\omega}{\partial p}+fu+\frac{\partial \Phi}{\partial y}+F_y=0 \\ \frac{\partial \Phi}{\partial p}=-\alpha=-\frac{R\Pi}{p}\theta \\ \frac{\partial \theta}{\partial t}+\frac{\partial \theta u}{\partial x}+\frac{\partial \theta v}{\partial y}+\frac{\partial \theta\omega}{\partial p}=\frac{\theta}{C_pT}Q=\frac{Q}{C_p\Pi} \end{cases}$$

where $\Pi=\frac{T}{\theta}=\left(\frac{p}{p_0}\right)^{R/C_p}$

Define $[A]=\frac{1}{2\pi}\int_0^{2\pi}A\,d\lambda$

Since $[v]\ll[u]$, we obtain approximate eq. describing zonally symmetric motion

$$\frac{\partial[v]}{\partial y}+\frac{\partial[\omega]}{\partial p}=0 \qquad (1)$$

$$\frac{\partial[u]}{\partial t}+\frac{\partial}{\partial y}[u][v]+\frac{\partial}{\partial p}[u][\omega]-f[v]+M=0 \;\checkmark \qquad (2)$$

$$[u]=-\frac{1}{f}\frac{\partial[\Phi]}{\partial y} \;\checkmark \qquad (3)$$

$$\frac{\partial[\Phi]}{\partial p}=-\frac{R\Pi}{p}[\theta] \qquad (4)$$

$$\frac{\partial[\theta]}{\partial t}+\frac{\partial[\theta][v]}{\partial y}+\frac{\partial[\theta][\omega]}{\partial p}-H=0 \;\checkmark \qquad (5)$$

where

$$M=+[F_x]+\left(\frac{\partial}{\partial y}[u'v']+\frac{\partial}{\partial p}[\omega'u']\right)$$

$$H=\frac{[Q]}{C_p\Pi}-\left(\frac{\partial}{\partial y}[v'\theta']+\frac{\partial}{\partial p}[\omega'\theta']\right)$$

Now, we have 3 diagonastic and 2 predict eqs

From geostrophic relation (3) and hydrostatic relation (4)

we can derive the thermal wind relation $\left(\frac{\partial}{\partial p}(3),\ \frac{\partial}{\partial y}(4)\right)$ subst.

$$\frac{\partial[u]}{\partial p}=\frac{R\Pi}{fp}\frac{\partial[\theta]}{\partial y} \qquad (6)$$

From contr. eq (1), we introduce streamfunction

$$[v]=\frac{\partial\psi}{\partial p},\quad [\omega]=-\frac{\partial\psi}{\partial y} \qquad (7)$$

Eq (2) & (5) can then be rewritten as

$$\frac{\partial[u]}{\partial t}+\frac{\partial\psi}{\partial p}\frac{\partial[u]}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial[u]}{\partial p}-f\frac{\partial\psi}{\partial p}+M=0 \qquad (8)$$

$$\frac{\partial[\theta]}{\partial t}+\frac{\partial\psi}{\partial p}\frac{\partial[\theta]}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial[\theta]}{\partial p}-H=0 \qquad (9)$$

$$\frac{\partial(8)}{\partial p}\rightarrow\cancel{\frac{\partial}{\partial p}\frac{\partial[u]}{\partial t}}+\overset{A}{\boxed{\frac{\partial^2\psi}{\partial p^2}\frac{\partial[u]}{\partial y}}}+\cancel{\frac{\partial\psi}{\partial p}\frac{\partial^2[u]}{\partial p\partial y}}\overset{B}{\boxed{-\frac{\partial^2\psi}{\partial y\partial p}\frac{\partial[u]}{\partial p}}}-\cancel{\frac{\partial\psi}{\partial y}\frac{\partial^2[u]}{\partial p^2}}\overset{A}{\boxed{-f\frac{\partial^2\psi}{\partial p^2}}}+\frac{\partial M}{\partial p}=0 \qquad (1$$

$$\frac{R\Pi}{fp}\frac{\partial(9)}{\partial y}\rightarrow\cancel{\frac{R\Pi}{fp}\frac{\partial}{\partial y}\frac{\partial[\theta]}{\partial t}}+\overset{B}{\boxed{\frac{R\Pi}{fp}\frac{\partial^2\psi}{\partial y\partial p}\frac{\partial[\theta]}{\partial y}}}+\cancel{\frac{R\Pi}{fp}\frac{\partial\psi}{\partial p}\frac{\partial^2[\theta]}{\partial y^2}}\underset{C}{\boxed{-\frac{R\Pi}{fp}\frac{\partial^2\psi}{\partial y^2}\frac{\partial[\theta]}{\partial p}}}$$

$$-\cancel{\frac{R\Pi}{fp}\frac{\partial\psi}{\partial y}\frac{\partial^2[\theta]}{\partial y\partial p}}\left(-\frac{\partial H}{\partial y}\right)=0 \qquad (1$$

(11)−(10) : by using of Eq (6),

$$\boxed{A\frac{\partial^2[\psi]}{\partial p^2}+B\frac{\partial^2[\psi]}{\partial y\partial p}+C\frac{\partial^2[\psi]}{\partial y^2}=\frac{\partial M}{\partial p}+\frac{\partial H}{\partial y}} \qquad ($$

momentum forcing

differential heating forcing

↳ Kuo-Eliassen equation

$$[\psi]\sim\underset{75\%}{\left(-\frac{\partial H}{\partial y}\right)},\ \underset{25\%}{\left(\frac{\partial M}{\partial p}\right)}$$

forcing by mid-latitudes

Internal tropical forcing (meridional gradient heat source)

where

$$A=f-\frac{\partial[u]}{\partial y}$$

$$B=2\frac{\partial[u]}{\partial p}$$

$$C=-\frac{R\Pi}{fp}\frac{\partial[\theta]}{\partial p}$$

B.C. : $[\psi]=0$ at $y=0$ and L

Discussion

① If no forcing term, $\frac{\partial H}{\partial y}=0$, $\frac{\partial M}{\partial p}=0$

when $B^2-4AC>0$ → Eq(12) is elliptic equation

The free convection is possible only if $B^2-4AC\leq 0$

② If there is forcing term, we have to simplify the coefficient A.B,C, Solve nonhomogeneous PDE, get ψ field and discuss the control factor of the Hadley cell.

※ Kuo-Eliassen eq. for zonally symmetric circulations

$$A\frac{\partial^2\psi}{\partial y^2}+2B\frac{\partial^2\psi}{\partial y\partial p}+C\frac{\partial^2\psi}{\partial p^2}=\frac{\partial H}{\partial y}+\frac{\partial M}{\partial p}$$

※ compare with above eq

$A=-\frac{1}{\rho\theta}\frac{\partial\theta}{\partial p}$: static stability (depth of circulation)

$B^2-AC>0$

$B=f\frac{\partial u}{\partial p}=\frac{1}{\rho\theta}\frac{\partial\theta}{\partial y}$: baroclinity → tilt of circulation

via thermal wind

$C=f^2-f\frac{\partial u}{\partial y}$: Inertial stability parameter.

A and B are usually regarded as positive definite quantities. The sign of C is in general positive although in the vicinity of strong anticyclonic flows as well as in the proximity of the equator its sign can often be negative.

⇒ Hadley cell is enhanced by

- a) a meridional gradient of the heat source $\left[\frac{\partial H}{\partial y}<0\right]$
- b) divergence of eddy flux of westerly momentum by the middle latitude waves at the boundaries of the tropical Hadley cell
- c) vertical convergence of flux of easterly momentum over the tropical upper troposphere

momentum forcing

$$M = \underbrace{fF_x}_{\text{frictional force}} + f\Big[\underbrace{\frac{\partial}{\partial y}[u'v']}_{\text{meridional eddy convergence of westerly momentum}} + \underbrace{\frac{\partial}{\partial p}[u'\omega']}_{\text{vertical convergence of westerly eddy momentum}}\Big]$$

East-West Circulation

- Rotational wind has no vertical motions.
- Divergent wind describes all up + down motion.
- $\mathbf{V}_H = \mathbf{V}_{Rot} + \mathbf{V}_{DIV} = \mathbf{V}_\psi + \mathbf{V}_\chi$

$\mathbf{V}_\psi = -\mathbf{k}\times\nabla\psi$ (rotational wind)

$\mathbf{V}_\chi = -\nabla\chi$ (divergent wind)

→ meridional momentum eq

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}uv + \frac{\partial}{\partial y}v^2 + \frac{\partial}{\partial p}v\omega + fu + g\frac{\partial Z}{\partial y} + F_y = 0 \quad \text{——} \quad (1)$$

define $[A] = \dfrac{1}{y_2-y_1}\displaystyle\int_{y_1}^{y_2} A\,dy$

$$\frac{\partial}{\partial t}[v] + \frac{\partial}{\partial x}[vu] + (v_2^2 - v_1^2) + \frac{\partial}{\partial p}[v\omega] + f[u] + g(Z_2 - Z_1) + [F_y] = 0$$

or

✓ $$\frac{\partial}{\partial t}[v] + \frac{\partial}{\partial x}[u][v] + \frac{\partial}{\partial p}[v][\omega] + \frac{\partial}{\partial x}[u'v'] + \frac{\partial}{\partial p}[v'\omega'] + f[u] + [F_y]$$
$$= -(v_2^2 - v_1^2) - g(Z_2 - Z_1) \quad \text{——} \quad (2)$$

Similarly, thermodynamic eq →

✓ $$\frac{\partial}{\partial t}[\theta] + \frac{\partial}{\partial x}[\theta][u] + \frac{\partial}{\partial p}[\theta][\omega] + \frac{\partial}{\partial x}[\theta'v'] + \frac{\partial}{\partial p}[\theta'\omega']$$
$$= \frac{\theta}{C_pT}[Q] - (\theta_2 v_2 - \theta_1 v_1) \quad \text{——} \quad (3)$$

$\because$ (θ/T is function of pressure only)

Let $\begin{cases} A = -(v_2^2 - v_1^2) - g(Z_2 - Z_1) \\ B = -(\theta_2 v_2 - \theta_1 v_1) \end{cases}$

$\dfrac{\partial}{\partial p}(2) \rightarrow$

✓ $$\frac{\partial}{\partial t}\frac{\partial}{\partial p}[v] + \frac{\partial^2}{\partial p\partial x}[v][u] + \frac{\partial^2}{\partial p^2}[v][\omega] + \frac{\partial}{\partial p}\Big[\frac{\partial}{\partial x}[v'u'] + \frac{\partial}{\partial p}[v'\omega']\Big]$$
$$+ f\frac{\partial}{\partial p}[u] + \frac{\partial}{\partial p}[F_y] = \frac{\partial A}{\partial p} \quad \text{——} \quad (4)$$

$\dfrac{\partial}{\partial x}(3) \rightarrow$

✓ $$\frac{\partial}{\partial t}\boxed{\frac{\partial}{\partial x}[\theta]} + \frac{\partial^2}{\partial x^2}[\theta][u] + \frac{\partial^2}{\partial x\partial p}[\theta][\omega] + \frac{\partial}{\partial x}\Big(\frac{\partial}{\partial x}[\theta'v'] + \frac{\partial}{\partial p}[\theta'\omega']\Big)$$
$$= \frac{\theta}{C_pT}\frac{\partial}{\partial x}[Q] + \frac{\partial}{\partial x}B \quad \text{——} \quad (5)$$

$\because$ Since $v_g = \dfrac{g}{f}\dfrac{\partial Z}{\partial x}$ ← geostrophic wind

$$\text{so } \frac{\partial v_g}{\partial p} = \frac{g}{f}\frac{\partial}{\partial x}\Big(\frac{\partial Z}{\partial p}\Big) \overset{\text{hydrostatic eq}}{=} -\frac{g}{f}\frac{\partial}{\partial x}\Big(\frac{1}{\rho g}\Big) \overset{\text{state eq}}{=} -\frac{R}{fp}\frac{\partial}{\partial x}T \overset{\text{Poisson eq}}{=} -\frac{R}{fp}\frac{\partial}{\partial x}\Big[\theta\underbrace{\Big(\frac{p}{p_0}\Big)^{\frac{R}{C_p}}}_{\to \frac{T}{\theta}}\Big]$$

$$= -\frac{R}{fp}\frac{T}{\theta}\frac{\partial\theta}{\partial x}$$

$$\frac{\partial[v_g]}{\partial p} = -\frac{R}{fp}\frac{T}{\theta}\frac{\partial[\theta]}{\partial x}$$

$$\frac{\partial[v_g]}{\partial p} = -d\frac{\partial[\theta]}{\partial x} \quad (\because \text{Let } d = \frac{R}{fp}\frac{T}{\theta}) \quad \text{——} \quad (6)$$

(6) → (5)

$$-\boxed{\frac{\partial}{\partial t}\frac{\partial}{\partial p}[v_g]} + d\frac{\partial^2}{\partial x^2}[\theta][u] + d\frac{\partial^2}{\partial x\partial p}[\theta][\omega] + d\frac{\partial}{\partial x}\Big(\frac{\partial}{\partial x}[\theta'v'] + \frac{\partial}{\partial p}[\theta'\omega']\Big)$$
$$= \frac{d\theta}{C_pT}\frac{\partial}{\partial x}[Q] + d\frac{\partial}{\partial x}B$$

using eq. (4)

$$-\Big(-\frac{\partial^2}{\partial p\partial x}[v][u] - \frac{\partial^2}{\partial p^2}[v][\omega] - \frac{\partial}{\partial p}\Big(\frac{\partial}{\partial x}[u'v'] + \frac{\partial}{\partial p}[v'\omega']\Big)$$
$$- f\frac{\partial}{\partial p}[u] - \frac{\partial}{\partial p}[F_y] + \frac{\partial A}{\partial p}\Big)$$
$$+ d\frac{\partial^2}{\partial x^2}[\theta][u] + d\frac{\partial^2}{\partial x\partial p}[\theta][\omega] + d\frac{\partial}{\partial x}\Big(\frac{\partial}{\partial x}[\theta'v'] + \frac{\partial}{\partial p}[\theta'\omega']\Big)$$
$$= \frac{d\theta}{C_pT}\frac{\partial}{\partial x}[Q] + d\frac{\partial B}{\partial x}$$

rearranging

$$\boxed{\frac{\partial^2}{\partial p\partial x}[v][u] + \frac{\partial^2}{\partial p^2}[v][\omega]} + f\frac{\partial}{\partial p}[u] + d\frac{\partial^2}{\partial x^2}[\theta][u] + d\frac{\partial^2}{\partial x\partial p}[\theta][\omega]$$
$$= -\frac{\partial}{\partial p}\Big(\frac{\partial}{\partial x}[v'u'] + \frac{\partial}{\partial p}[v'\omega']\Big) - d\frac{\partial}{\partial x}\Big(\frac{\partial}{\partial x}[\theta'v'] + \frac{\partial}{\partial p}[\theta'\omega']\Big)$$
$$- \frac{\partial}{\partial p}[F_y] + \frac{d\theta}{C_pT}\frac{\partial}{\partial x}[Q] + \frac{\partial A}{\partial p} + d\frac{\partial B}{\partial x} \quad \text{——} \quad (7)$$

Further rearranging,

$\because$
$$d\frac{\partial^2}{\partial x^2}[\theta][u] = d\frac{\partial}{\partial x}\Big(\frac{\partial}{\partial x}[\theta][u]\Big) = d\frac{\partial}{\partial x}\Big([\theta]\frac{\partial}{\partial x}[u] + [u]\frac{\partial}{\partial x}[\theta]\Big)$$
$$d\frac{\partial^2}{\partial x\partial p}[\theta][\omega] = d\frac{\partial}{\partial x}\Big([\theta]\frac{\partial}{\partial p}[\omega] + [\omega]\frac{\partial}{\partial p}[\theta]\Big)$$

Summing

$$d\frac{\partial^2}{\partial x^2}[\theta][u] + d\frac{\partial^2}{\partial x\partial p}[\theta][u] = d\frac{\partial}{\partial x}\Big([\theta]\Big(\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega]\Big)$$
$$+ [u]\frac{\partial}{\partial x}[\theta] + [\omega]\frac{\partial}{\partial p}[\theta]\Big)$$

Similarly

$$\frac{\partial^2}{\partial p\partial x}[v][u] + \frac{\partial^2}{\partial p^2}[v][\omega] = \frac{\partial}{\partial p}\Big([v]\Big(\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega]\Big) + [u]\frac{\partial}{\partial x}[v] + [\omega]\frac{\partial}{\partial p}[v]\Big)$$

Let $\begin{cases} H = -d\Big(\dfrac{\partial}{\partial x}[\theta'v'] + \dfrac{\partial}{\partial p}[\theta'\omega']\Big) + \dfrac{d\theta}{C_pT}[Q] \\ F = -\Big(\dfrac{\partial}{\partial x}[v'u'] + \dfrac{\partial}{\partial p}[v'\omega'] + [F_y]\Big) \end{cases}$

$$\frac{\partial}{\partial p}\Big([v]\Big(\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega]\Big) + [u]\frac{\partial}{\partial x}[v] + [\omega]\frac{\partial}{\partial p}[v]\Big) + f\frac{\partial}{\partial p}[u]$$
$$+ d\frac{\partial}{\partial x}\Big([\theta]\Big(\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega]\Big) + [u]\frac{\partial}{\partial x}[\theta] + [\omega]\frac{\partial}{\partial p}[\theta]\Big)$$
$$= \frac{\partial F}{\partial p} + \frac{\partial H}{\partial x} + \frac{\partial A}{\partial p} + d\frac{\partial B}{\partial x} \quad \text{——} \quad (8)$$

$\because$ From conti. eq →

$$\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega] = (v_2 - v_1)$$

using thermal wind

$$d\frac{\partial}{\partial x}\Big([u]\frac{\partial}{\partial x}[\theta]\Big) + \frac{\partial}{\partial p}\Big([u]\frac{\partial}{\partial x}[v]\Big) = d\frac{\partial}{\partial x}[u]\frac{\partial}{\partial x}[\theta] + \frac{\partial}{\partial p}[u]\frac{\partial}{\partial x}[v]$$
$$d\frac{\partial}{\partial x}\Big([\omega]\frac{\partial}{\partial p}[\theta]\Big) + \frac{\partial}{\partial p}\Big([\omega]\frac{\partial}{\partial p}[v]\Big) = d\frac{\partial}{\partial x}[\omega]\frac{\partial}{\partial p}[\theta] + \frac{\partial}{\partial p}[\omega]\frac{\partial}{\partial p}[v]$$

(8) →

$$\frac{\partial}{\partial p}\Big([v]\Big(\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega]\Big)\Big) + d\frac{\partial}{\partial x}\Big([\theta]\Big(\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega]\Big)\Big)$$
$$+ d\frac{\partial}{\partial x}[u]\frac{\partial}{\partial x}[\theta] + \frac{\partial}{\partial p}[u]\frac{\partial}{\partial x}[v] + d\frac{\partial}{\partial x}[\omega]\frac{\partial}{\partial p}[\theta] + \frac{\partial}{\partial p}[\omega]\frac{\partial}{\partial p}[v] + f\frac{\partial}{\partial p}[u]$$
$$= \frac{\partial F}{\partial p} + \frac{\partial H}{\partial x} + \frac{\partial A}{\partial p} + d\frac{\partial B}{\partial x} \quad \text{——} \quad (9)$$

$\because$ Since y-averaged mass continuity eq is

$$\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega] = v_2(x,p) - v_1(x,p) = G(x,p)$$

And if G is a continuous function, then

$$\frac{\partial}{\partial x}[u] + \frac{\partial}{\partial p}[\omega] = \frac{\partial}{\partial x}\int_{x_0}^{x} G(\lambda,p)\,d\lambda$$

or $$\frac{\partial}{\partial x}\Big([u] - \int_{x_0}^{x} G(\lambda,p)\,d\lambda\Big) + \frac{\partial}{\partial p}[\omega] = 0$$

if we define $[u'] = [u] - \underbrace{\displaystyle\int_{x_0}^{x} G(\lambda,p)\,d\lambda}_{G'} \longrightarrow [u] = [u'] + G'$

then $\dfrac{\partial}{\partial x}[u'] + \dfrac{\partial}{\partial p}[\omega] = 0$

$$f\frac{\partial}{\partial p}[u] + d\frac{\partial}{\partial x}[u]\frac{\partial}{\partial x}[\theta] + \frac{\partial}{\partial p}[u]\frac{\partial}{\partial x}[v] = d\frac{\partial}{\partial x}[u']\frac{\partial}{\partial x}[\theta] + \frac{\partial}{\partial p}[u']\frac{\partial}{\partial x}[v]$$
$$+ d\frac{\partial G'}{\partial x}\frac{\partial}{\partial x}[\theta] + \frac{\partial G'}{\partial p}\frac{\partial}{\partial x}[v] + f\frac{\partial G'}{\partial p} + f\frac{\partial}{\partial p}[u']$$

(9) →

$$d\frac{\partial}{\partial x}[u']\frac{\partial}{\partial x}[\theta] + \frac{\partial}{\partial p}[u']\frac{\partial}{\partial x}[v] + d\frac{\partial}{\partial x}[\omega]\frac{\partial}{\partial p}[\theta] + \frac{\partial}{\partial p}[\omega]\frac{\partial}{\partial p}[v] + f\frac{\partial}{\partial p}[u']$$
$$= \frac{\partial F}{\partial p} + \frac{\partial H}{\partial x} + \frac{\partial A}{\partial p} + d\frac{\partial B}{\partial x} - \frac{\partial}{\partial p}([v]G) - d\frac{\partial}{\partial x}([\theta]G)$$
$$- d\frac{\partial G'}{\partial x}\frac{\partial}{\partial x}[\theta] - \frac{\partial G'}{\partial p}\frac{\partial}{\partial x}[v] - f\frac{\partial G'}{\partial p}$$

$\because$ (defining streamfunction $\quad [u'] = \dfrac{\partial\psi}{\partial p}\ ;\ [\omega] = -\dfrac{\partial\psi}{\partial x}$)

The LHS →

$$\alpha\frac{\partial^2\psi}{\partial x\partial p}\frac{\partial}{\partial x}[\theta] + \frac{\partial^2\psi}{\partial p^2}\frac{\partial}{\partial x}[v] - \alpha\frac{\partial^2\psi}{\partial x^2}\frac{\partial}{\partial p}[\theta] - \frac{\partial^2\psi}{\partial x\partial p}\frac{\partial}{\partial p}[v] + f\frac{\partial^2\psi}{\partial p^2}$$

$$= -\alpha\frac{\partial}{\partial p}[\theta]\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial}{\partial p}[v]\frac{\partial^2\psi}{\partial x\partial p} + \left(f + \frac{\partial}{\partial x}[v]\right)\frac{\partial^2\psi}{\partial p^2}$$

$$\therefore\ \boxed{\begin{aligned} & -\alpha\frac{\partial}{\partial p}[\theta]\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial}{\partial p}[v]\frac{\partial^2\psi}{\partial x\partial p} + \left(f+\frac{\partial}{\partial x}[v]\right)\frac{\partial^2\psi}{\partial p^2} \\ & = \frac{\partial F}{\partial p} + \frac{\partial H}{\partial x} \\ & \quad + \frac{\partial A}{\partial p} + \alpha\frac{\partial B}{\partial x} \\ & \quad - \frac{\partial}{\partial p}([v]G) - \alpha\frac{\partial}{\partial x}([\theta]G) \\ & \quad - \alpha\frac{\partial G'}{\partial x}\frac{\partial}{\partial x}[\theta] - \frac{\partial G'}{\partial p}\frac{\partial}{\partial x}[v] - \frac{f\partial G'}{\partial p} \end{aligned}}$$

Zonally asymmetric features of tropics

- seasonal or monthly mean weather maps
- climatological charts in middle latitudes do not display the transient disturbances.
- both daily and monthly mean charts over the tropics
  - subtropical highs
  - equatorial troughs
  - monsoon troughs
  - trades of the two hemispheres

Climatological means carry much of the variance of the total motion field in the tropics. Thus an understanding of the maintenance of the time averaged zonally asymmetric feature of the tropics is important.

- Gradient level winds (≈ 1km level) (low-level)

→ The principal asymmetric features

(1) The subtropical highs
(2) The tropical convergence zones
(3) The southeast and northwest trade wind systems
(4) The Asian monsoon flows of the summer and winter seasons
(5) The monsoon trough of the summer season
(6) The heat lows over the deserts
(7) The cross-equatorial flow over the Indian Ocean – Arabian Sea during northern summer (≈ 35 knots)
(8) The slightly stronger intensity of the trades over the winter hemisphere
(9) A strong zonal asymmetry in the location of the vortices and stream-line convergence zones (near the ITCZ) over the oceanic and land areas

- motion field in the upper troposphere (≈ 200mb) (upper-level)

Salient features of the winter season

– subtropical westerly jet stream : a quasi-stationary 3-wave pattern
↳ latitude ≈ 27°N

① The Tibetan and west African high pressure areas
② The mid-Pacific trough
③ The mid-Atlantic trough
④ The tropical easterly jet over Asia and equatorial Africa
⑤ The Mexican high

The zonal asymmetry of 200mb during northern summer

- Temperature field

land-ocean distributions.
Summer : land warmer than ocean
winter : ocean 〃 〃 land

near the Earth's surface : the sensible heat flux
near the 200mb 〃 : deep convective and subsidence warming

The amplitude of the zonal asymmetry decreases rapidly with height. The phase reverses above the tropical tropopause, the troughs becoming warm and the ridges becoming cold in the lower stratosphere.

East/west circulations in the tropics

Time-averaged east-west circulations are essentially "divergent motion"
↳ also Hadley.

$\underset{\sim}{V} = \underset{\sim}{V}_\psi + \underset{\sim}{V}_\chi$ → $\underset{\sim}{V}_\chi$: divergent part, $\underset{\sim}{V}_\psi$: rotational part

$\overline{\underset{\sim}{V}_\chi} = -\nabla\overline{\chi}$: time mean velocity potential $\overline{\chi}$

Intensity of the Hadley and east/west circulations

$$I_H = -\frac{1}{L}\oint\frac{\partial\overline{\chi}}{\partial y}dx \longrightarrow \text{varies along } y$$

$$I_E = -\frac{1}{y_2-y_1}\int_{y_1}^{y_2}\frac{\partial\overline{\chi}}{\partial x}dy \longrightarrow \text{varies along } x$$

$$\nabla^2\overline{\chi} = -\nabla\cdot\overline{\underset{\sim}{V}} \longrightarrow \text{obtain } \overline{\chi}$$

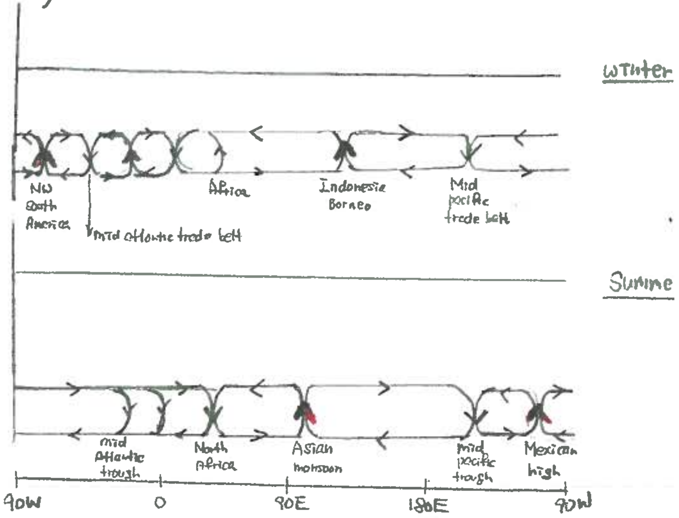
⇒ draw the geometry of $\overline{\chi}$ and typical streamlines of the divergent part of the moti

→ divergent circulations are present in the zonal as well as the meridional planes. *(upper-level)*

Northern Summer : The major center of the east/west circulations is found on the northern part of the Bay of Bengal.
The divergent outflow regions are the region of the Asian summer monsoons and the Pacific coast of southern Mexico.

Northern winter : The major center → Indonesia-Borneo
the northwestern part of South America, central Africa and Indonesia.

- Intensity of east/west circulation



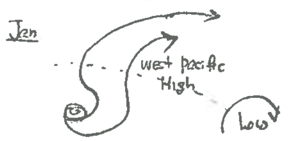
- These vertical circulations are capable of generating eddy kinetic energy on the scales of these circulations.
- Tropical planetary scale quasi-stationary waves acquire kinetic energy by this process

- moisture field (850mb)

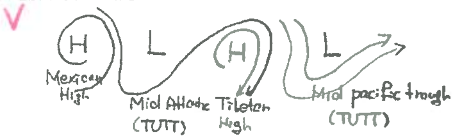
During the northern winter
– 3 regions of large specific humidity
(northwestern part of South America, equatorial Africa, Indonesia) → look at above figu
During the northern summer
– 2 regions
(monsoon region, equatorial eastern Pacific) →

The largest values of specific humidity are found over the land areas and not over the oceanic tropics. This is related to larger temperatures of land areas which can hold large amounts of moisture prior to saturation, the monsoon belts being the moistest in this sense

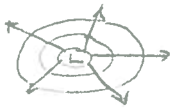
- Sea-level pressure
- Other parameters
  - Sea-surface temperature
  - Height of base and of the top of the trade wind inversion
  - Monthly mean cloud amounts
  - Satellite digital cloud brightness charts
  - Orography, mountain heights
  - Albedo of the Earth's surface
  - Tropical monthly rainfall
  - Monthly mean total solar radiation reaching the Earth's surface
  - Net OLR
  - Monthly mean net solar radiation absorbed by the troposphere
- 200mb streamlines

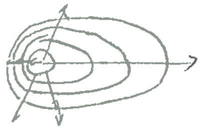


N. Summer Northern Hemisphere (high level)

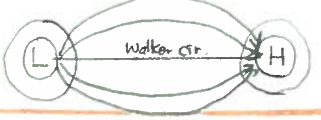


- Velocity potential (200mb)





In meteorology, take into account all directions of Walker Cir

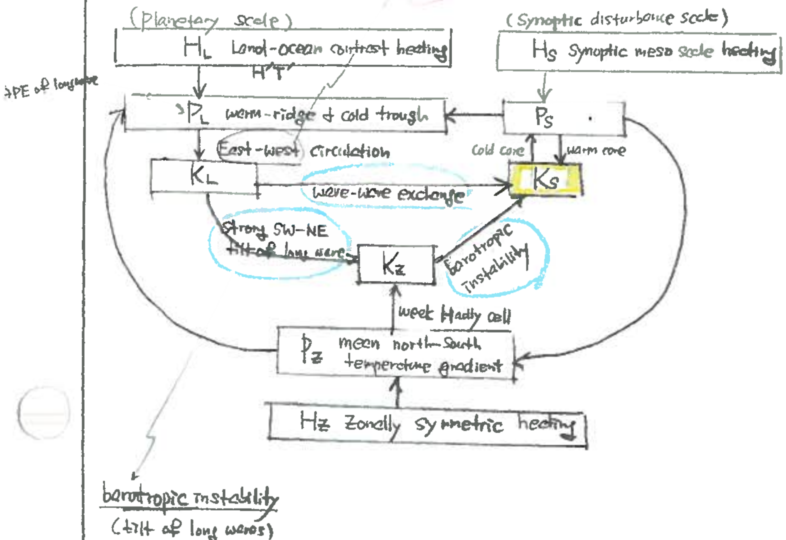


Tropical general circulation, Northern Summer

How does Ks get the K.E. from the other processes?

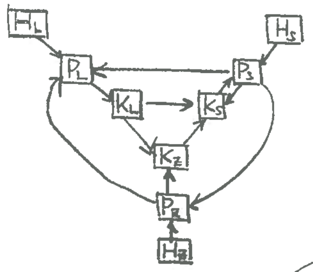
where
L : long
S : short
Z : Zonal

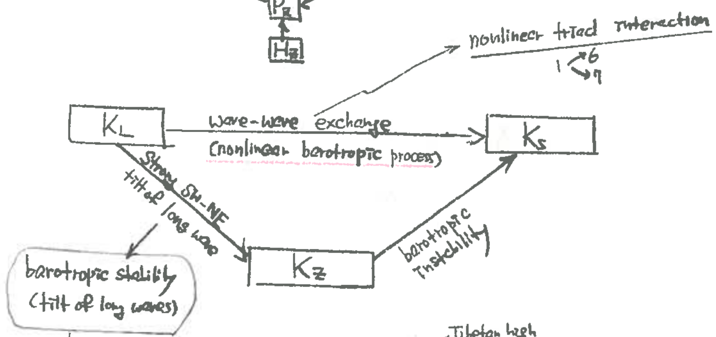
$(\frac{\overline{W'T'}}{P})$

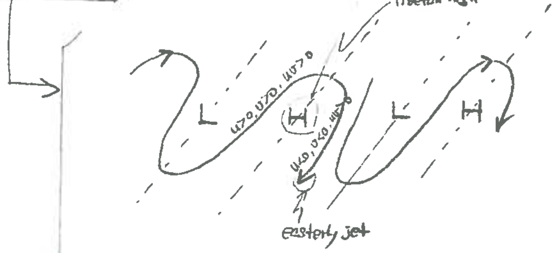


< Tropical Energetics >

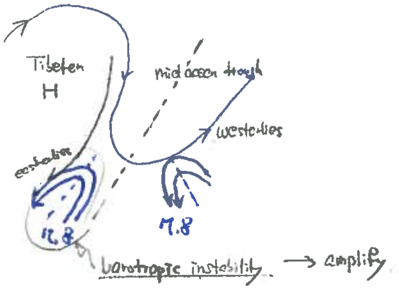
* draw it again?







SW to NE tilt, you are transporting westerly momentum northward



barotropic instability → amplify

Covariances

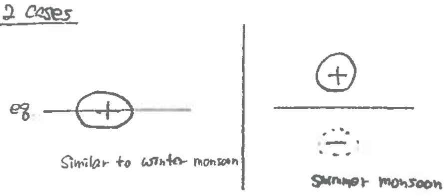
1. vertical velocity & temp $-\frac{\overline{W'T'}}{P}$: warm air rising relatively cold air sinking >0
2. diabatic heating & temp. $\overline{H'T'}$: heating where it is warm relatively cooling where it is cold >0
3. Covariance of flux of westerly momentum in a region of Zonal easterlie eddy motions will goes barotropicity

$$\frac{\partial R_E}{\partial t} = +\overline{[U]}\frac{\partial}{\partial y}\overline{[U'V']}$$

- Zonally asymmetric, ultralong wave heating (H_L) plays an important role in generating potential energy (P_L) on the scale of these long waves; furthermore Zonally asymmetric ascending and descending motions most likely should account for a generation of kinetic energy on the scale of the long waves (K_L), i.e., a transfer of energy such that $H_L \longrightarrow P_L \longrightarrow K_L$
- We can conjecture on the energetics of tropical atmospheric motions as ~~follows~~ Synoptic scale eddies are formed as a result of barotropic instability or CISK and these obtain part of their kinetic energy from the zonal flow and from the quasi-stationary long waves. The kinetic energy of the zonal flow is replenished by a transfer of kinetic energy from the long waves and by the conversion of zonal available potential energy to zonal kinetic energy due to the Hadley circulatio The long waves, in their turn, have their K.E replenished by the conversion of APE by the large-scale east-west overturnings. The potential energy of the long waves is in part supplied by the synoptic scale waves, which implies that these smaller

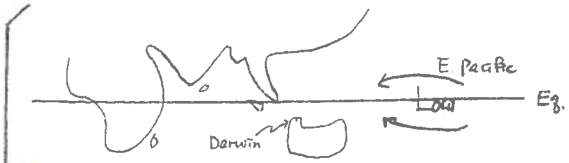
- heat-induced tropical circulation → Gill's model

→ 2 cases



※ refer to other notes

① El Niño and Southern Oscillation



- Sequence of events (or scenario)

1. Low-High pressure anomalies
2. Cyclonic winds around the Low in each hemisphere leads to an enhancement of trades.
3. Piling of water over the western (equatorial) ocean.
   ※ Slacking of the trades (westerly-wind anomaly)
4. Initiation of an equatorial Kelvin wave
   (eastward propagation (in the ocean) of a trapped equatorial Kelvin wave)
5. East-west differential in the structure of the thermocline.
   (Thermocline overturning)
6. Reflection of the Kelvin wave as a Rossby-gravity wave
7. Initiation of the Elnino warming.
   (Initiation of warm water, coastal east Pacific)
8. Spread of the warm water anomaly toward the central equatorial Pacific
8'. Shift of the east-west circulation
9. Enhancement of deep cumulus convection over the equatorial Pacific (equatorial convection)
10. Maximum amplitude of the trade winds.
11. Setting up of a Pacific-North-American climate Pattern
12. Floods and Droughts over the tropics of beyond

- Southern Oscillation (SO)

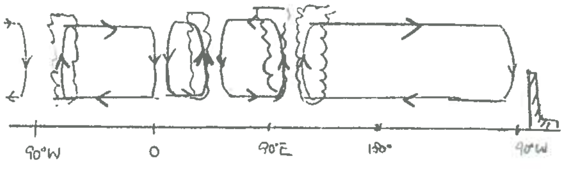
→ When pressure is high in the Pacific Ocean it tends to be low in the Indian ocean from Africa to Australia: these conditions are associated with low temperature in both these areas and rainfall varies in the opposite direction to pressure.

Index of the SO <SOI> : the anomaly of the pressure difference between Tahiti and Darwin (Tahiti − Darwin)

Below normal SOI → above normal SST.

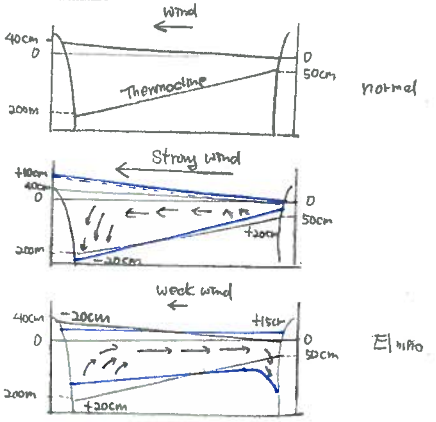
- Walker Circulation

Large-scale zonal, west-east overturning in the equatorial plane.

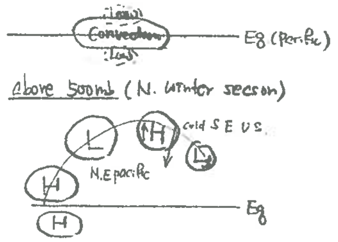


→ Large scale tropical convection acts as a major driving force of the atmospheric circulation through the release of latent heat in the mid-troposphere (diabatic heat source).

- Thermocline



- PNA pattern



Mechanism of the PNA

① Stationary Rossby wave as seen from shallow water eqs.
  → advective nonlinearities

② Full climate model
  → Baroclinic dynamics account for the vorticities of the lows + highs of PNA whereas the advective nonlinears tend to kill this system (PNA)

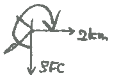
- African drought
- westerly wind burst.
- Zebiak and cane article

② Monsoon

- Winter Monsoon

period : Nov. 15 − Mar. 15

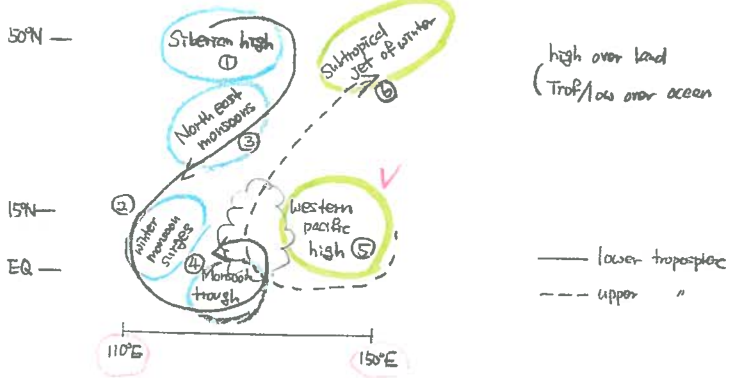
→ depth : shallow (~2km)
veers with height



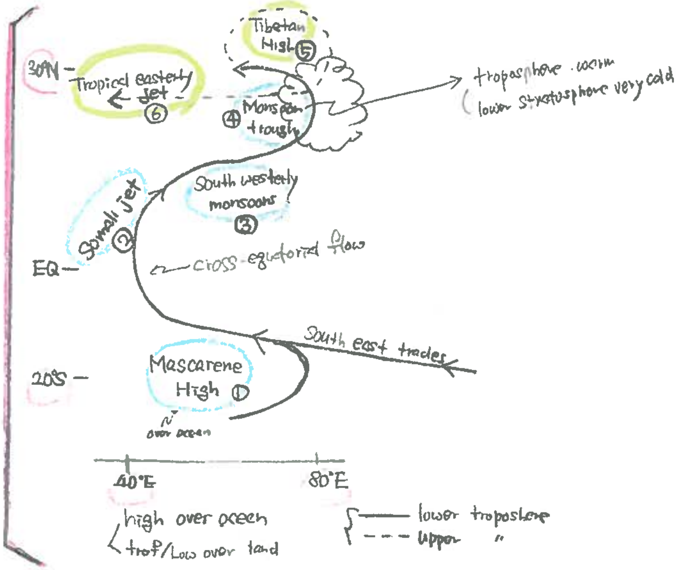
- Important question for Monsoon

Where is heating and cooling?
Where is the differential heating?

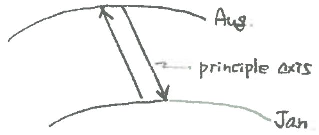
- Winter monsoon (elements of the monsoon)



★ Summer monsoon



- annual cycle of the monsoon



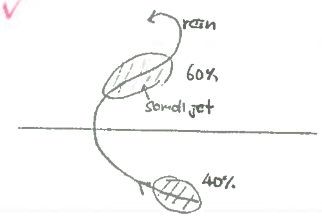
- Latent heat flux

※ Bowen ratio : the ratio of sensible to latent heat fluxes at the surface

$$= \frac{\text{Sensible heat flux}}{\text{Latent heat flux}} = 0.01$$

→ larger values of latent heat flux
E. of Somali jet
Souther Hemi Tradewind belt.

✓



Where's moisture from?
60% of moisture comes from N.H
40% of " " " S.H

★ SST for Monsoon

Water Cooling ← wind speed is impotent element
29°C ⟶ 26°C
↳ It takes just 3~4 days

- onset - active - Break Monsoon

- Dominant scales of oscillations of the Monsoon

1. Diurnal
2 4 to 6 days Monsoon lows (1004mb)
direction : westwerd or North-westward
speed : 5 to 7° longitude/day
weak weather : 2"/day
3 Quasi biweekly is 15 days to 20 days
also the time scale of monsoon depression
3000km scale E-W
2000km scale N-S
passage is from east to west.
from 5° to 10° long. /day
precip. : 5"-8"/day

4. 30 to 50 days oscillation (mostly in lower troposphere)
(Madden-Julian Time scale)

① Meridionally propagating over Asia ⟶ lower troposphere
Scale : 3000km
Troughs + ridges lines are east-west elongated
propagates at the rate of 1° lat /day
Originate near 5°S

② Planetary scale divergent wave (mostly upper troposphere)
wave #s 1 + 2
go west to east around the globe in rougly 40 days

- dry + wet spells of the Monsoon are very closely related to the passage of these elongated troughs and ridges.

* How to extract 30 to 50 days oscillation from u, v data? ← at 850mb
daily data (90days)

→ use "time filter"
need digitized data @ grid points
Apply 30-50 day band pass filter of u, v ⇒ we want low freq. field.
make a map.

5. Elniño Time scale

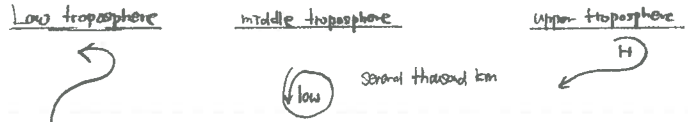
4 to 6 years
El niño years : found to be years with below normall rainfall
In general El nino year : less than normal monsoon rainfall
La nina year : more " " " "

6. Decadal time Scale

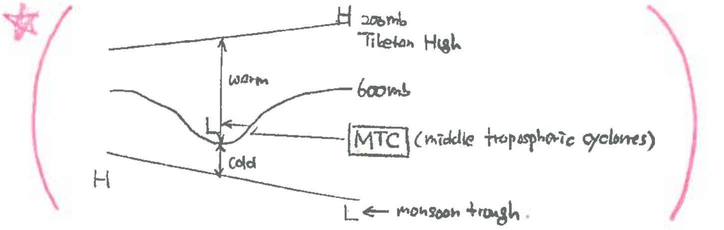
50 ~ 60 years
ex) 1950 1980
↑ above normal ↑ below normal

- Monsoon depression is tilted Southward with height.

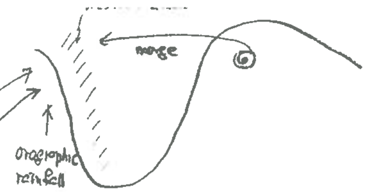
- Mid-tropospheric cyclones of the monsoon



rain producer
6" to 8"/day
most seen in ① Arabian Sea coast
② Southern Indochina
③ around N. Australia
It must have
cold core below : evap. of falling rain + adiabatic asce
warm core aloft : downdrafts



- These MTC's may form from debris or remnents of Bay of Bengal depression.
- Mesoconvective organization sustains mid-tropospheric disturbance.
- Orographic rainfall coalesces with remnants of depression to maintain organization,



- **Theory of formation**

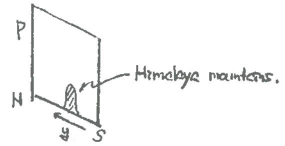
  Cyclonic circulation dies out in the lower levels;
  mid-level cyclone remains.

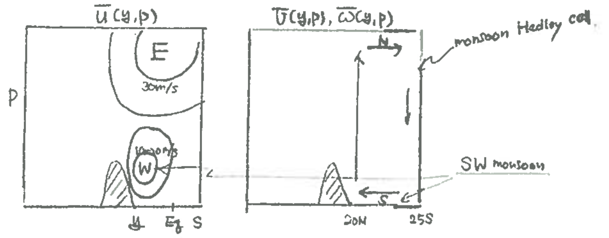
Maintenance of a Monsoon System by differential heating!

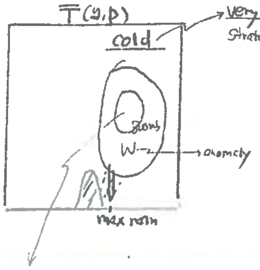
- **How does differential heating drive the monsoon?**

  ↳ using an inequality argument

- From zonally symmetric climate model (no $\frac{\partial}{\partial x}$ term in model)







very cold : reason for cold stratosphere

Stratosphere (※ Stratospheric fountain)

① overshooting cloud tops
(if we define these belong to the stratosphere)

② evaporation of water

latent heat release

↓

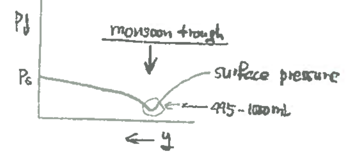
downward motion (sensible heat)

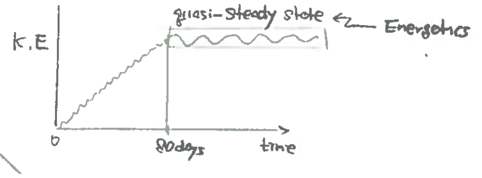
↓

produce warm core (in tropics)

warm (near) tropopause

On the average, the warmest average do not occur at the updrafts where latent heating is maximized but in areas of downdraft adjacent to the updrafts where the latent heating air is further heated by compression





★ < differential heating ⟶ monsoon >

proof

In the absence of ① heat sources & sinks (G), ② Friction (F_ψ, F_χ) ③ boundary fluxes (B_ψ, B_χ)

$$\longrightarrow \frac{\partial}{\partial t}(K_\psi + K_\chi + P) = 0$$

* Steady-state and no boundary flux

$$\overline{\langle K_\chi \cdot K_\psi \rangle} - \overline{D_\psi} = 0 \quad \text{———} \; ①$$

$$-\overline{\langle K_\chi \cdot K_\psi \rangle} + \overline{\langle P \cdot K_\chi \rangle} - \overline{D_\chi} = 0 \quad \text{———} \; ②$$

$$-\overline{\langle P \cdot K_\chi \rangle} + \overline{G} - \overline{D_P} = 0 \quad \text{———} \; ③$$

look below (6), (7), & (8)

This leads to the Inequality argument

① → There must be a dissipation of rotational K.E
hence $\overline{D_\psi} > 0$, hence $\overline{\langle K_\chi \cdot K_\psi \rangle} > 0$
hence energy must go from divergent to the rotational motion

② → $-\overline{D_\chi}$ has to be negative (<0)
since $-\overline{\langle K_\chi \cdot K_\psi \rangle}$ is negative (<0)
hence $\overline{\langle P \cdot K_\chi \rangle}$ has to be positive (>0)

③ → Dissipation of APE > 0
we know that $-\overline{\langle P \cdot K_\chi \rangle} < 0$
hence $G > 0$

∴ G has to be positive to maintain the Monsoon in steady-state.

$$\left(\because G = \int_m H'T' \, d\underset{\uparrow \text{mass}}{m}\right)$$

※ Let's look at details

- **Zonally symmetric monsoon**

Zonal eq of motion

$$\frac{\partial u}{\partial t} = -v\frac{\partial u}{\partial y} - \omega\frac{\partial u}{\partial p} + fv + F_x \quad \text{———} \; (1)$$

meridional eq of motion

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial y} - \omega\frac{\partial v}{\partial p} - fu - g\frac{\partial z}{\partial y} + F_x \quad \text{———} \; (2)$$

Eq of continuity

$$\frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \quad \text{———} \; (3)$$

∵ { v : entirely divergent
u : entirely rotational

First law of thermodynamics

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial y} - \omega\frac{\partial T}{\partial p} - \frac{RT}{C_p p}\omega + \sum \frac{H_i}{C_p} \quad \text{———} \; (4)$$

all kinds of heating

Moisture conservation

$$\frac{\partial q}{\partial t} = -v\frac{\partial q}{\partial y} - \omega\frac{\partial q}{\partial p} + E - P_r \quad \text{———} \; (5)$$

※ Rotational wind has no vertical motions.
Divergent wind describe all up & down motion.

$$\underset{\sim}{V}_H = \underset{\sim}{V}_\psi + \underset{\sim}{V}_\chi$$

$$\begin{cases} \underset{\sim}{V}_\psi = -\mathbf{k} \times \nabla\psi & \text{(rotational wind)} \\ \underset{\sim}{V}_\chi = -\nabla\chi & \text{(divergent wind)} \end{cases}$$

⟹ energetics of the above system.

∵ $K_\psi = \frac{u^2}{2}$, $K_\chi = \frac{v^2}{2}$

$\langle K_\psi \cdot K_\chi \rangle$: energy exchange from the rotational to the divergent K.E.

- Zonal kinetic energy (rotational kinetic energy)

$$\frac{\partial K_\psi}{\partial t} = \langle K_\chi \cdot K_\psi \rangle - D_\psi + B_\psi \quad \text{———} \; (6)$$

boundary flux term

dissipation

- Meridional kinetic energy (divergent K.E)

$$\frac{\partial K_\chi}{\partial t} = -\langle K_\chi \cdot K_\psi \rangle + \langle P \cdot K_\chi \rangle - D_\chi + B_\chi \quad \text{———} \; (7)$$

APE

$$\because \langle P \cdot K_\chi \rangle = \int_m -\frac{\omega' T'}{p} \, dm$$

- Internal plus potential energy

$$\frac{\partial P}{\partial t} = -\langle P \cdot K_\chi \rangle + G - D_P + B_P \quad \text{———} \; (8)$$

↳ Generation (differential heating)

$$\because \int (\;)\, dm = \frac{1}{g}\int_p \int_y (\quad)\, dy\, dp$$

- The maintenance of a statistically steady monsoon

(Inequality statements)

a) In a statistical steady state, $\frac{\partial}{\partial t}K_\psi$ is small.

$-D_\psi < 0 \longrightarrow \langle K_\chi \cdot K_\psi \rangle > 0$

b) In a statistical steady state, $\frac{\partial}{\partial t}K_\chi$ is small

$-D_\chi < 0$, $-\langle K_\chi \cdot K_\psi \rangle < 0 \longrightarrow \langle P \cdot K_\chi \rangle > 0$.

c) $-D_P < 0$, $-\langle P \cdot K_\chi \rangle < 0 \longrightarrow G > 0$

∴ there must be a net generation of energy G to maintain the system.

G : differential heating
(covariance of the diabatic heating rate H and the temperature)

⇒ the maintenance of a steady state monsoon $(\frac{\partial}{\partial t}K_\psi \approx 0.0)$ requires a net barotropic energy exchange $\langle K_\chi \cdot K_\psi \rangle$, which in turn requires a net Hadley-type overturning $\langle P \cdot K_\chi \rangle$ and is facilitated by a differential heating G that drives this system.

→ This system describes several features of the monsoon

⟨Southern trades, the Mascarene high, cross-equatorial flow, the southwest monsoon, the monsoon trough, the monsoon rains near 20°N, the warm troposphere above the monsoon trough, the Tibetan high, the tropical easterly jet system⟩

- 3-D differential heating that drives the monsoon

ψ–χ interactions framework

Vorticity eq → no pressure gradient term (no solenoidal term)

$$\frac{\partial \zeta}{\partial t} + V_H \cdot \nabla(\zeta + f) + \omega\frac{\partial \zeta}{\partial p} + \underbrace{(\zeta + f)\nabla \cdot V_H}_{\text{divergent term}}$$

$$+ \underbrace{k \cdot \nabla\omega \times \frac{\partial V}{\partial p}}_{\text{tilting term}} = F_\zeta \qquad \text{①}$$

Divergence eq

$$\frac{\partial D}{\partial t} + \nabla\cdot(V_H \cdot \nabla V_H) + \nabla\cdot\left(\omega\frac{\partial V}{\partial p}\right) + \nabla(f k \times V_H) + \nabla^2\Phi = F_D \qquad \text{②}$$

First law of thermodynamics

$$\frac{\partial}{\partial t}C_pT + V_H \cdot \nabla C_pT + \omega S = G + D_T \qquad \text{③}$$

where $S = \text{stability} = C_p\frac{\partial T}{\partial p} - \alpha$

$G = \text{Generation}$

$D_T = \text{dissipation}$

Continuity eq

$$D + \frac{\partial\omega}{\partial p} = 0 \qquad \text{④}$$

Hydrostatic law

$$\alpha + \frac{\partial\Phi}{\partial p} = 0 \qquad \text{⑤}$$

Definition

$V = V_\psi + V_\chi$

where $V_\psi = k \times \nabla\psi$; $u_\psi = -\frac{\partial\psi}{\partial y}$; $v_\psi = \frac{\partial\psi}{\partial x}$

$V_\chi = -\nabla\chi$; $u_\chi = -\frac{\partial\chi}{\partial x}$; $v_\chi = -\frac{\partial\chi}{\partial x}$

$\zeta = k \cdot \nabla \times V = k \cdot \nabla \times V_\psi = \nabla^2\psi$ ∵ $\nabla \times V = \zeta k$

$D = \nabla \cdot V_\chi = -\nabla^2\chi$

The total K. E. per unit mass (K) is

$$K = \frac{1}{2}V_H \cdot V_H = \frac{1}{2}(V_\psi + V_\chi)\cdot(V_\psi + V_\chi)$$

$$= \frac{1}{2}(V_\psi \cdot V_\psi) + \frac{1}{2}(V_\chi \cdot V_\chi) + V_\psi \cdot V_\chi$$

$$= \frac{1}{2}|\nabla\psi|^2 + \frac{1}{2}|\nabla\chi|^2 - J(\psi, \chi)$$

where $J(\psi,\chi) \equiv (k \times \nabla\psi)\cdot\nabla\chi$

$$K_\psi \equiv \frac{1}{2}|\nabla\psi|^2, \quad K_\chi \equiv \frac{1}{2}|\nabla\chi|^2$$

So, $K = K_\psi + K_\chi - J(\psi, \chi)$

∵ tilting term

$$k \cdot \nabla\omega \times \frac{\partial V}{\partial p} = k \cdot \nabla\omega \times \frac{\partial V_\psi}{\partial p} + k \cdot \nabla\omega \times \frac{\partial V_\chi}{\partial p}$$

∵ vector identity $(A \cdot B \times C = B \cdot C \times A)$

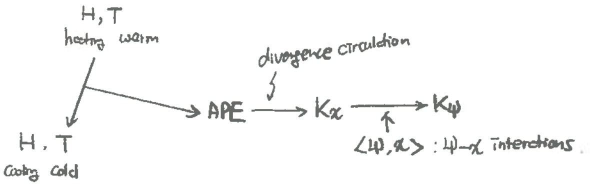
$$= \nabla\omega \cdot \frac{\partial V_\psi}{\partial p} \times k + \nabla\omega \cdot \frac{\partial V_\chi}{\partial p} \times k$$

$$= \nabla\omega \cdot \frac{\partial}{\partial p}\underbrace{(k \times \nabla\psi) \times k}_{\nabla\psi} + \nabla\omega \cdot \frac{\partial}{\partial p}(-\nabla\chi) \times k$$

$$= \nabla\omega \cdot \nabla\frac{\partial\psi}{\partial p} + \nabla\omega \cdot k \times \nabla\frac{\partial\chi}{\partial p}$$

$$= \nabla\omega \cdot \nabla\frac{\partial\psi}{\partial p} + J\left(\frac{\partial\chi}{\partial p}, \omega\right)$$

continue ○○○○○ (생략요)

- 

$$\frac{\partial}{\partial t}K_\psi = \langle K_\chi \cdot K_\psi \rangle + D_\psi$$

4 terms (ψ, χ)

$$\frac{\partial}{\partial t}K_\chi = \langle APE \cdot K_\chi \rangle - \langle K_\chi \cdot K_\psi \rangle + D_\chi$$

$-\frac{\omega' T'}{P}$ H'T'

$$\frac{\partial}{\partial t}APE = -\langle APE \cdot K_\chi \rangle + G + D_{APE}$$

⇒ we can use "inequality argument" like 2-D case.

Full eq ψ–χ interactions

$V = k \times \nabla\psi - \nabla\chi$

$$\overline{\overline{Q}} = \frac{\frac{1}{g}\iiint Q\,dx\,dy\,dp}{\frac{1}{g}\iiint dx\,dy\,dp}$$

most important. ψ–χ interactions

$$\frac{\partial}{\partial t}\overline{\overline{K_\psi}} = \overline{\overline{f\nabla\psi\cdot\nabla\chi}} + \overline{\overline{\nabla^2\chi\frac{|\nabla\psi|^2}{2}}} + \overline{\overline{\nabla^2\psi\nabla\psi\cdot\nabla\chi}} + \overline{\overline{\omega J\left(\psi, \frac{\partial\chi}{\partial p}\right)}} + \overline{\overline{D_\psi}} + B_\psi$$

$$= \underbrace{\langle K_\chi \cdot K_\psi \rangle}_{\text{so}(>0)} + \underbrace{D_\psi}_{(<0)} + B_\psi \qquad \overline{\overline{\frac{R}{P}\omega T}}$$

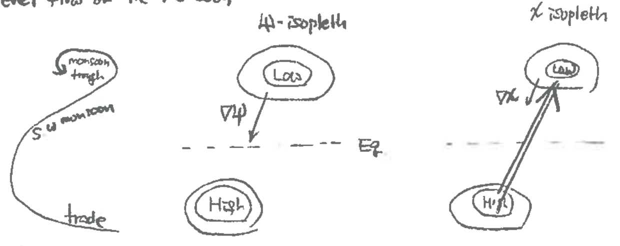
$$\frac{\partial}{\partial t}\overline{\overline{K_\chi}} = -\overline{\overline{\chi\nabla^2\Phi}} - \boxed{\psi\text{–}\chi \text{ interactions}} + \overline{\overline{D_\chi}} + B_\chi$$

$$= \underbrace{-\langle K_\chi \cdot K_\psi \rangle}_{(<0)} + \underbrace{\langle APE \cdot K_\chi \rangle}_{\text{so}(>0)} + \underbrace{D_\chi}_{(<0)} + B_\chi$$

$$\frac{\partial}{\partial t}APE = \overline{\overline{\chi\nabla^2\Phi}} + \overline{\overline{G}} + \overline{\overline{D_{APE}}} + B_{APE} \qquad -\overline{\overline{\frac{R}{P}\omega T}}$$

$$= \underbrace{-\langle APE \cdot K_\chi \rangle}_{(<0)} + \underbrace{G}_{\text{so}(>0)} + \underbrace{D_{APE}}_{(<0)} + B_{APE}$$

*Low level flow of the monsoon



⇒ "alignment of ∇ψ & ∇χ" is very important!

$\nabla\psi\cdot\nabla\chi \neq 0$

• Intraseasonal oscillation of the monsoon

time scale : 30 to 50 days.

Madden-Julian time scales

- SLP
- precipitation anomalies
- OLR anomalies
- 200mb divergence field

→ Monsoonal intraseasonal oscillation swings N & S. a little while propagating zonally.

Band pass filter (Butterworth filter)

filtered weather maps on the time scale of 30 to 50 days

planetary scale SLP wave that propagates from west to east roughly 40 days.

• If we have SLP data, how do we pull out the Madden-Julian time scale?

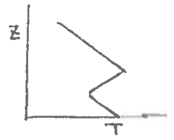
- time series data of SLP in western Pacific.
- applied band pass filter to each time series seperately.
- Make filtered weather maps on the 30-50 day time scale by projecting the phase
- Planetary scale SLP wave that propagate W to E around the world in 30-50 days
- They did the same thing with precip. anomalies & OLR anomalies on that time scale

* look at other notes !

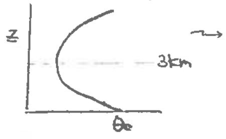
ⓔ The trade-wind inversion

• Inversion & conditional instability over the tropics

Temp inversion



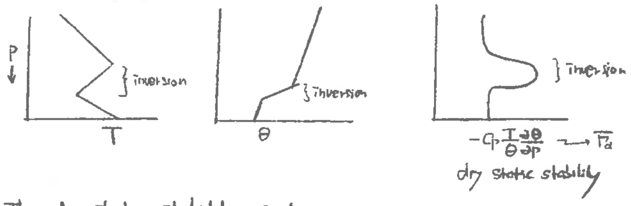
Convective instability
→ conditional



→ nearly everywhere almost continously exist.

$\frac{\partial}{\partial t}$(feature) = horizontal advection of the feature
+ vertical " " " "
+ Sources maintaining the feature
− sinks destroying the feature

Trade wind inversion
Monsoon inversion



• The dry static stability equations

$$\boxed{C_p\frac{I}{\theta}\frac{d\theta}{dt} = \sum H_i} \quad \text{——— (1)}$$

↳ First law of thermo.

$$\frac{\partial\theta}{\partial t} = -\nabla\cdot\theta V - \frac{\partial}{\partial p}\omega\theta + \frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\sum H_i \quad \text{——— (2)}$$

↳ flux form using conti. eq & poisson's eq.

⟨ Applying an averaging $\overline{(\;)}$ over a scale larger than the cumulus scale but smaller than that of the synoptic-scale systems. ⟩

$\therefore -\nabla\cdot\overline{\theta V} - \frac{\partial}{\partial p}\overline{\theta\omega}$

assume $\overline{\theta V} = \overline{\theta}\,\overline{V} + \cancel{\overline{\theta' V'}}$ → small
$\overline{\theta\omega} = \overline{\theta}\,\overline{\omega} + \overline{\theta'\omega'}$

$-\nabla\cdot\overline{\theta}\,\overline{V} - \frac{\partial}{\partial p}\overline{\theta}\,\overline{\omega} - \frac{\partial}{\partial p}\overline{\theta'\omega'}$

$\because$ Conti. eq. $\nabla\cdot\overline{V} + \frac{\partial\overline{\omega}}{\partial p} = 0$

$= -\overline{V}\cdot\nabla\overline{\theta} - \overline{\omega}\frac{\partial}{\partial p}\overline{\theta} - \frac{\partial}{\partial p}\overline{\theta'\omega'}$

$$\frac{\partial\overline{\theta}}{\partial t} = -\overline{V}\cdot\nabla\overline{\theta} - \overline{\omega}\frac{\partial\overline{\theta}}{\partial p} - \frac{\partial}{\partial p}\overline{\omega'\theta'} + \frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\sum_i\overline{H_i}$$

$\because \boxed{\overline{\Gamma_d} \equiv -\frac{\partial\overline{\theta}}{\partial p}}$: usually positive

look back ?

$$\frac{\partial\overline{\Gamma_d}}{\partial t} = \underbrace{-\overline{V}\cdot\nabla\overline{\Gamma_d}}_{\text{horiz adv.}} \underbrace{-\overline{\omega}\frac{\partial\overline{\Gamma_d}}{\partial p}}_{\text{vertical adv.}} + \underbrace{\frac{\partial^2}{\partial p^2}\overline{\omega'\theta'}}_{\text{turbulent mixing flux}} \underbrace{-\overline{\Gamma_d}\frac{\partial\overline{\omega}}{\partial p}}_{\text{Subsidence inversion effect}} \underbrace{-\frac{\partial}{\partial p}\left[\frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\sum\overline{H_i}\right]}_{\text{heating term}}$$

just convey

turbulent mixing flux ↳ elevating the inversion

⟹ **Dry static stability eq.**

where $\sum_i H_i = H_{sen} + H_{Rad} + H_{COND} + H_{evap}$

H_{sen} ↑ sensible heat

H_{evap} → only cloud evaporation is included here

H_{COND} → the rate of condensating heat non-precipitating clouds is set to zero

H_{Rad}, H_{evap} → Sharpening the inversion

$-\overline{\Gamma_d}\frac{\partial\overline{\omega}}{\partial p}$: divergence term ← most important term

positive below 500mb

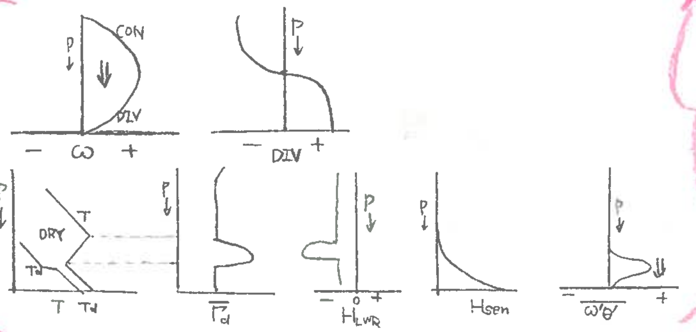
This term will produce inversions everywhere in the tropics.

$\overline{\Gamma_d}$ → generally positive.

$-\frac{\partial\overline{\omega}}{\partial p} > 0$: subsidence

→ Largest source term that generates $\overline{\Gamma_d}$ over eastern oceans is

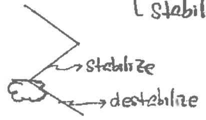
$-\overline{\Gamma_d}\frac{\partial\overline{\omega}}{\partial p}$



→ Large scale divergence → stabilize $\frac{\partial\overline{\Gamma_d}}{\partial t} > 0$

$-\overline{\Gamma_d}\frac{\partial\overline{\omega}}{\partial p} > 0$

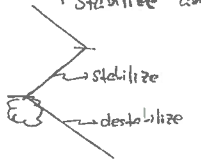
Cloud top radiative cooling { destabilize below inversion base $\frac{\partial\overline{\Gamma_d}}{\partial t} < 0$
stabilize above inversion base $\frac{\partial\overline{\Gamma_d}}{\partial t} > 0$



Sensible heat flux → destabilize above surface $\frac{\partial\overline{\Gamma_d}}{\partial t} < 0$

Vertical eddy flux of heat → destabilize below inversion base $\frac{\partial\overline{\Gamma_d}}{\partial t} < 0$

Cloud evaporation { destabilize below inversion base $\frac{\partial\overline{\Gamma_d}}{\partial t} < 0$
Stabilize above inversion base $\frac{\partial\overline{\Gamma_d}}{\partial t} > 0$



⇒ diagnostic approach with ECMWF data sets

① Parameterize $\sum_i H_i$

② use large scale data to calculate all bar terms

③ Calculate $\overline{\omega'\theta'}$ by solving $\frac{\partial^2\overline{\omega'\theta'}}{\partial p^2} = F(p)$ as a second order ordinary differential equation between surface and inversion top

④ Time average each term in three layers

- i) below inversion
- ii) inversion layer
- iii) above inversion

Make maps of each term. Maps will show geographical source region where dry static stability is large.

- $\overline{\Gamma_d}$ is almost always positive. $\times \cdot \nabla \cdot \overline{V} + \frac{\partial \overline{\omega}}{\partial p} = 0$ $\quad \frac{\partial \overline{\Gamma_d}}{\partial t} = -\overline{\Gamma_d}\frac{\partial \overline{\omega}}{\partial p}$

↓ motion (low-level divergence) $-\frac{\partial \overline{\omega}}{\partial p}$ is positive

dry static stability is increased, $\overline{\Gamma_d}$ increases.

where? in subtropical highs + associated with monsoons

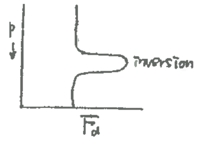
For this term alone, inversion will be infinitesimally small near the sfc of the ocean.

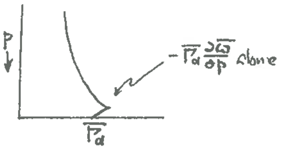
We usually see inversions km's above ocean. So, we have to examine the other terms.

- Examine the turbulent mixing term alone

Important term which explains the lifting of the inversion layer ✓

$$\frac{\partial \overline{\Gamma_d}}{\partial t} = \frac{\partial^2}{\partial p^2}(\overline{\omega'\theta'}) \rightarrow \text{destabilize below the inversion base.}$$



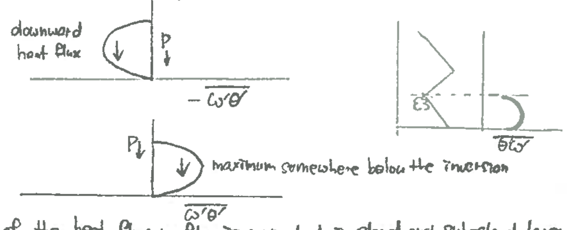


What is $\overline{\omega'\theta'}$?

Upward heat flux $= -\overline{\omega'\theta'}$

We must have a heat flux so that it kills the inversion right near the sfc + produce an inversion 2-3km up.

Intuition



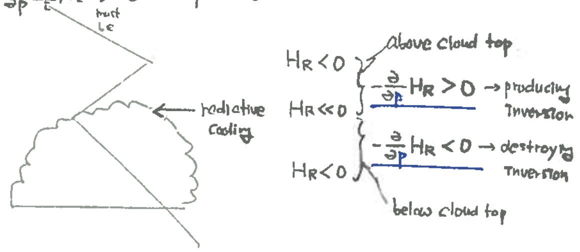
Shape of the heat flux profile is important in cloud and sub-cloud layer.

- Examine heating terms

What are they doing to the inversion?
→ $H_{RAD} + H_{EVAP}$

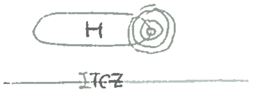
$$-\frac{\partial}{\partial p}\left[\underbrace{\frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}}_{\text{take outside}}\sum_i H_i\right] \approx -\frac{1}{C_p}\left(\frac{P_0}{P}\right)^{R/C_p}\frac{\partial}{\partial p}\sum_i H_i$$

$$-\frac{\partial}{\partial p}\sum_i H_i \underset{\text{must be}}{>} 0 \text{ to produce an inversion}$$



Hevap acts much the same way.

※ main occurrence region



- moist static stability ($\overline{\Gamma_m}$)

The moist static stability eq. (= eq. for conditional instability)

$$\frac{d}{dt}(gZ + C_pT + Lq) = LE_B + H_R + H_{sen}$$

(surface moisture flux) (radiative processes) (flux of sensible heat of the lower layer)

$$\frac{d}{dt}E_m = LE_B + H_R + H_{sen}$$

$$\frac{\partial \overline{E_m}}{\partial t} = -\overline{V}\cdot\nabla\overline{E_m} - \overline{\omega}\frac{\partial}{\partial p}\overline{E_m} - \frac{\partial}{\partial p}\overline{\omega' E_m'} + L\overline{E_B} + \overline{H_R} + \overline{H_{sen}}$$

$$\frac{\partial \overline{\Gamma_m}}{\partial t} = \overset{①}{-\overline{V}\cdot\nabla\overline{\Gamma_m}} \overset{②}{- \overline{\omega}\frac{\partial}{\partial p}\overline{\Gamma_m}} \overset{③}{+ \frac{\partial^2}{\partial p^2}\overline{\omega' E_m'}} - \frac{\partial}{\partial p}\{\overset{⑥}{L\overline{E_B}} + \overset{⑤}{\overline{H_R}} + \overline{H_{sen}}\}$$
$$-\overline{\Gamma_m}\frac{\partial \omega}{\partial p} ④$$

note that condensation or cloud evaporation is not included here.

⇒ this occurs every tropics.

Term ① Horiz. adv. of moist static stability at 1 level
must also consider differential adv. at different levels

Term ② vert. adv. of moist static stability

Term ③ Turbulent mixing, largely from deep cumulus convection
erodes instability ⇒ stabilizing effect. (deep cumulus convection stabilize)

$$\frac{\partial \overline{\Gamma_m}}{\partial t} = \frac{\partial^2}{\partial p^2}\overline{\omega' E_m'} \qquad \overline{\Gamma_m} = -\frac{\partial}{\partial p}(gZ + C_pT + Lq)$$

Below 700 mb $\overline{\Gamma_m} < 0$
Above 700 mb $\overline{\Gamma_m} > 0$

Stabilizing ⇒ want to make $\overline{\Gamma_m}$ less negative.

so $\frac{\partial \overline{\Gamma_m}}{\partial t}$ must be > 0 (positive) more stable

so we want $\frac{\partial^2}{\partial p^2}\overline{\omega' E_m'} > 0$

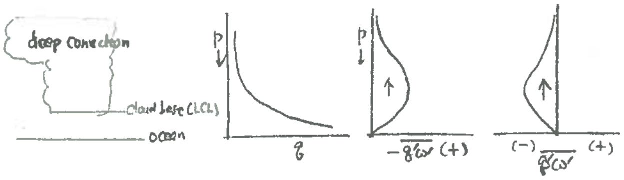
what is E_m'

$$E_m' = \underbrace{gZ' + C_pT'}_{\text{very small}} + Lq' \quad \text{↳ large + more important}$$

$$\therefore \frac{\partial^2}{\partial p^2}\overline{E_m'\omega'} \approx L\frac{\partial^2}{\partial p^2}\overline{q'\omega'}$$

$$\frac{\partial \overline{\Gamma_m}}{\partial t} = \frac{\partial^2}{\partial p^2}\overline{E_m'\omega'} \approx L\frac{\partial^2}{\partial p^2}\overline{q'\omega'} \quad ✓$$

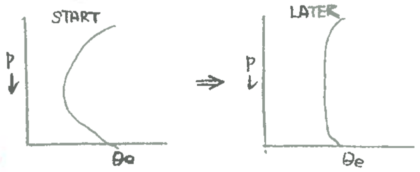
Definition : $-\overline{q'\omega'}$ is upward energy flux of moisture



⇒ deep cumulus convection erodes conditional instability.

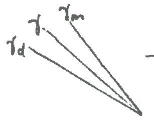
For deep convective regions.

Hence $\frac{\partial \overline{\Gamma_m}}{\partial t} > 0 \longrightarrow$ stabilizing effect.



- Conditional instability → How to produce or destroy?
- Basic math to develop $\overline{\Gamma_m}$ eq.

→ definition of stability (conditional instability)



$$\rightarrow \begin{cases} -\frac{\partial \theta}{\partial p} > 0 \\ -\frac{\partial \theta_e}{\partial p} < 0 \end{cases} \quad \text{or} \quad \begin{cases} -\frac{\partial}{\partial p}(gZ + C_pT) > 0 \\ -\frac{\partial}{\partial p}(gZ + C_pT + Lq) < 0 \end{cases}$$

Identity (유도! prelim exam!)

$$\begin{cases} -C_p\frac{T}{\theta}\frac{\partial \theta}{\partial p} \approx -\frac{\partial}{\partial p}(gZ + C_pT) \\ -C_p\frac{T}{\theta}\frac{\partial \theta_e}{\partial p} \approx -\frac{\partial}{\partial p}(gZ + C_pT + Lq) \end{cases}$$

INL

adiabatic motion

$$\frac{d\theta}{dt} = 0 \quad : \text{exact}$$

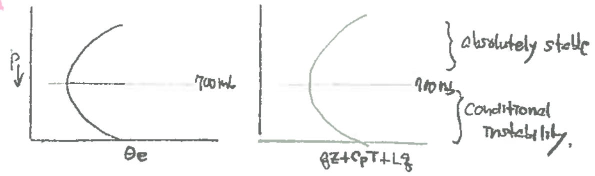
$$\frac{d}{dt}(gz + C_pT) = 0$$

↳ approximation (inexact)

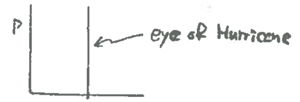
moist adiabatic motion

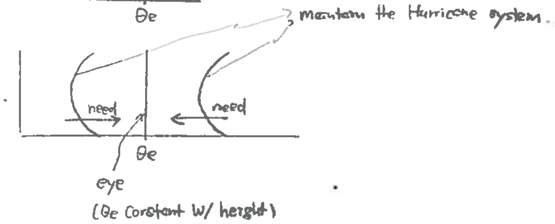
$$\frac{d\theta_e}{dt} = 0$$

$$\frac{d}{dt}(gz + C_pT + Lq_s) = 0$$

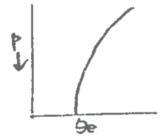


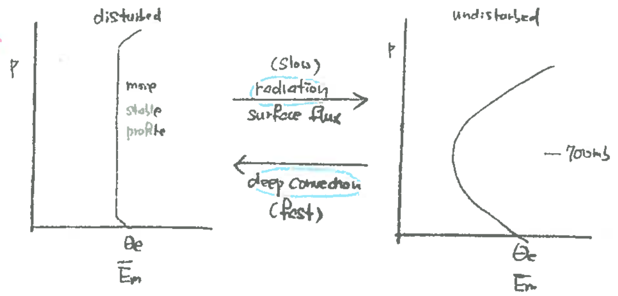
※ In order to maintain a hurricane





In extratropics

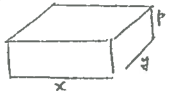




term ④

$$\frac{\partial \overline{\Gamma}_m}{\partial t} = -\overline{\Gamma}_m \frac{\partial \omega}{\partial p}$$: very small term

※ θe minimum → important !



θe ≈ θ_{e_1} minimum

at some later time

θe ≈ θ_{e_2} ← strong min

no boundary interactions

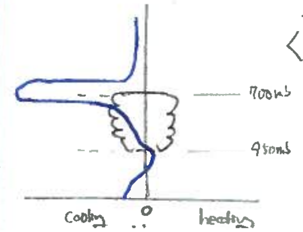
RADIATIVE COOLING is the only process from with the box

which can produce a stronger minimum in θe

new values of θe minimum in a closed box can only come from "radiative cooling"

Term ⑤

$$\frac{\partial \overline{\Gamma}_m}{\partial t} = -\frac{\partial}{\partial p}\overline{H_R}$$



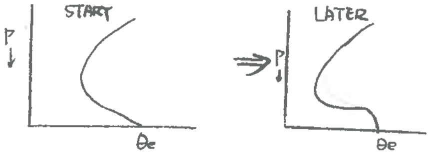
in the cloud layer $-\frac{\partial \overline{H_R}}{\partial p} < 0$ → destabilizing

below the cloud $-\frac{\partial \overline{H_R}}{\partial p} > 0$ → stabilizing

within the cloud layer $-\frac{\partial}{\partial p}H_R < 0$

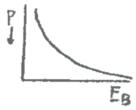
Hence $\frac{\partial \overline{\Gamma}_m}{\partial t} < 0$ destabilizing sharper θe profile

Below the cloud : $-\frac{\partial}{\partial p}H_R > 0$ hence $\frac{\partial \overline{\Gamma}_m}{\partial t} > 0$ stabilizing

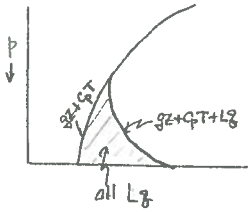


Term ⑥ surface evaporation term

$$\frac{\partial \overline{\Gamma}_m}{\partial t} = -L\frac{\partial \overline{E}_B}{\partial p}$$: surface flux of latent heat + evaporation



$\frac{\partial \overline{E}_B}{\partial p} > 0$ hence $\frac{\partial \overline{\Gamma}_m}{\partial t} < 0$ destabilizing



"Conditional instability arises due to the presence of moisture"

— Contribution from Lq.

- If sfc evaporation is very large ⇒ slope will be greater

⇒ conditional instability will increase.

✱ Sfc. evaporation is important in restoring conditional instability

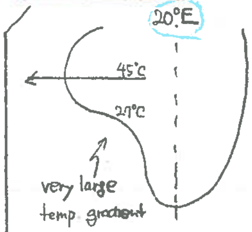
⇒ 2 major points.

Deep cumulus convection destroys conditional instability

Radiation + sfc fluxes restore it! → These eventually win!

African waves

- African waves → related to Hurricane.

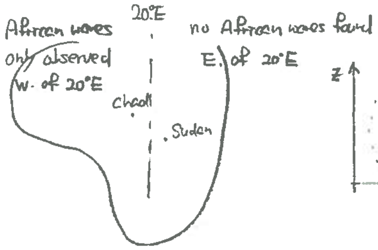


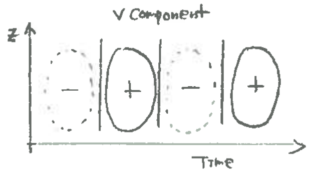
20°E cut off point

African waves are all Rossby-gravity waves

which propagate westward

No free baroclinic instability over Africa.



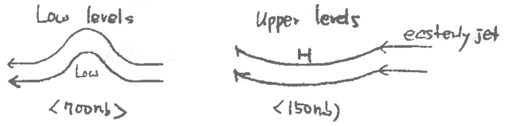


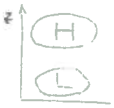
frequency of African waves = 3 to 5 days. max amplitude ~ 700mb

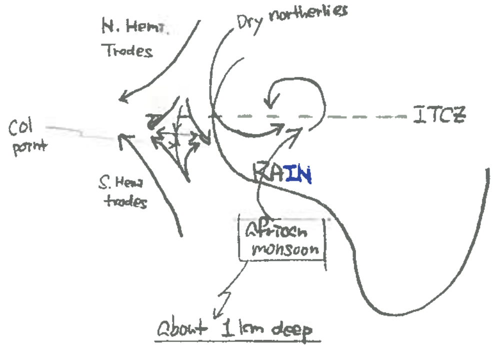
wavelength ~ 3000 km (zonal)

~ 2000 km (meridional)

extend through depth of troposphere

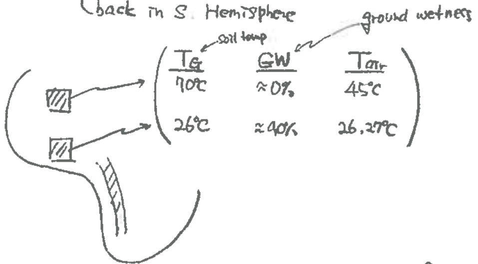






flow becomes easterly with height

Usually veer in N. Hemisphere
——— Eq
back in S. Hemisphere

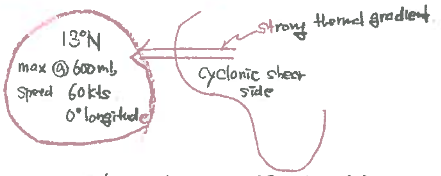


Tg	GW	Tair
70°C	≈0%	45°C
26°C	≈40%	26,27°C

- Difference between African Monsoon and African waves.

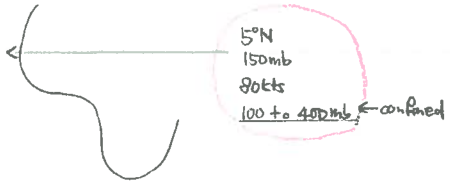
Climatological African monsoon is being enhanced by the African waves.
African monsoon oscillates around climatology,

- AEJ (African easterly jet)

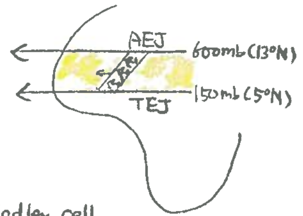


→ cyclonic shear side (South side) is most important
 ↳ This is where African waves develop.
→ Strongest winds found near 0° lon.
→ confined to 400~800mb

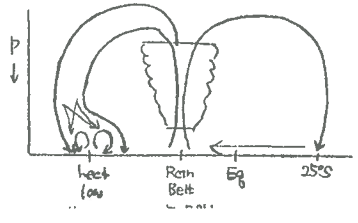
- TEJ (Tropical easterly jet)



- See AEJ + TEJ together
  - all African squall-lines are confined to between AEJ and TEJ
  - ✓ propagate at speed 2 times that of African waves (10° lon/day)



- Local Hadley cell



What is the mechanism of the African waves?

Procedure.

We have

High Resolution Regional model →
- Dynamics
- Land sfc
- Cumulus convection
- dry convection
- PBL surface fluxes
- Radiative transfer (dust, aerosols, clouds)
- Air-sea interaction

→ reasonable fcst of an African waves for ≃ 3days.

→ Gross energetics → 3eqs.

i) What is the role of horizontal shear?
ii) " " " " vertical " ?
iii) " " ... convection?

After running the regional model ⇒ compute gross energetics.
⇒ graph the energetics which are most important.

- Gross energetics
  - Basic energy quantities.

Zonal (mean) APE : P_M
eddy APE : P_E
Zonal (mean) K.E : K_M
eddy K.E. : K_E

- Energy eqs.

Zonal potential energy

$$\frac{\partial P_M}{\partial t} = G(P_M) - \boxed{C(P_M, P_E)} - C(P_M, K_M) + B(P_M)$$

eddy potential energy

$$\frac{\partial P_E}{\partial t} = G(P_E) + \boxed{C(P_M, P_E)} - C(P_E, K_E) + B(P_E)$$

Zonal kinetic energy

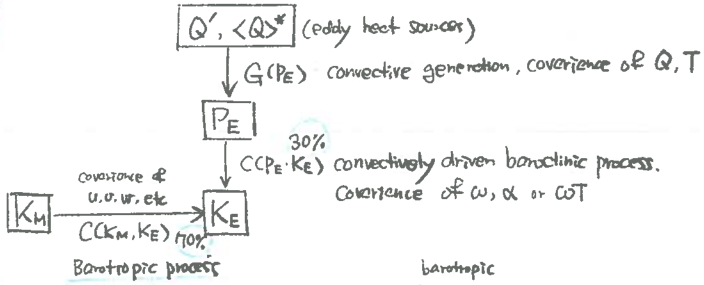
$$\frac{\partial K_M}{\partial t} = C(P_M, K_M) + C(K_E, K_M) - D(K_M) + B(K_M)$$

eddy kinetic energy

★ $$\frac{\partial K_E}{\partial t} = C(P_E, K_E) - C(K_E, K_M) - D(K_E) + B(K_E)$$

↳ the most important term in Africa waves

Interest in K_E : several processes help maintain K_E



Easterly wave driven by Convection + Shear flow

$P_E \rightarrow K_E$ (Convection): ↑ important in hurricane

barotropic $K_M \rightarrow K_E$ (Shear flow): ↑ most important for easterly waves (70%)

- Some properties of the barotropic flows (barotropic dynamics)

domain — no net mass flux in or out

→ Parcel invariants

ζ_a = conserved
All powers of vorticity are parcel invariants.

→ Domain invariants

$\overline{\overline{\zeta_a}} = \text{const}$ $\overline{\overline{\zeta_a^n}} = \text{const}$ $\overline{\overline{K}} = \text{const}$

barotropic vorticity eq.

$$\frac{\partial \zeta_a}{\partial t} = -u\frac{\partial \zeta_a}{\partial x} - v\frac{\partial \zeta_a}{\partial y} \text{ ——— ①}$$

$\frac{\zeta_a^{n-1}}{n} * ① \rightarrow$

$$\frac{\partial}{\partial t}\zeta_a^n = -u\frac{\partial}{\partial x}\zeta_a^n - v\frac{\partial}{\partial y}\zeta_a^n$$

$$\frac{\partial}{\partial t}\zeta_a^n = -\frac{\partial}{\partial x}u\zeta_a^n - \frac{\partial}{\partial y}v\zeta_a^n$$

$$\frac{\partial}{\partial t}\iint \zeta_a^n\,dxdy = 0$$ if the normal wind component is zero at boundary.

⟶ look at NWP note.

Same as for kinetic energy.

→ energy exchanges of the barotropic model

Start with zonal eq. of motion (barotropic)

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + fv - g\frac{\partial z}{\partial x}$$

↓ flux form $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\right)$

$$\frac{\partial u}{\partial t} = -\frac{\partial uu}{\partial x} - \frac{\partial uv}{\partial y} + fv - g\frac{\partial z}{\partial x}$$

Furthermore $v = \frac{\partial \psi}{\partial x}$

$[\ ] \equiv$ zonal mean

$$\frac{\partial}{\partial t}[u] = -\frac{\partial}{\partial y}[u'v'] \quad \text{as } [uv] = [u]\overset{\text{zero}}{[v]} + [u'v']$$

meaning: Zonal flow acceleration = convergence of eddy flux momentum.

multiply by $[u]$ gives

$$\left\{\begin{aligned} \frac{\partial \overline{K_Z}}{\partial t} &= \langle K_E \cdot K_Z\rangle = -\overline{[u]\frac{\partial}{\partial y}[u'v']} \\ \frac{\partial \overline{K_E}}{\partial t} &= \langle K_Z \cdot K_E\rangle = \overline{[u]\frac{\partial}{\partial y}[u'v']} = -\overline{\frac{\partial [u]}{\partial y}[u'v']} \\ \frac{\partial \overline{K_T}}{\partial t} &= \frac{\partial}{\partial t}\left\{\overline{K_E}+\overline{K_Z}\right\} = 0 \end{aligned}\right.$$

where, $\widetilde{(\ )}$: meridial average
$[\ ]$: zonal average
$\overline{(\ )}$: time average

$$\frac{\partial \overline{K_E}}{\partial t} = \overline{\underbrace{[u]}_{<0}\underbrace{\frac{\partial}{\partial y}[u'v']}_{<0}} \longrightarrow \frac{\partial \overline{K_E}}{\partial t} > 0 \text{ by barotropic process.}$$

SW to NE tilt

eddy flux of westerly momentum moves northward

(convergence of flux of momentum into easterly jet.
jet weakens ⇒ wave amplifies as it gains energy.
easterly wave will amplify.)

· non-linear triad

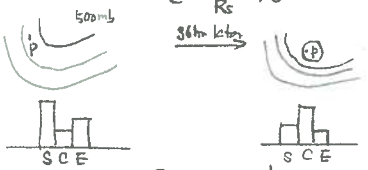
Barotropic dynamics

$$\frac{d\zeta_a}{dt} = 0$$

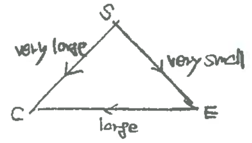
$$\frac{d}{dt}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f\right) = 0$$

$$\rightarrow \text{another } \frac{d}{dt}(S + C + \overset{\text{earth vorticity}(=f)}{E}) = 0$$

$$\because \left\{\begin{aligned} S &= -\frac{\partial V}{\partial n} &&: \text{shear vorticity} \\ C &= \frac{V}{R_s} &&: \text{curvature vorticity} \end{aligned}\right.$$



S+C+E = const.



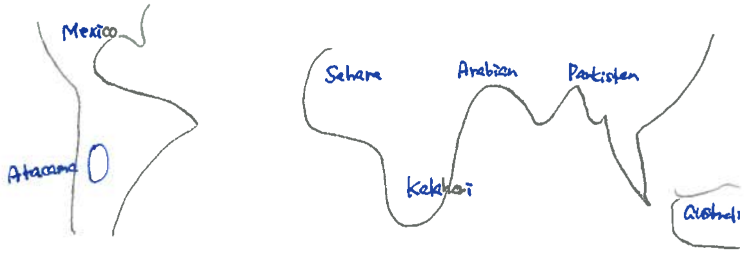
$$\frac{d\zeta_a}{dt} = 0, \quad \frac{d}{dt}(S+C+E) = 0$$

$$\left\{\begin{aligned} \frac{dS}{dt} &= \langle E\cdot S\rangle + \langle C\cdot S\rangle \\ \frac{dC}{dt} &= \langle E\cdot C\rangle - \langle C\cdot S\rangle \\ \frac{dE}{dt} &= -\langle E\cdot S\rangle - \langle E\cdot C\rangle \end{aligned}\right.$$

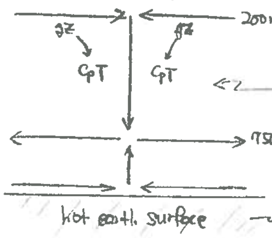
* there's more notes ? → look at exam answer.

← look (more Note)

⊙ Heat lows of the tropics



heat low : very shallow



→ towards early morning, heat low will disappear
⇒ just have a column of sinking air
⇒ atmosphere stabilizes

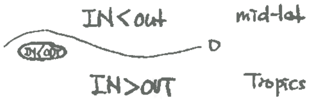
← no (deep) cumulus convection, no rain

→ desert, very semi-arid
central pressure ~ 1003mb
(pressure osc. ~ 3mb
semi-diurnal pressure osc ~0.5 ~0.75m

lapse rate near the ground → superadiabatic
Atmosphere wishes to set up a stable/neutral lapse rate
Energy is passed up via dry convection.
Teleconnection/Response — sinking motion teleconnected to rain areas.

· usually, daytime in tropics incoming > outgoing (radiation)
in extratropics 〃 < 〃

Exception
Desert areas incoming < outgoing.



⊙ Why is the trade-inversion so strong where there is cold upwelling region?

Cold SSTs along the coast-line are due to cold advection and upwelling associated with alongshore winds. These cold SSTs cause extra subsidence. If the SSTs are very low, the inversion extend further downwards and may even touch the sea surface. In the dry static stability equation, if the divergence term $\left(-\overline{\Gamma_d}\frac{\partial \overline{\omega}}{\partial p}\right)$ alone were influencing the stability of a parcel, we might write $\frac{\partial \overline{\Gamma_d}}{\partial t} = -\overline{\Gamma_d}\frac{\partial \overline{\omega}}{\partial p}$

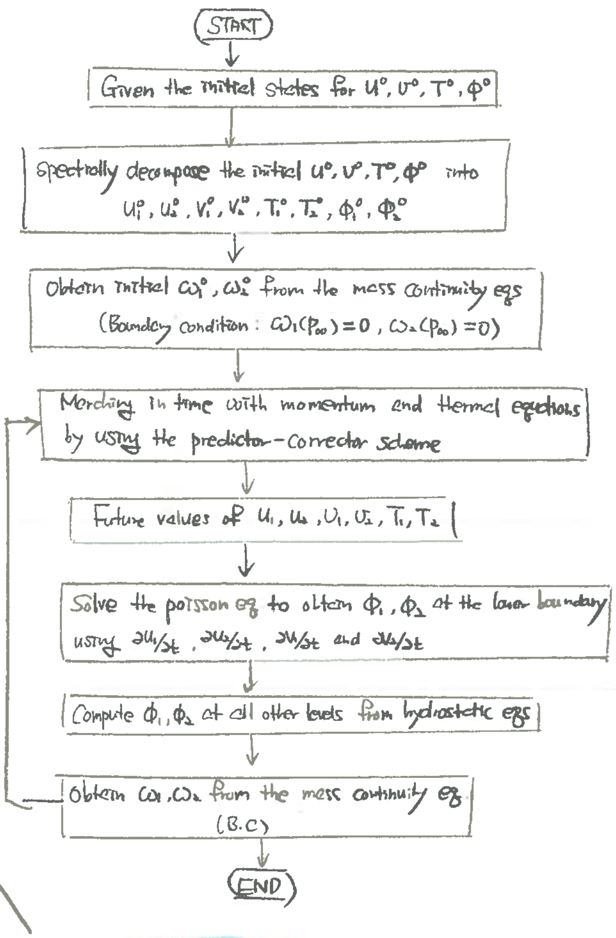
In downward motion (low-level divergence), $-\frac{\partial \overline{\omega}}{\partial p}$ is positive. $\overline{\Gamma_d}$ is almost always positive. But if the SSTs are low, $\overline{\Gamma_d}$ will be more positive. Therefore dry static stability $(\overline{\Gamma_d})$ will be increased more than usual

- ※ The tropical oceanic PBL tends to have neutral lapse rate due to turbulent processes. The large-scale subsidence and cold SSTs tend to lower the inversion base right upto the sea-level, if possible. The PBL turbulence tends to destroy the inversion in the PBL and to lift it up to the top of the PBL. As a result of these two conflicting tendencies, the inversion base is higher than the sea level but is lowest along the cost-lines of North America, South America, north Africa and south Africa

If we consider the sensible heat flux in the dry static stability equation, we find that sensible heat fluxes are smaller in low SSTs region. Therefore the destabilizing effect of sensible heat flux will be smaller and the inversion base will be lower in the low SSTs region. The heating terms of dry static stability eq. also affect this mechanism

According to the above, we can conclude that the inversion is so strong where there are cold SSTs.

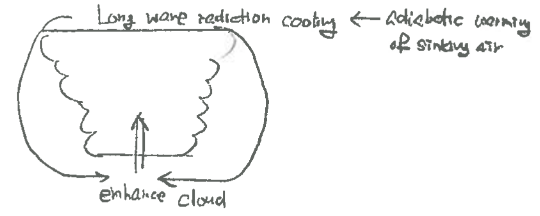
⊙ Draw flowchart for initial value approach to the problem of combined barotropic/baroclinic instability.



END TMI

Sea Breeze and diurnal change

- Arritt nomograms
- Diurnal change
  - most ocean : early morning
  - most land ; afternoon shower
- DNS (Day vs Night Radiation-Subsidence)

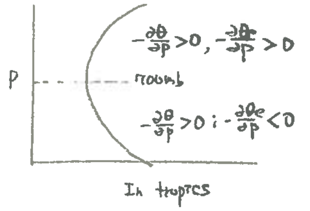


- How to model nocturnal enhancement of rain from longwave radiative destabilization
  - ① Long wave radiative transfer algorithm which include cloud radiation
  - ② Incorporate its effect on nonconvective precip. algorithm and convection precip. algorithm.

- Non-convective rain ⟶ look at NWP note!

Dynamic ascent of absolute stable saturated air
↑ not buoyancy driven

$$-\frac{\partial\theta}{\partial p}>0 \; ; \; -\frac{\partial\theta_e}{\partial p}>0$$

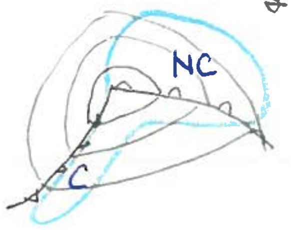


In tropics

$H_{NC} = -L\frac{dq_s}{dt}$: non-convective heating

Total (non-convective) heating $= -\frac{1}{g}\int_{P_B}^{P_T} H_{NC}\,dp$

Total non-convective rain $= -\frac{1}{g}\int_{P_B}^{P_T} \frac{H_{NC}}{L}\,dp$



$$H_{NC} = -L\frac{dq_s}{dt} \simeq -L\omega\frac{\partial q_s}{\partial p}\Big|_M \leftarrow \text{measured at moist adiabat}$$

$$R_{NC} \approx -\frac{1}{g}\int_{P_B}^{P_T} \omega \frac{\partial q_s}{\partial p}\,dp$$

↳ most difficult

★ There are detail handouts for non-convective rain. (look at it)

Convection related topics

- Show that $\underbrace{\frac{d}{dt}(gZ+C_pT)=0}_{\text{Approximate}}$ and $\underbrace{\frac{d}{dt}\theta=0}_{\text{exact}}$ (adiabatic motion)

dry static energy $= gZ+C_pT$

moist static energy $= gZ+C_pT+Lq$

Eqs of motion

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\omega\frac{\partial u}{\partial p} = fv-g\frac{\partial Z}{\partial x}+F_x \quad \text{———} ①$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\omega\frac{\partial v}{\partial p} = -fu-g\frac{\partial Z}{\partial y}+F_y \quad \text{———} ②$$

$u*① + v*② \Rightarrow \quad (\because K=\frac{u^2+v^2}{2})$

$$\frac{dK}{dt} = -g\mathbf{V}\cdot\nabla Z+\mathbf{V}\cdot E$$

$$= -g\left[\frac{dZ}{dt}-\frac{\partial Z}{\partial t}-\omega\frac{\partial Z}{\partial p}\right]+\mathbf{V}\cdot E$$

$\because$ Hydrostatic law $\frac{RT}{p} = -g\frac{\partial Z}{\partial p}$

$$\frac{d}{dt}(gz+K) = g\frac{\partial z}{\partial t} - \frac{RT}{p}\omega + \underset{\sim}{V}\cdot\underset{\sim}{F}$$

∵ First law of thermodynamics. $\left(c_p\frac{dT}{dt} - \alpha\frac{dp}{dt} = H\right)$

$$\left(c_p\frac{dT}{dt} = \frac{RT}{p}\omega + \boxed{\sum_i H_i} \right)$$

↳ sum of different heat sources and sinks

$$\frac{d}{dt}(gz + c_pT + \cancel{K}) = \cancel{g\frac{\partial z}{\partial t}} + \sum_i H_i + \cancel{\underset{\sim}{V}\cdot\underset{\sim}{F}}$$

Note that $c_pT \gg K$, neglect $\underset{\sim}{V}\cdot\underset{\sim}{F}$, $g\frac{\partial z}{\partial t}$

$$\therefore \frac{d}{dt}(gz+c_pT) = \sum_i H_i$$

In the absence of heat sources and sinks.

$$\frac{d}{dt}(gz+c_pT) = 0$$

cf. the conservation of potential temp.

$$c_p\frac{T}{\theta}\frac{d\theta}{dt} = 0$$

- Show $c_p\frac{T}{\theta}\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}(gz+c_pT)$: dry static-stability relation

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p}$$

$$\downarrow$$

$$\frac{\partial \ln\theta}{\partial p} = \frac{\partial \ln T}{\partial p} - \frac{R}{c_p}\frac{\partial \ln p}{\partial p}$$

hydrostatic eq.

$$\rightarrow c_p\frac{T}{\theta}\frac{\partial\theta}{\partial p} = c_p\frac{\partial T}{\partial p} - \frac{RT}{p} = c_p\frac{\partial T}{\partial p} + g\frac{\partial z}{\partial p} = \frac{\partial}{\partial p}(c_pT+gz)$$

- Show $c_p\frac{T}{\theta_e}\frac{\partial\theta_e}{\partial p} = \frac{\partial}{\partial p}(gz+c_pT+Lq)$

$\rightarrow$ when $\frac{d\theta_e}{dt} = 0$, approximately $\frac{d}{dt}(gz+c_pT+Lq) = 0$

- given $z(p)$, $T(p)$, $q(p)$ find $z(P_{LCL})$, $T(P_{LCL})$, $q(P_{LCL})$, $\theta(P_{LCL})$ and P_{LCL}

$gz_{LCL}+c_pT_{LCL}$, θ_{LCL}, q_{LCL} —— find LCL → (p, T, q, z)

$\quad\;\;\,\shortparallel \qquad\qquad\quad\;\, \shortparallel \qquad \shortparallel$

$gz+c_pT$, θ_{LCL}, q —— $p, T, \underset{q}{RH}, z$

PROGRAM EXAMPLE

```
   E0 = gZ0 + CPT0
   PLCL = P0 - 1.   ← 1mb
   DO  20  K =1, 1000
    TLCL = THETA * (PLCL/1000)**(2/7.)  ←  T=θ(P/P0)^(R/Cp)
    ZLCL = ← g ∂z/∂p = -RT/p ;  g (ZL-Zu)/(PL-P0) = -R ((T0+TL)/2)/((P0+PL)/2)
    ED = g*ZLCL + Cp*TLCL
    IF(ED. GT. E0)  go to 21
    PLCL = PLCL - 1
20 continue
21 continue
```

- Given $T(p)$, $q(p)$, find $\theta_e(p)$

$$\theta_e = \theta\exp\frac{Lq_s}{c_pT}$$ ← the value at LCL.

- Show $\frac{d}{dt}(gz+c_pT+Lq) = LE_0 + H_{sen} + H_{RAD}$

ocean

→ note : no latent heating on right hand side

From $\frac{d}{dt}(gz+c_pT) = \sum_i H_i$ ———— ①

where $H_i = H_{sen} + H_{evc} + H_{RAD} + H_{CON}$

H_{EVC} : heating due to evaporation of cloud matter

H_{sen} : sensible heat flux from the ocean

H_{CON} : condensation heating

H_{RAD} : rate of radiative heating

$$H_{CON} = -L\frac{dq}{dt} = -L(E_0+E_c-P)$$

∵ conservation of moisture. evap. from cloud $E_c = -H_{EVC}/L$

$$\frac{dq}{dt} = E_0 + E_c - \boxed{P} \rightarrow \text{precip.}$$

evap. from ocean

$$\therefore L\frac{dq}{dt} = LE_0 - H_{CON} - H_{EVC} \quad \text{———— ②}$$

so. ①+② →

$$\frac{d}{dt}(gz+c_pT+Lq) = LE_0 + H_{SEN} + H_{RAD}$$

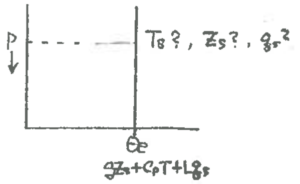
- Show how you would construct a moist adiabat

using i) $gz_s + c_pT_s + Lq_s = \text{const}$

ii) $\theta_e = \text{const}$

given p, T, q, z

solve for p, T, q, z along an entire adiabat.



$gz_s + c_pT + Lq_s = E_m = \text{const}$ along the vertical

$gz_s + c_pT + Lq_s = E_m$ ——— ①

$\frac{\partial}{\partial p}gz_s = -\frac{R}{p}T(1+q_s)$ ——— ②

$e_s = 6.11\exp\frac{a(T-273.16)}{T-b}$ ——— ③

$q_s = 0.622\frac{e_s}{p-0.378e_s}$ ——— ④

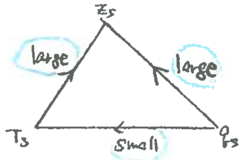
4 eqs, 4 unknowns

z_s, T_s, q_s, e_s

——————— $p-\Delta p$, guess T_u (know)

——————— LCL, P, T_L

- nonlinear triad <in moist adiabat ascents>



900mb ——→ 200mb

LCL

Lq_s is going to decrease

$$\frac{dgz_s}{dt} = -\langle(gz_s)\cdot(c_pT)\rangle - \langle(gz_s)\cdot Lq_s\rangle$$

$$\frac{dc_pT}{dt} = +\langle gz_s\cdot c_pT\rangle + \langle Lq_s\cdot c_pT\rangle$$

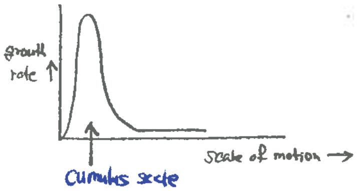
$$+)\;\frac{dLq_s}{dt} = +\langle gz_s\cdot Lq_s\rangle - \langle Lq_s\cdot c_pT\rangle$$

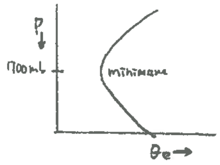
$$\frac{d}{dt}(gz_s+c_pT+Lq_s) = 0$$

ine

- Conditional instability of the first kind (1961)

basic eqs where large scale conditional instability

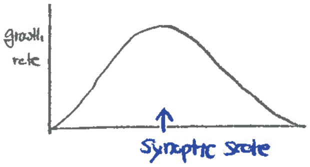




* there is just cumulus scale

- CISK (1964)

Cumulus + large scale co-exist



※ heating function is independent of vertical velocity in free atmosphere.

- Cumulus parameterization ⟶ look at NWP note

- kuo scheme

Q_1 = apparent heat source

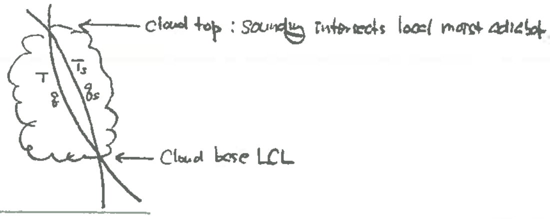
$$= \frac{\partial \bar{S}}{\partial t} + \bar{V}\cdot\nabla\bar{S} + \bar{\omega}\frac{\partial \bar{S}}{\partial p} \quad \text{where} \quad \bar{S} = g\bar{z} + C_p\bar{T}$$

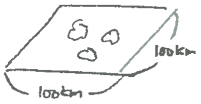
large scale mean

Q_2 = apparent moisture sink.

$$= -L\left(\frac{\partial \bar{q}}{\partial t} + \bar{V}\cdot\nabla\bar{q} + \bar{\omega}\frac{\partial \bar{q}}{\partial p}\right)$$

kuo cloud





$$Q = \frac{1}{g}\int_{p_T}^{p_B}\left\{\frac{C_p(\bar{T}_s-\bar{T})}{L\Delta\tau} + \frac{\bar{q}_s-\bar{q}}{\Delta\tau}\right\}dp$$

⟶ maximum amount of moisture supply needed to form clouds everywhere on the grid square

$$I = -\frac{1}{g}\int_{p_T}^{p_B}\left(\nabla\cdot\bar{q}\bar{V} + \frac{\partial}{\partial p}\bar{q}\bar{\omega}\right)dp$$

⟶ available supply of moisture is usually $\ll Q$

$a = \frac{I}{Q}$: fraction area of clouds.

First law

$$C_p\left(\frac{\partial \bar{T}}{\partial t} + \bar{V}\cdot\nabla\bar{T} + \omega\frac{\partial \bar{T}}{\partial p}\right) = +\frac{R\bar{T}}{p}\bar{\omega} + \sum_i \overline{H_i}$$

↓ contains convective heating

$a\frac{C_p(T_s-T)}{\Delta\tau}$

→ original kuo scheme

convective heating $H_c = aC_p\frac{T_s-T}{\Delta\tau}$ ✓

total convective rain $R_c = \frac{1}{g}\int_{p_T}^{p_B}\frac{C_p(T_s-T)}{L\Delta\tau}dp$ ✓

moistening $M_c = \frac{a(q_s-q)}{\Delta\tau}$ ✓

- modified Kuo scheme

heating (x, y, p, t)
moistening (〃)
rain rates (x, y, t)

<conditions>

① Conditional instability has to be present.

$$-\frac{\partial\theta}{\partial p} > 0$$

$$-\frac{\partial\theta_e}{\partial p} < 0$$

② There should be a supply of moisture.

⟹ look at NWP note.

* Note for modified Kuo scheme.

- 2 conditions ① Conditionally unstable ⟶ $-\frac{\partial}{\partial p}(gz+C_pT+Lq) < 0$
  ② a net supply of moisture convergence
- the large-scale moisture supply (I_L)

$$I_L = -\frac{1}{g}\int_{p_T}^{p_B}\omega\frac{\partial q}{\partial p}dp \longrightarrow \text{a close measure of the rainfall rate.}$$

- To provide sufficient moisture for the moistening of the vertical column and to account for the observed rainfall on large scale.

η : a mesoscale moisture convergence parameter
b : a moistening parameter

- Total moisture supply (I)

$$I = I_L(1+\eta)$$

- rainfall rate (R) and moistening rate (M)

$R = I(1-b) = I_L(1+\eta)(1-b)$
$M = Ib = I_L(1+\eta)b$

- the supply of moisture required to produce a cloud over a unit area.

$$Q = \frac{1}{g}\int_{p_T}^{p_B}\frac{q_s-q}{\Delta\tau}dp + \frac{1}{g}\int_{p_T}^{p_B}\left[\frac{C_pT(\theta_s-\theta)}{L\theta\Delta\tau} + \omega\frac{C_pT}{L\theta}\frac{\partial\theta}{\partial p}\right]dp$$

↳ Cloud time scale

$$= Q_q + Q_\theta$$

- $I = I_q + I_\theta$

$I_q = Ib = I_L(1+\eta)b = M$
$I_\theta = I(1-b) = I_L(1+\eta)(1-b) = R$

- The thermal & humidity eqs.

$$\frac{\partial\theta}{\partial t} + V\cdot\nabla\theta + \omega\frac{\partial\theta}{\partial p} = a_\theta\left(\frac{\theta_s-\theta}{\Delta\tau} + \omega\frac{\partial\theta}{\partial p}\right)$$

$$\frac{\partial q}{\partial t} + V\cdot\nabla q = a_q\left(\frac{q_s-q}{\Delta\tau}\right)$$

where $a_\theta = \frac{I_\theta}{Q_\theta} = \frac{I(1-b)}{Q_\theta} = \frac{I_L(1+\eta)(1-b)}{Q_\theta} = \frac{R}{Q_\theta}$

$a_q = \frac{I_q}{Q_q} = \frac{I_b}{Q_q} = \frac{I_L(1+\eta)b}{Q_q} = \frac{M}{Q_q}$

- The parameterization is closed if b & η are determined from known large-scale variables.

using a screening multiregression analysis

$\frac{M}{I_L} = a_1\zeta + b_1\bar{\omega} + C_1$

$\frac{R}{} = a_2\zeta + b_2\bar{\omega} + C_2$

where ζ : relative vorticity at the 700mb level
$\bar{\omega}$: vertically integrated velocity.

$$\left\{\begin{array}{l} \frac{R}{I_L} = (1+\eta)(1-b) \\ \frac{M}{I_L} = b(1+\eta) \end{array}\right. \longrightarrow \text{obtain } b + \eta$$

- Total convective precipitation (P_{con})

$$P_{con} = \frac{1}{g}\int_{P_T}^{P_B} a_\theta C_p \frac{T}{\theta}\frac{\theta_c-\theta}{L\Delta\tau}\,dp$$

- apparent heat source (Q_1)

$$Q_1 = a_\theta\left[C_p\frac{T}{\theta}\frac{(\theta_c-\theta)}{\Delta\tau} + \omega C_p\frac{T}{\theta}\frac{\partial\theta}{\partial p}\right] + C_p\frac{T}{\theta}[\overset{\text{radiation}}{H_R} + \overset{\text{sensible heat.}}{H_S}]$$

- apparent moisture sink (Q_2)

$$Q_2 = -L\left(a_q\frac{q_c-q}{\Delta\tau} + \omega\frac{\partial q}{\partial p}\right)$$

⊙ Arakawa-Schubert cumulus parameterization

- Outline

The Arakawa-Schubert cumulus parameterization scheme is based on the thermodynamic interactions among idealized clouds and environment. The clouds result from the moisture flux from the sub-cloud mixed layer. The clouds experience entrainment over their depth and detrainment from their tops. The entrainment dilutes the air inside clouds and thus limits their growth. Higher the entrainment, lesser will be the height to which the cloud grows and vice versa. The cloud type is characterized by a parameter which is inversely proportional to cloud top height (or directly proportional to entrainment rate). The cloud environment responds to the growth of cloud by a sinking motion and diabatic warming. This tends to inhibit the development of clouds in the ensemble. The large-scale processes in the atmosphere (advection, radiation, vertical velocity) on the other hand may help increase the convective instability and cloud growth, or in some cases they may decrease the convective instability and inhibit cloud growth. The cloud ensemble is maintained under a thermodynamic equilibrium of cloud and environmental forcings. A complex set of thermodynamic eqs for the clouds and enviroment is developed describing these processes. Their solution leads to determination of rainfall rate, diabatic heating and moistening rates by the convective process.

- Definitions + basic concepts.

Cloud ensemble : The convective clouds are assumed to be existing as an ensemble over a unit horizontal area (which may by grid area of a numerical model). This area must be large enough to contain an ensemble of cumulus clouds but small enough to cover only a fraction of a large-scale disturbance. Also, the individual clouds have time-scale much smaller than that of large-scale disturbance. The existence of such on area and cloud time scale are basic assumptions in the scheme.

If σ_c is the total cloud area and σ_i the area of i-th cloud, then

$$\sigma_c = \sum_i \sigma_i$$

$$\sigma_c \ll 1$$

Dry static energy : $S = C_pT + gz$ (1)

Moist static energy : $h = C_pT + gz + Lq = S + Lq$ (2)

Saturated static energy : $h^* = C_pT + gz + Lq^*$ ↳ saturated specific humidity.

$= S + Lq^*$ (3)

* virtual static energy of cloud

$$S_v = S + C_pT(\delta q - \ell) \quad (4) \qquad \left(\begin{array}{l}\delta = 0.608 \\ \ell = \text{cloud liquid water contents.}\end{array}\right.$$

vertical cloud mass flux through σ_i

$$M_i = \int_i \rho\omega\, d\sigma \quad \text{or} \quad M_i = \rho\omega_i\sigma_i \qquad (5)$$

Total cloud mass flux : $M_c = \sum_i M_i$ (6)

Downward mass flux between clouds (from mass continuity)

$$-\tilde{M} = M_c - \rho\overline{\omega} \qquad (7)$$

Entrainment rate : $E = \sum_i E_i$ (8)

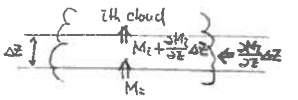
Detrainment rate : $D = \sum_i D_i$ (9)

Therefore

$$E_i - D_i = \frac{\partial M_i}{\partial z} + \rho\frac{\partial\sigma_i}{\partial t} \qquad (10)$$

If steady-state in area, $\frac{\partial\sigma_i}{\partial t} = 0$, then

$$E_i - D_i = \frac{\partial M_i}{\partial z} \qquad (11)$$



and if there is no detrainment

$$E_i = \frac{\partial M_i}{\partial z} \qquad (12)$$

We characterize a cloud type in the ensemble by a parameter λ, where λ takes the value (positive) from 0 to λ_{max} for different cloud types in the ensemble. Then the interval $(\lambda, \lambda + d\lambda)$ denotes a sub-ensemble of clouds with parameter λ.

For such a cloud sub-ensemble, (12) may be written as

$$E(\lambda, z) = \frac{\partial M(\lambda, z)}{\partial z}$$

$$\text{or} \quad \frac{E(\lambda, z)}{M(\lambda, z)} = \frac{1}{M(\lambda, z)}\frac{\partial M(\lambda, z)}{\partial z} \equiv \mu(\lambda, z) \qquad (13)$$

is the fractional entrainment rate at level z in cloud type λ.

If $M_B(\lambda)$ is the cloud mass flux at the cloud base $(z = z_B)$, then

$$\eta(\lambda, z) = \frac{M(\lambda, z)}{M_B(\lambda)} \qquad (14)$$

is defined as normalized cloud mass flux at level z.

From (13) + (14)

$$\mu(\lambda, z) = \frac{1}{M(\lambda z)}\frac{\partial M(\lambda z)}{\partial z} = \frac{M_B(\lambda)}{M(\lambda, z)}\frac{\partial}{\partial z}\frac{M(\lambda, z)}{M_B(\lambda)}$$

$$\text{or}^{*} \quad \frac{1}{\eta(\lambda, z)}\frac{\partial\eta(\lambda, z)}{\partial z} = \mu(\lambda, z) \qquad (15)$$

is the normalized mass flux in cloud type λ and level z which from (13) is equal to fractional entrainment in the cloud.

To simplify the matter, Arakawa and Schubert assumed that for a cloud type λ, fractional entrainment rate is constant and equal to λ itself.

So that (15) may be written as

$$\frac{1}{\eta(\lambda, z)}\frac{\partial\eta(\lambda, z)}{\partial z} = \lambda \qquad (16)$$

on integrating from cloud base (z_B) to level z gives.

$$\eta(\lambda, z) = \eta(\lambda, z_B)\, e^{\lambda(z - z_B)}$$

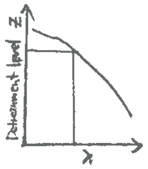
$$\text{or} \quad \eta(\lambda, z) = e^{\lambda(z - z_B)} \quad (\because \text{as } \eta(\lambda, z_B) = 1) \qquad (17)$$

From (13) and (17) we see that for $\lambda = 0$, fractional entrainment rate $\frac{E(\lambda, z)}{M(\lambda, z)} = 0$ and the normalized cloud mass flux $\eta(\lambda z) = 1$

In such a case the cloud rises without any dilution due to absense of environmental entrainment and attains the height of its top, the level at which detrainment is assumed to take place. The cloud types with higher values of parameter λ, have higher rate of entrainment. Due to dilution of cloud air by dry air entraining from the environment, the cloud development is inhibited and clouds rise to lower heights or have a lower detrainment level. Thus parameter λ can be interpretted in various physical implication

→ λ is a measure of height of the top of the cloud or the detrainment level and is inversely proportional to the height of detrainment level.

→ λ can also be interpreted as a measure of fraction of entrainment i.e. directly proportional to fractional entrainment



Cloud types with lower detrainment level and larger fractional entrainment have larger λ, while those taller clouds with higher detrainment level and smaller amount of entrainment

Denoting '—' by area mean value, 'i' the ith cloud value and '~' the environment value, the relationship between cloud and environment parameters can be given as follows;

$$\bar{S} = (1-\sigma_c)\tilde{S} + \sum_i \sigma_i S_i \quad \text{or} \quad \bar{S} = \tilde{S} + \sum_i (S_i - \tilde{S})\sigma_i \qquad (18)$$

$$\bar{q} = (1-\sigma_c)\tilde{q} + \sum_i \sigma_i q_i \quad \text{or} \quad \bar{q} = \tilde{q} + \sum_i (q_i - \tilde{q})\sigma_i \qquad (19)$$

$$\text{as} \quad \sum_i \sigma_i = \sigma_c$$

Using above definitions and assumptions, the equations for the moisture and the dry and moist static energy budgets for the cloud type and environment are written. The value of cloud parameter λ is obtained by solving these budget eqs iteratively under the condition that at the cloud top, the moist static energy of the cloud is equal to the static energy of the environment. Once the value of λ is known various properties of cloud ensemble in terms of λ can be calculated, as

i) level of cloud top or the detrainment level.
ii) Vertical normalized mass flux in cloud at all levels.
iii) Buoyancy of cloud air at different levels.
iv) updraft speed inside the cloud
v) detrainment from the cloud tops.

※ usually λ (0.01 ~ 0.2)

- Cloud work function → for $M_B(\lambda)$

However, to calculate these and other quantities in physical units, we need to know cloud base mass flux $M_B(\lambda)$. An assumption of a dynamic quasi-balance between cloud and a environment is made to obtain $M_B(\lambda)$. This need calculation of cloud work function $A(\lambda)$ which is equal to kinetic energy generated per unit mass flux, and is a measure of efficiency for generation of K.E. by the convective system. $A(\lambda)$ is given by

$$\underset{\substack{\uparrow \\ \text{unit } m^2s^{-2}}}{A(\lambda)} = \int_{Z_B}^{Z_D(\lambda)} \frac{g}{C_p \bar{T}(z)} \eta(z,\lambda) \overbrace{\left[S_{vc}(z,\lambda) - \bar{S}_v(z)\right]}^{\text{buoyancy}} dz \qquad (20)$$

Cloud work function $A(\lambda)$ is an important parameter in the Arakawa-Schubert scheme. In particular it represent

i) rate of generation of K.E. by the cloud buoyancy
ii) It is a property of environment for a particular value of λ
iii) A positive value of $A(\lambda)$ indicates presence of convective instability in the atmosphere ☆

In (20), S_{vc} and $\bar{S}_v$ are the virtual static energy of cloud and the large-scale atmosphere respectively. Their difference is a measure of buoyancy of cloud. The observed cloud work function is compared with its climatological value A(clim.). If

$$A(\lambda) - A(\text{Clim.}) > 0 \qquad (21)$$

then the convective cloud can exist. If this difference is less than 0, the the cloud is not unstable enough to exist and is removed from the ensemble.

- Quasi-equilibrium assumption → $\left[\frac{d}{dt}A(\lambda)\right]_c + \left[\frac{d}{dt}A(\lambda)\right]_{\overset{\text{large scale}}{L}} \simeq 0$

Cumulus convection tends to destroy the convective instability of environment as follows.

- Subsidence warms the environment which reduces buoyancy of cloud updraft
- " dries the atmosphere which inhibit cloud growth.
- " also pushes down the top of PBL from which moist mass flux occurs into cloud

By tending to destroy the instability of cloud free environment each cloud reduces the buoyancy of its own and also of other clouds.

If $K(\lambda, \lambda')$ is the change in kinetic energy of cloud type λ, due to influence of other cloud-type λ', per unit cloud base mass flux of cloud λ', then the time rate change of work function $A(\lambda)$ of cloud λ due to all other cloud types in the ensemble is given by

$$\left[\frac{dA(\lambda)}{dt}\right]_c = \int_0^{\lambda_{max}} K(\lambda,\lambda') M_B(\lambda') d\lambda' \qquad (22)$$

As cloud-cloud interaction reduces convective instability $\left[\frac{dA(\lambda)}{dt}\right]_c$ will be negative. $K(\lambda, \lambda')$ is called the kernel of the integral equation in (22).

The cloud type λ also experience change in $A(\lambda)$ due to changes in convective instability due to processes in large-scale atmosphere like advection, vertical motion, diabatic heating etc.

Let $\left[\frac{dA(\lambda)}{dt}\right]_{LS} = F(\lambda) \qquad (23)$

$F(\lambda)$ being effect of large scale atmosphere on cloud λ.

Arakawa-Schubert scheme makes an important assumption for mathematical solution of the scheme. It assumes that the negative contribution from cloud-cloud interaction is balanced by contribution $F(\lambda)$ from cloud-large scale atmosphere interactions.

i.e. $$\int_0^{\lambda_{max}} K(\lambda,\lambda') M_B(\lambda') d\lambda' + F(\lambda) = \boxed{\left[\frac{d}{dt}A(\lambda)\right]_c + \left[\frac{d}{dt}A(\lambda)\right]_{LS} = 0} \qquad (23)$$

This assumption serves as closure for the solution of the scheme.

Eq. (23) is Fredholm's integral eq. This is written in finite difference form and solved using simple method or variational method to obtain cloud base mass flux $M_B(\lambda)$

From this the total cloud mass flux due to all types of clouds is obtained as

$$M_c(z) = \int_0^{\lambda_D} M_B(\lambda) \eta(z,\lambda) d\lambda \qquad (24)$$

- Heating and rainfall rates

The energy (static energy) and moisture budget eqs for large scale atmosphere can be written as

$$\rho\frac{\partial\bar{S}}{\partial t} = \underbrace{\overset{\text{detrainment}}{D}(\hat{S} - \bar{S} - L\hat{l})} + \underline{M_c\frac{\partial\bar{S}}{\partial z}} - \underline{\rho \underline{V}\cdot\nabla\bar{S} - \rho\bar{\omega}\frac{\partial\bar{S}}{\partial z}} + \bar{Q}_R \qquad (25)$$

$$\rho\frac{\partial\bar{q}}{\partial t} = \underline{D(\hat{q}^* - \bar{q} + \hat{l})} + \underline{M_c\frac{\partial\bar{q}}{\partial z}} - \underline{\rho\underline{V}\cdot\nabla\bar{q} - \rho\bar{\omega}\frac{\partial\bar{q}}{\partial z}} \qquad (26)$$

Here $\hat{()}$ refers to the value at detrainment level where the cloud buoyancy vanishe. $\hat{l}$ is liquid water content at the detrainment level. In (25) $\bar{Q}_R$ are radiation effects.

- The first underlined term on the RHS is total contribution from detrainment from cloud tops
- The second term is the contribution from cloud-large-scale atmospheric interactions
- The third term represents the contribution from large-scale dynamics.

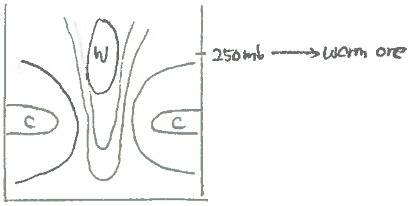
From (25) + (26) we can get the net changes in $\bar{T}$ and $\bar{q}$ over a model time step. Changes due to convective process are from first two terms. From these heating due to cumulus convection and precipitation rates can be calculated

A part of falling precipitation is parameterized to evaporate in dry environment, thus providing the moistening of atmosphere due to convection. Contribution from large-scale dynamics may be positive or negative and net heating and moistening/or drying depends upon the total contribution from all terms.

Hurricane

• Structure of mature Hurricane

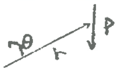
vertical distribution of temperature anomaly



* Why does the wind not change with height?

In cylindrical coord.

$$\frac{u^2}{r} + fu = -g\frac{\partial Z}{\partial r}$$ gradient



$$\frac{RT_v}{p} = -g\frac{\partial Z}{\partial p}$$ hydrostatic

$$\frac{2u}{r}\frac{\partial u}{\partial p} + f\frac{\partial u}{\partial p} = \frac{R}{p}\frac{\partial T_v}{\partial r}$$

gradient wind shear

$$\left.\frac{\partial u}{\partial p}\right|_{gr} = \frac{\frac{R}{p}\frac{\partial T_v}{\partial r}}{\frac{2u}{r} + f}$$

geostrophic wind shear from $\left(\begin{array}{l} fu = -g\frac{\partial Z}{\partial r} \\ \frac{RT_v}{p} = -g\frac{\partial Z}{\partial p} \end{array}\right.$

$$f\left.\frac{\partial u}{\partial p}\right)_{geos} = \frac{R}{p}\frac{\partial T_v}{\partial r}$$

$$\left.\frac{\partial u}{\partial p}\right)_{geos} = \frac{\frac{R}{p}\frac{\partial T_v}{\partial r}}{f}$$

If $\frac{\partial T_v}{\partial r}$ is given

$$\frac{\left.\frac{\partial u}{\partial p}\right)_{geos}}{\left.\frac{\partial u}{\partial p}\right)_{grad}} = \frac{\frac{2u}{r} + f}{f} \left(= 1 + \frac{2u}{rf}\right)$$

∴ geostrophic wind shear is 40 times larger than gradient wind shear.

※ $f \approx 0.7\times10^{-4}\,sec^{-1}$ in hurricane

$u \approx 30\,ms^{-1}$

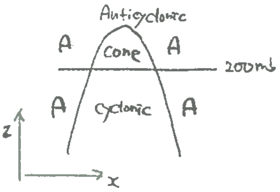
$r \approx 20\times10^3\,m$.

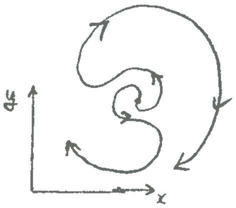
$$\frac{2u}{r} \approx \frac{2\times30}{2\times10^4} = 3\times10^{-3}\,s^{-1}$$

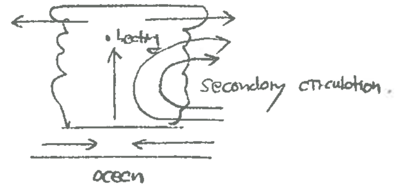
$$\frac{2u}{r} \gg f$$

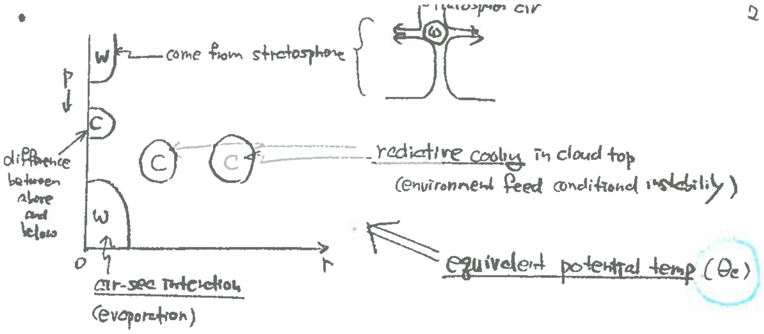
$$\frac{2u}{rf} \approx \frac{3\times10^{-3}}{0.7\times10^{-4}} \approx \frac{3}{7}\times10^{2} \approx 40$$

• Inner rain area is important

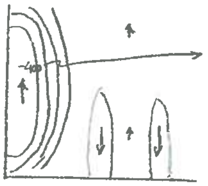








• vertical motion (ω)



$$\ddot{\omega} = -400\,mb/day \simeq \frac{400mb}{10^5 sec}$$

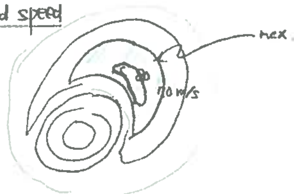
$\simeq 4\times10^{-3}\,mb/sec$

$\sim 5\,cm/sec$ (too small)

⟶ Azimutally averaged

• RH

• 200mb wind speed



• Local cylindrical coordinate

From the tangent plane eqs of motion, we can transform them into cylindrical coords.

$$\left(\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right.$$

$V_r = \frac{dr}{dt}$ (radial velocity) ; $V_\theta = r\frac{d\theta}{dt}$ (tangential velocity)

$$u = \frac{dx}{dt} = \frac{d}{dt}(r\cos\theta) = \frac{dr}{dt}\cos\theta - r\sin\theta\frac{d\theta}{dt} = V_r\cos\theta - V_\theta\sin\theta \quad \text{—— (1)}$$

$$v = \frac{dy}{dt} = \frac{d}{dt}(r\sin\theta) = \frac{dr}{dt}\sin\theta + r\cos\theta\frac{d\theta}{dt} = V_r\sin\theta + V_\theta\cos\theta \quad \text{—— (2)}$$

$$\frac{du}{dt} = \frac{dV_r}{dt}\cos\theta - V_r\sin\theta\frac{d\theta}{dt} - \frac{dV_\theta}{dt}\sin\theta - V_\theta\cos\theta\frac{d\theta}{dt} \quad \text{—— (3)}$$

$$\frac{dv}{dt} = \frac{dV_r}{dt}\sin\theta + V_r\cos\theta\frac{d\theta}{dt} + \frac{dV_\theta}{dt}\cos\theta - V_\theta\sin\theta\frac{d\theta}{dt} \quad \text{—— (4)}$$

If we let the x axis coincide with r so $\theta = 0$,

$\cos\theta = 1$. $\sin\theta = 0$

(3) + (4) reduce to

$$\frac{du}{dt} = \frac{dV_r}{dt} - V_\theta\frac{d\theta}{dt} = \frac{dV_r}{dt} - \frac{V_\theta^2}{r} \quad \text{—— (5)}$$

$$\frac{dv}{dt} = \frac{dV_\theta}{dt} + V_r\frac{d\theta}{dt} = \frac{dV_\theta}{dt} + \frac{V_rV_\theta}{r} \quad \text{—— (6)}$$

From tangent plane eqs. $\left(\begin{array}{l} \frac{du}{dt} = fv - \frac{1}{\rho}\frac{\partial P}{\partial x} \\ \frac{dv}{dt} = -fu - \frac{1}{\rho}\frac{\partial P}{\partial y} \end{array}\right.$

$$\Rightarrow \frac{dV_r}{dt} - \frac{V_\theta^2}{r} = fV_\theta - \frac{1}{\rho}\frac{\partial P}{\partial r} \quad \text{—— (7)}$$

$$\frac{dV_\theta}{dt} + \frac{V_rV_\theta}{r} = -fV_r - \frac{1}{\rho}\frac{\partial P}{r\partial\theta} \quad \text{—— (8)}$$

or

$$\frac{dV_r}{dt} - \frac{V_\theta^2}{r} - fV_\theta = -g\frac{\partial Z}{\partial r} \quad \text{—— (9)}$$

$$\frac{dV_\theta}{dt} + \frac{V_rV_\theta}{r} + fV_r = -g\frac{\partial Z}{r\partial\theta} \quad \text{—— (10)}$$

* total derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V_\theta\frac{\partial}{r\partial\theta} + V_r\frac{\partial}{\partial r} + \omega\frac{\partial}{\partial p}$$

- Conservation of angular momentum.

Absolute angular momentum in cylindrical coord. is defined

$$* \quad M = V_\theta r + \frac{f_0 r^2}{2} \rightarrow \frac{\cancel{2}\Omega \sin\theta\, r}{\cancel{2}} r$$

The first term is angular momentum about an axis, and the second term is momentum due to the earth's rotcion.

$$\frac{dM}{dt} = -g\frac{\partial Z}{\partial \theta} + F_\theta r$$

The first term is a pressure torque, and the second term is a frictional torque. If the storm is symmetric, then the pressure torque is zero.

Relating vorticity to angular momentum we obtain

$$\zeta_a = \frac{1}{r}\frac{dM}{dr}$$

The conservation of angular momentum does not imply conservation of absolute vorticity because convergence and divergence effects are not taken into account.

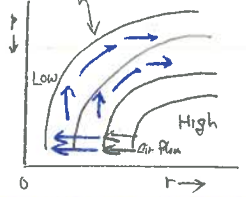
If we assume a symmetric storm so there are no pressure torques, and frictional torques are negligible (which is really not a valid assumption) we see that

$$V_\theta r + \frac{f_0 r^2}{2} = C$$

For a given V_θ at a radius r, we can calculate the wind distribution for the entire storm. The velocity goes to ∞ at $r=0$.
Thus we see that frictional forces are neglegible. These frictional torques arise from eddy motions on the cumulus scale as well as friction with the surface.

- Introduction to angular momentum

$$\boxed{M = ur + \frac{f_0 r^2}{2}}$$: angular momentum per unit mass gain where u is tangential velocity (azimuthally averaged)



lack of conservation in the inflowing air
close to conservation in the outflow layer

$$\boxed{\underbrace{\frac{dM}{dt}} = \underbrace{-g\frac{\partial Z}{\partial \theta}} + \underbrace{F_\theta r}}$$

$\frac{dM}{dt}$ → 3-D change in angular momentum following a parcel

$F_\theta r$ → Frictional torques (surface + cloud friction)

$-g\frac{\partial Z}{\partial \theta}$ → Pressure torques

Surface · atmosphere

$$F_\theta = -g\frac{\partial}{\partial p}\textcircled{\tau_\theta}\ \underbrace{-gr\frac{\partial}{\partial p}\overline{u'\omega'}}$$

← 계산 요소 (residual로)

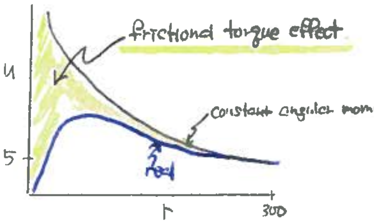
τ_θ → surface frictional stress $C_D \rho V_\theta \sqrt{V_\theta^2 + V_r^2}$ (60%)

$-gr\frac{\partial}{\partial p}\overline{u'\omega'}$ → vertical eddy flux of relative momentum by subgrid scale inside { Clouds / sub cloud layer } 40%

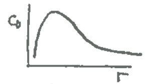
$$M = ur + f_0\frac{r^2}{2}$$

Take a parcel of air at $r = r_1$ (300km), $u \approx 5\,ms^{-1}$

$r = 10km \longrightarrow u = ?$
$r = 0 \longrightarrow u = \infty$



<goal> to get C_D



- Gross angular momentum budget

$$\frac{dM}{dt} = -g\frac{\partial Z}{\partial \theta} + F_\theta r \quad \text{———} \quad ①$$

∴ Derive ①

$$M = V_\theta r + f_0 r^2/2$$

$$\frac{dM}{dt} = r\frac{dV_\theta}{dt} + V_\theta\frac{dr}{dt} + f_0 r\frac{dr}{dt} = r\frac{dV_\theta}{dt} + V_\theta V_r + f_0 r V_r$$

$$\because \left(\frac{dV_\theta}{dt} + \frac{V_r V_\theta}{r} + fV_r = -g\frac{\partial Z}{r\partial\theta} + F_\theta\right) \leftarrow \text{eq of motion in } \theta \text{ including fricti}$$

$$= r\left[\cancel{-\frac{V_r V_\theta}{r}} \cancel{-fV_r} - g\frac{\partial Z}{r\partial\theta} + F_\theta\right] \cancel{+V_\theta V_r} \cancel{+ f_0 r V_r}$$

$$= -g\frac{\partial Z}{\partial \theta} + rF_\theta$$

advective form in cylindrical coord.

$$\frac{\partial M}{\partial t} = -V_\theta\frac{\partial M}{r\partial\theta} - V_r\frac{\partial M}{\partial r} - \omega\frac{\partial M}{\partial p} - g\frac{\partial Z}{\partial\theta} + F_\theta r \quad \text{———} \quad ②$$

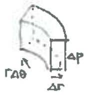
Flux form →

$$\frac{\partial M}{\partial t} = -\frac{\partial M V_\theta}{r\partial\theta} - \frac{1}{r}\frac{\partial M V_r r}{\partial r} - \frac{\partial M\omega}{\partial p} - g\frac{\partial Z}{\partial\theta} + F_\theta r \quad \text{———} \quad ③$$

which uses mass continuity eq

$$\frac{\partial V_\theta}{r\partial\theta} + \frac{1}{r}\frac{\partial V_r r}{\partial r} + \frac{\partial\omega}{\partial p} = 0 \quad \text{———} \quad ④$$

Integrate term by term



$$-\frac{1}{g}\int_{p=0}^{p_s}\int_{r_1}^{r_2}\oint_\theta (\quad) r\,d\theta\,dr\,dp$$

$$☆ \quad \frac{\partial \overline{\overline{\overline{M}}}^{p}}{\partial t} = -\overline{\overline{(M_2 V_{r_2} r_2 - M_1 V_{r_1} r_1)}}^{p}_{\theta} + \overline{\overline{\overline{F_\theta}}}^{\theta}_{r}\,\pi(r_2^2 - r_1^2) \quad \text{———} \quad ⑤$$

$$✓ \quad F_\theta = -g\frac{\partial \tau_\theta}{\partial p} \quad \text{———} \quad ⑥$$

τ_θ is the surface wind stress

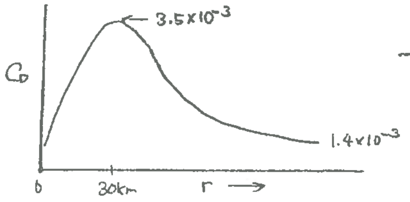
$$\tau_\theta = C_D \rho V_\theta \sqrt{V_r^2 + V_\theta^2} \quad \text{———} \quad ⑦$$

In equation ⑤ everything is known from data except $\overline{\overline{\overline{F_\theta}}}$ which is deduced as a residual.

$$\overline{\overline{\overline{F_\theta}}} = -g\frac{\overline{\overline{\tau_\theta}} - 0}{\Delta p} = -g\frac{\overline{\overline{\tau_\theta}}}{\Delta p} \rightarrow \text{entire atmosphere}$$

hence we know $\overline{\overline{\tau_\theta}}$. In equation ⑦, we can now solve for C_D

Then you can plot C_D as a function of r



→ This does not include cloud.

We can also plot τ_θ as a function of $\sqrt{V_\theta^2 + V_r^2}$ i.e. the total wind speed that shows that the stress increases very rapidly at high wind speed.

- How to find $\overline{M'\omega'}(r,\theta,p)$ from angular momentum budget

$$\frac{dM}{dt} = -g\frac{\partial Z}{\partial \theta} + \boxed{F_\theta} r$$

$$\hookrightarrow F_\theta = -g\frac{\partial \tau_\theta}{\partial p} \quad \begin{cases} \tau_{\theta o} = C_D \rho V_\theta \sqrt{V_\theta^2+V_r^2} & \text{surface} \\ \tau_\theta = -\overline{u'\omega'} \end{cases}$$

The same eq ③ is integrated over a much smaller mass element.

$$dm = -\frac{1}{g}\int_{p_1}^{p_2}\int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2} r\,dr\,d\theta\,dp$$

Over the mass element we can obtain $\overline{\overline{\overline{F_\theta r}}}$ as a residual agcm

mass element integration gives us

$$\frac{\partial \overline{\overline{\overline{M}}}}{\partial t} = \text{Tangent covergence of flux}$$
+ Radial " " "
+ large scale vertical convergence of flux
+ $\overline{\overline{\overline{F_\theta r}}}$

Here the only unknown is $\overline{\overline{\overline{F_\theta r}}}$ because all else can be calculated from large scale data sets.

We can write
$$\underline{F_\theta r} = -g\frac{\partial \tau_\theta}{\partial p} r$$
$$= -g\frac{\partial}{\partial p}\overline{u'\omega'}\, r$$
$$= \underline{-g\frac{\partial}{\partial p}\overline{M'\omega'}}$$

Here we have added zero which is $\frac{\partial}{\partial p} f\frac{r^2}{2}$ (which is zero)

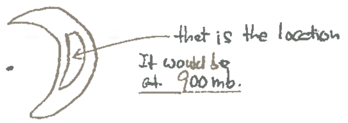
This enables us to find $\frac{\partial}{\partial p}\overline{M'\omega'}$ as a residual which in turn gives us $\overline{M'\omega'}(r,\theta,p)$ since $\overline{M'\omega'} = 0$ at the top of the stratosphere and is described by the similarity fluxes at the bottom of the atmosphere.

What is $-\overline{M'\omega'}$ physically?

It is the cloud contribution in the free atmosphere, it is the surface friction contribution at the lower boundary.

- We have a forecast of hurricane intensity which looks reasonable. How do we go about diagnosing it?

1. Go to the map location of the wind maximum



2. Prepare data in storm relative coordinate.
3. Construct backward trajectories.

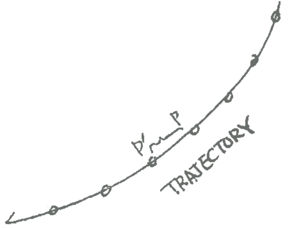
Along the trajectories, interpolate and obtain best values for

$$-g\frac{\partial Z}{\partial \theta}$$

and $\quad F_\theta r$

from your model which made the intensity forecast in the first place. You also know $\frac{dM}{dt}$ from the trajectory.

You should have a residue-free budget. You can then find why the intensity change occurred.



Thus at points P & P' along the trajectory (of relative flow) you know $\frac{dM}{dt}$, $-g\frac{\partial Z}{\partial \theta}$ and $F_\theta r$

They are to be in near balance or else you have done something wrong.

This simply says that

$$M(p') = M(p) - \left\{\frac{g\frac{\partial Z}{\partial \theta})_p + g\frac{\partial Z}{\partial \theta})_{p'}}{2}\right\}\Delta t - \left\{\frac{F_\theta r)_p + F_\theta r)_{p'}}{2}\right\}\Delta t$$

Note that
$$M(p') = (V_\theta r + \frac{f_0 r^2}{2}) \text{ at } p'$$
$$M(p) = (V_\theta r + \frac{f_0 r^2}{2}) \text{ at } p$$

hence the change in intensity is given by

$$\left(V_\theta(p') = V_\theta(p)\frac{r(p)}{r(p')} + \frac{\frac{f_0 r^2}{2})_p - \frac{f_0 r^2}{2})_{p'}}{r(p')} - \frac{g\frac{\partial Z}{\partial \theta}\big|_p + g\frac{\partial Z}{\partial \theta}\big|_{p'}}{r(p')} - \frac{F_\theta r|_{p'} + F_\theta r)_p}{r(p')} \right)$$

intensity eq.

- Potential vorticity & Angular momentum in hurricanes (for intensity forecast)

- Diabatic Potential Vorticity equation
  → complete Ertel potential vorticity eq

$$\frac{d}{dt}\Big(\underbrace{-\zeta_{a\theta}\, g\frac{\partial \theta}{\partial p}}_{\text{isentropic PV}}\Big) = \left(-\zeta_{a\theta}\, g\frac{\partial \theta}{\partial p}\right)\frac{\partial}{\partial \theta}\frac{d\theta}{dt} + \left\{\boxed{\nabla\frac{d\theta}{dt}}\cdot\frac{\partial(\underset{\sim}{V}\times \mathbb{k})_\theta}{\partial \theta}\right\} g\frac{\partial \theta}{\partial p} - \underbrace{\{\nabla\cdot(\underset{\sim}{F}\times\mathbb{k})\} g\frac{\partial \theta}{\partial p}}_{\text{friction term}}$$

vertical differential of diabatic heating

lateral differential of diabatic heating (very large cross ITCZ)

Generally, positive in N.H/negative in S.H.

where isentropic absolute vorticity is given by

$$\zeta_{a\theta} = \frac{\partial v}{\partial x}\Big)_\theta - \frac{\partial u}{\partial y}\Big)_\theta + \frac{u}{a}\tan\phi + f$$

and potential vorticity

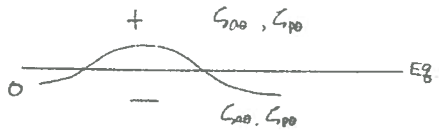
$$\zeta_{p\theta} = -g\zeta_{a\theta}\frac{\partial \theta}{\partial p}$$

$-\frac{\partial \theta}{\partial p}$ is generally positive definite

※ main Question?

$$\left\langle \frac{\text{PVA}}{\text{each term}} = \text{magnitude?} \right\rangle$$

advection



- Order of magnitudes
  - potential vorticity ($\zeta_{p\theta}$) ⟶ unit : $kg^{-1}m^2 deg\, s^{-1}$

$$g = 9.81\ ms^{-2} \;:\; (\zeta_\theta + f) \approx 10^{-4}s^{-1}$$
$$\frac{\partial \theta}{\partial p} \approx 5°C/100mb \approx 5\times10^{-4}\ kg^{-1} deg\, ms^2$$
$$\zeta_{p\theta} = -g\zeta_{a\theta}\frac{\partial \theta}{\partial p} \approx \underline{10^{-7}\ kg^{-1}\ m^2 s^{-1}\ deg}.$$ over the tropics

  - Advection of PV : $-\underset{\sim}{V}\cdot\nabla\zeta_{p\theta}$ (PVA)

$$|\underset{\sim}{V}| \approx 10ms^{-1} \;:\; \nabla = \frac{1}{300km} \approx 0\ 3\times10^{-5}m^{-1}$$ for large scale

$$-\underset{\sim}{V}\cdot\nabla\zeta_{p\theta} \approx \underline{3\times10^{-12}}\ m^2 s^{-2}\ deg\ kg^{-1}$$

  - differential heating along the vertical

$$\zeta_{p\theta}\frac{\partial}{\partial \theta}\frac{d\theta}{dt}$$

$$\left(\frac{d\zeta_{p\theta}}{dt}\right) = \underbrace{\zeta_{p\theta}\frac{\partial}{\partial \theta}\frac{d\theta}{dt}}_{\text{in N.H}\ \zeta>0\ \text{lower troposphere}}$$

$$\boxed{\frac{\partial}{\partial\theta}\frac{d\theta}{dt} > 0}$$

$$\frac{d\theta}{dt} < 0$$ —— Cloud top cooling

$$\boxed{\frac{\partial}{\partial\theta}\frac{d\theta}{dt} < 0}$$

$\frac{\partial}{\partial\theta}\frac{d\theta}{dt}$ is greater than zero over the lower troposphere and is less than zero above the maximum heating in the upper troposphere.

Since $\zeta_{p\theta}$ is generally positive in the N.H., the effect of such a convective heating is to generate PV in the lower troposphere and to destroy PV in the upper troposphere.

Large values of $\zeta_{p\theta}\frac{\partial}{\partial\theta}\frac{d\theta}{dt}$ can be expected where the vertical gradient of the apparent heat source, Q_1, is large since $Q_1 = C_p\left(\frac{p}{p_0}\right)^{R/C_p}\frac{d\theta}{dt}$.

ex) ICTZ, monsoon + tropical depression, tropical waves, tropical squall + non squall systems, general monsoon rainfall systems, mid tropospheric cyclones etc

– Differential lateral heating and generation of PV

Creterion ① large lateral gradient of heating
② Strong wind shear

$$\left(\nabla\frac{d\theta}{dt}\cdot\frac{\partial(\mathbf{V}\times\mathbf{k})}{\partial\theta}\right)g\frac{\partial\theta}{\partial p}$$

no clouds $\frac{d\theta}{dt} < 0$

ITCZ $\frac{d\theta}{dt} \gg 0$

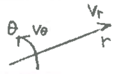
} important hear

– Frictional contribution

$$-\{\nabla\cdot(\mathbf{E}\times\mathbf{k})\}g\frac{\partial\theta}{\partial p}$$

$$\mathbf{E} = -g\frac{\partial\tau}{\partial p}$$

✱ What is the relationship between PV + AM ?



$$\boxed{\zeta_a = \frac{\partial V_\theta}{\partial r} - \frac{\partial V_r}{r\partial\theta} + \frac{V_\theta}{r} + f_0}$$ ← derive ?

large (↑ shear term) small large (↑ curvature term)

$$\zeta_a = \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} + f_0 = \frac{1}{r}\frac{\partial(V_\theta r)}{\partial r} + f_0$$

$$M = V_\theta r + \frac{f_0 r^2}{2}$$

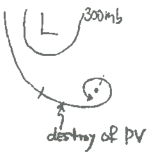
$$\frac{1}{r}\frac{\partial M}{\partial r} = \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} + f_0 = \frac{1}{r}\frac{\partial}{\partial r}(V_\theta r) + f_0$$

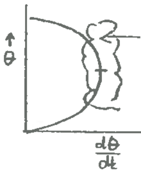
$$\therefore \zeta_a \approx \frac{1}{r}\frac{\partial M}{\partial r}$$

Therefore.

$$\boxed{PV = -\zeta_a g\frac{\partial\theta}{\partial p} \approx -\frac{1}{r}\frac{\partial M}{\partial r}g\frac{\partial\theta}{\partial p}}$$

→ generation of PV → generation of gradient of AM

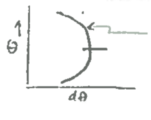




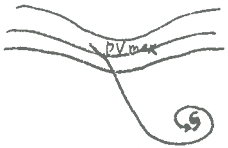
$\frac{\partial}{\partial\theta}\frac{d\theta}{dt} < 0 \rightarrow \frac{d}{dt}\zeta_{p\theta} < 0$.

- PV continue

$$PV \longrightarrow \frac{d\zeta_{p\theta}}{dt} = \overset{>0}{+\zeta_{p\theta}}\,\overset{<0}{\frac{\partial}{\partial\theta}\frac{d\theta}{dt}} + \cdots$$



$\frac{\partial}{\partial\theta}\frac{d\theta}{dt} < 0$



in upper level

PV will reduce from convective heating.

$$\zeta_{\theta a} = \frac{1}{r}\frac{\partial M}{\partial r}$$

$$\zeta_{p\theta} = -g\frac{\partial\theta}{\partial p}\zeta_{\theta a} = -g\frac{\partial\theta}{\partial p}\frac{1}{r}\frac{\partial M}{\partial r}$$

Following parcel motion, processes that reduce $\zeta_{p\theta}$ will reduce $\frac{1}{r}\frac{\partial M}{\partial r}$

Hence, $\frac{\partial M}{\partial r}$ must reduce rapidly along inflowing parcels.

Outer angular momentum goes in somewhat unabated hurricane will intensify (angular momentum conservation)

Entire circulation below that region will intensify (hydrostatics)

- Lack of convection along inflow channels in low levels.

$$\frac{dM}{dt} = -g\frac{\partial z}{\partial\theta} + F_\theta r$$

↑ Cloud vertical eddy monentum flux

$$-\frac{\partial}{\partial p}\overline{M'\omega'}$$

※ upper level development want to convection
(lower 〃 〃 〃 〃 cloud free.

- We covered up to now PV + AM

other issues ① PVA ② SST ③ concentric eye walls.

- PVA (positive vorticity adv)

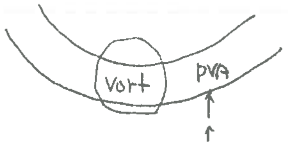
analogy of Q-G ω-eq.

$$\underbrace{\sigma\nabla^2\omega + f_0^2\frac{\partial^2\omega}{\partial p^2}}_{\propto -\omega} = -\frac{\partial}{\partial p}\mathbf{V}\cdot\nabla\zeta_a - \pi\nabla^2(\mathbf{V}\cdot\nabla T)$$

↑↓ Large PVA aloft
small PVA below
} $-\frac{\partial}{\partial p}\mathbf{V}\cdot\nabla\zeta_a > 0$

forcing function > 0

$\omega < 0$



→ There exist a similar eq in isentropic coord.

- SST

if already warm SST exist

many clouds in warm SSTA region will kill hurricane

← if there is cloud around hurricane, it will grow

- Formation of Hurricane

1. SST > 26°C
2. preexisting lower-troposphere cyclonic vorticity
3. Weak wind sheer (vertical)
4. R H 500mb > normal.

✱ there is handout for details

- Given the model output, why did the storm form?

1 Energetics

2 Kuo-Eliassen eqs. ⟶ eddy flux + ★differential heat

which one is more important?

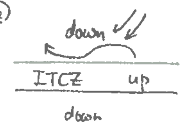
3. Angular momentum principle

– Energetics

Initial growth

vertical shear + convection } ① $H_E \longrightarrow A_E \longrightarrow K_E$

②



Local Hedley cell

$$H_Z \longrightarrow A_Z \underset{\substack{\uparrow \\ \text{Hedley}}}{\longrightarrow} K_Z \underset{\substack{\uparrow \\ \text{barotropic dynamics.}}}{\longrightarrow} K_E$$

③ $H_E \longrightarrow A_E \longrightarrow K_E$

Planetary Boundary layer in the tropics

- preamble.

Land-air
ocean-air) interfaces, fluxes, how they vary with height

Similarity theory.

Bulk-aerodynamic

⟶ look at NWP note

* There's a hand out ⟶ look exam questions.!

- Tropical cyclones

Genesis regions/conditions.

① Warm SST's + deep oceanic mixed layer (>26°C)

ATL → easterly waves are main source, then upper level low
SH → monsoon trough

② significant values of ζ_a (cyclonic) in lower troposphere.

③ Weak vertical sheer over disturbances. Note that shear is greater during El Niño and so fewer storms.

④ mean $w > 0$ and moist at midlevels.

· Aircraft recon show that the following features near eye.

①

1 max vertical velocity + radial wind
2 max tangential wind
3 max rain.

② extensive area of stratiform rain

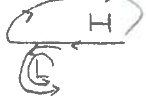
→ an order of magnitude more area covered by stratiform precip. than convective precip. but stratiform precip. only accounts for 60% of total storm precip

③ eyewall convection slopes outward w/ height. Steeper slopes associated w/ stronger storms + smaller eyes. In the eyewall updrafts tend to follow lines of constant angular momentum that slope outward

angular momentum

$$\frac{dM}{dt} = \text{pressure torque} + \text{friction torque} \; ; \; M = \overset{\text{relative arr}}{ur} + \underset{\text{planetary arr}}{\frac{fr^2}{2}}$$

→ subtropical high is a source of AM.



→ pressure + friction torques work to keep winds bounded (finite)

→ If there are many clouds in the inflow chanels into a hurricane, the clouds take AM from the inflow air and therefore deplete AM.

→ The pressure gradient force is not entirely symmetric in a hurricane. When $-g\frac{\partial z}{\partial \theta}$ is negative, it deplete AM

- Scale analysis of the large-scale tropical boundary layer (PBL)

Zonal eq. of motion

$$\underline{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial Z}{\partial x} + F_x}$$

$$T + A + C = \underbrace{P + F}_{\text{dominant terms}}$$

⟶ vertical advection is assumed small.

non-dimensionalize ⟶

$$\begin{cases} u = Uu' \\ \frac{\partial}{\partial x} = (U/\beta)^{-1/2}\frac{\partial}{\partial x'} \\ f = \beta y \\ \frac{\partial}{\partial t} - \boxed{\omega}\frac{\partial}{\partial t'} \end{cases}$$

usual beta plane approximation

└ Characteristic frequency.

$$\omega U\left(\frac{\partial u'}{\partial t'}\right) + \frac{U^2}{(U/\beta)^{1/2}}\left[u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right] - fUv' = P + F$$

or

$$\omega\left(\frac{\partial u'}{\partial t'}\right) + (U\beta)^{1/2}\left[u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right] - \beta y v' = \frac{P}{U} + \frac{F}{U}$$

⟶ three time scales

ω^{-1}, $(U\beta)^{-1/2}$ and $(\beta y)^{-1}$

Let's consider the following three cases.

i) If $\omega < \beta y$ and $(U\beta)^{1/2} < \beta y$, then C, P, F are dominant.

⇒ Ekman balance ($C = P + F$)

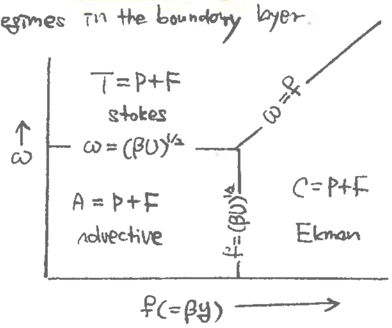
ii) $f = \beta y < (U\beta)^{1/2}$ and $\omega < (U\beta)^{1/2}$

⇒ advective or drift boundary layer ($A = P + F$)

iii) $\omega > \beta y$, $\omega > (U\beta)^{1/2}$

⇒ Stokes regime ($T = P + F$)

⇒ Regimes in the boundary layer.



cf) In mid-lat. the Ekman balance is a characteristic features in PBL

N.H : veering
S.H : backing

END