MET 5510C Examination

(Take-home ; open book)

by

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I. During WWI when the "Norwegian School" was formed, the concept of a cyclone was about 100 years old, weather maps has been routinely analyzed and used as a basis for rudimentary weather forecasts for about 50 years, several meteorologists had made reference to polar and tropical air currents associated with cyclones and some models suggested two wind shift lines in cyclones; yet fronts had not been discovered and described. In a relatively short time the "Norwegian School" made major contributions to improved description of the existence of fronts and their relationship to cyclones :
(a) Summarize concisely these major descriptive discoveries and advances accomplished by the "Norwegian School".

Summary>>

- Relation between fronts and cyclone : They discovered that there are fronts in relation to cyclone and idealized that a front was bounded by two oppositely directed straight currents of air of different temperatures (tropical air and polar air). They thought that there was no mixing of air within the zone. This idea of fronts was related to the frontal cyclones which were observed on their charts. These frontal cyclones were associated with the formation of extratropical cyclones. They regarded fronts as asymptote of confluence lines. The idea of steering line and squall line are also ~~brought~~ in by them.

  not always opposite

  mod in center

- A life cycle of a cyclone : They explained the life cycle of cyclone from the formation to destruction. The initial formation of a cyclone was given by two oppositely directed currents of air of different temperature separated by a straight line. The warm air bulges out toward the cold side with a low pressure center forming at the top of projecting tongue of warm air. The warm tongue is identical to warm sector of the cyclone and the ascending air in this sector is the warm front. And the descending air in other sector is the cold front. Further growth of the amplitude of the warm and cold fronts leads the cold air near the front to reach the cold air from its front and thereby to undercut the warm sector. This stage was said to be occluded front.
- Cyclone families and polar front : A series of cyclones formed on one and the same polar front, each cyclone follows a track of the proceeding cyclone. The mother cyclone partly destroys the surface of discontinuity so that it serves as a steering surface for the next one. They also described that the formation and movement of fronts were linked to the global circulation of the atmosphere. They said that cyclones were links in the interchange of air between the polar regions and the equatorial zone. This interchange, which was effected continuously in the zone of the trade winds, took the irregular and intermittent character of cyclonic motions in the latitudes outside the highs limiting the trade wind belt.

(b) Discuss briefly why these efforts were so quickly successful as contrasted to the lack of success during the previous 50-100 years.

Discussion>>

- Fist of all, their philosophy "not to let pass a single phenomenon or singularity on the weather map without attempting a physical explanation of it" was in their success.

- They tried to establish a very dense network of observation to understand the atmosphere.
- Fundamentally scientific approach - accurate description and better methods of analysis ; The wind fields were analyzed with the streamline analysis and convergence and divergence of streamlines were emphasized. They thought that if the initial state of the atmosphere was known, then if is possible to forecast.
- They tried to understand the scientific methods by applying Helmholtz and Margules theories. They considered density, temperature and winds being discontinuous so that instability(shearing instability : Helmholtz theory) establishes, the straight line separating warm and cold air will roll up. The amplitude of this wave-like pattern will grow until the cold air undercuts the warm air. If this amplification of the fronts continues, the fronts will be occluded. They applied Margules theory to understand the equilibrium of the slope of the fronts. The motion that the warm air is rising and the cold air is sinking will be related to conversion of available potential energy to kinetic energy generating the direct circulation.

good

In addition to, the "Norwegian School" used their improved descriptive knowledge in combination with certain physical and theoretical ideas to develop what they believed to be improvements in the understanding of the development and behavior of cyclones and fronts.
(c) Summarize concisely the important new concepts (theories and hypotheses) developed by this group about the development of cyclones and fronts, their subsequent life-cycle and their relation to the general circulation of the atmosphere.

Summary>>

- Fronts
  * Asymptote is basic feature of the atmospheric current.
  * Approximation as zero-order discontinuity in density, temperature and wind fields.
  * Material surface - i.e. no exchange of air across discontinuity
  * Formed primarily as a result of 2-dimensional, horizontal, kinematic motions, primarily deformation and convergence.
- Cyclones
  * Applying shearing instability to the formation of cyclone (Helmholtz theory)
  * Wave formation to vortex development (Margules theory)
- The subsequent life-cycle and their relation to the general circulation of the atmosphere :
  * The life cycle of cyclone was summarized in part (a).
  * Without synoptic cyclones, there will be a diffrant ~~no~~ general circulation.
  * The source of energy for the general circulation of the atmosphere lies in the contrast of temperature between the polar and the equatorial regions. The system of motion which is comprised under the name "general circulation", tends to smoothe this contrast by bringing polar air to tropical regions, and vice versa. The formation and movement of fronts were linked to the global circulation of the atmosphere. Cyclones were links in the interchange of air between the polar regions and the

V. Bjerknes circ theorem for baroclinic fluids equatorial zone. This interchange, which was effected continuously in the zone of the trade winds, took the irregular and intermittent character of cyclonic motions in the latitudes outside the highs limiting the trade wind belt.

(d) Discuss briefly the dominant physical processes used explicitly (and sometimes implicitly) by the Norwegians in explaining the structure and behavior of frontal cyclones.

* Cyclone is not close system *

Discussion>>

- The dominant physical processes used explicitly by the Norwegians are 2-dimensional, horizontal, kinematic motions, primarily deformation and convergence. They concluded that frontal cyclones grow when there is conversion of potential energy to kinetic energy (direct circulation) .
- Only convergence and deformation can produce ~~or~~ destroy ~~cyclone~~ fronts. In 2-dimensional flow, rotation can not produce or destroy ~~cyclone~~ fronts.
- The physical processes important in the generation of weather phenomena in frontal system were
  * the ascent of the warm air stream at front, producing extensive belts of moderate rain.
  * the changes of state within the principal air streams, primarily
    - chilling during pole-ward travel of the warm current, producing low layer clouds and perhaps slight rains of small drops(drizzle), and
    - warming during tropic-ward travel of the(descending) cool current, producing scattered clouds and perhaps brief heavy rains(showers)

good, comprehensive answer —

II. **After WWII, the improved upper-air network of observations over North America provided the opportunity for new studies of strong baroclinic developments involving frontogenesis, cyclogenesis and the resulting fronts and cyclones. In particular, Newton and Reed and Sanders carried out investigations using expressions analogous to those originally developed by Miller :**

Newton :

$$\frac{d}{dt}\left(\frac{\partial\theta}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right) - \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} - \frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}$$

$$\frac{d}{dt}\zeta_a = -\zeta_a Div + \frac{\partial u}{\partial z}\frac{\partial w}{\partial y}$$

Reed & Sanders

$$\frac{d}{dt}\left(-\frac{\partial\theta}{\partial y}\right) = -\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right) + \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}$$

$$\frac{d}{dt}\left(-\frac{\partial u}{\partial y}\right) = -g\left[\left(f - \frac{\partial u}{\partial y}\right)\frac{\partial}{\partial p}(\rho w) + \frac{\partial u}{\partial p}\frac{\partial}{\partial y}(\rho w)\right] - \frac{df}{dt}$$

(a) Give a physical interpretation of the terms in these expressions, including the coordinate-system and sign conventions employed.

Solution>>

* Newton chose the coordinates such that the x-axis lies along a streamline at the level under consideration.

- $\frac{d}{dt}\left(\frac{\partial\theta}{\partial y}\right)$: Frontogenesis following 3-dimension air motion with time.

  Approximation $\frac{d}{dt}(\nabla_H\theta) \approx \frac{d}{dt}\left(\frac{\partial\theta}{\partial y}\right)$ is applied.

  In Newton's convention, frontolysis was > 0 since $\frac{\partial\theta}{\partial y} < 0$

- $\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right)$: Cross-front gradient of diabatic heating. If adiabatic, =0

  If $\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right) > 0$, reduce the horizontal temperature gradient, so frontolysis.

- $-\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}$: Cross-front shearing effect on thermal advection.

  A positive value implies more rapid cold advection by the strong wind

  $-\frac{\partial u}{\partial y} = \frac{1}{2}(\zeta - \delta_{shearing})$ in xy plane shear on the warm so it changes $\frac{\partial \theta}{\partial y}$ (frontolysis)

- $-\frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y}$: Cross-front fluence effect. on what?
- $-\frac{\partial w}{\partial y}\frac{\partial \theta}{\partial z}$: Cross-front "tipping" what? effect due to differential vertical motion.

  A positive value means upward motion and thus adiabatic cooling of the air on the warm side of the front. This will lead to the weakening of the horizontal gradient of potential temperature and hence frontolysis.

- $\frac{d}{dt}\zeta_a$: The total rate of change of vertical component of absolute vorticity with time following the air parcel in 3-dimensions.
- $-\zeta_a Div$: Generation of vorticity by horizontal divergence. If there is positive horizontal divergence, the area enclosed by a chain of fluid parcel will increase in time; and if circulation is to be conserved the average absolute vorticity of the enclosed fluid must decrease. [illegible]
- $\frac{\partial u}{\partial z}\frac{\partial w}{\partial y}$: This term represents vertical vorticity which is generated by the "tilting" of horizontally oriented components of vorticity into the vertical by a non-uniform vertical motion field. We consider a region where the x-component of velocity is increasing with height so that there is a component of shear vorticity oriented in the negative y direction. If at the same time there is a vertical motion field in which w decreases motion will tend to tilt the vorticity vector initially oriented parallel to y so that it has a component in the vertical.

* Reed and Sanders chose the coordinates such that the x and y axes are taken normal and parallel to the horizontal temperature gradient, respectively.

- The term $\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial x}$ vanished because $\frac{\partial \theta}{\partial x} = 0$. The physical interpretation for each term is similar to that given for Newton's equations except that a positive value of the term would imply a strengthening of the horizontal gradient of potential temperature and therefore an intensification of the front(frontogensis). Reed and Sanders chose the sign convention such that positive value implies intensification. That is because Reed and Sanders were studying frontogenesis whereas Newton was studying frontolysis and his sign convention was chosen to give weakening of the front for the positive values of the terms. probably
- $\frac{d}{dt}\left(-\frac{\partial u}{\partial y}\right)$: The total rate of change of vertical relative vorticity with time following the air parcel in 3-dimensions.

- $-g\left(f-\dfrac{\partial u}{\partial y}\right)\dfrac{\partial}{\partial p}(\rho w)$: Generation of vorticity by horizontal divergence.

  Horizontal divergence in p-coordinate is denoted by such that

$$\frac{\partial}{\partial p}\left(\frac{dp}{dt}\right)\approx\frac{\partial}{\partial p}\left(w\frac{\partial p}{\partial t}\right)=\frac{\partial}{\partial p}(-g\rho w)=-g\frac{\partial}{\partial p}(\rho w)$$

- $-g\left(\dfrac{\partial u}{\partial p}\dfrac{\partial}{\partial y}(\rho w)\right)$: Vertical shear term.

  This term represents the effect of horizontal gradient of the vertical motion transforming vorticity about horizontal axis to that of the vertical axis.

(b) Discuss the relative importance of the magnitudes of the various physical processes represented by these expressions that resulted from these studies, and comment briefly on the differences in methodology used by the different investigators.

Discussion>>

- Newton

$$\frac{d}{dt}\left(\frac{\partial\theta}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right)-\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}-\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}$$

$$+1.39 \quad = \quad 0 \quad + \quad 0.41 \quad + \quad 0.66 \quad + \quad 0.54 \quad (\times 10^{-9}\ \mathrm{deg\ m^{-1}\ sec^{-1}})$$

  The changes of the horizontal temperature gradient along the trajectory are due to ? the air advection, ?lateral difluence of air and so on. The horizontal gradient of diabatic cooling or heating is negligible.

$$\frac{d}{dt}\zeta_a=-\zeta_a Div+\frac{\partial u}{\partial z}\frac{\partial w}{\partial y}$$

$$-1.91 = -\ 0.67 - 1.07 \quad (\times 10^{-9}\ \mathrm{sec^{-2}})$$

  The decrease of vorticity in the vertical axis is accomplished predominantly by the vertical tipping term and little more than one third is through divergence.

- Reed and Sanders

$$\frac{d}{dt}\left(-\frac{\partial\theta}{\partial y}\right)=-\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right)+\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}$$

$$+7 \quad = \quad 0 \quad - \quad 2 \quad + \quad 9 \quad ({}^{\circ}C/\mathrm{100km/12h})$$

  The tipping effect due to differential vertical motion contributes to the frontogenesis while the fluence term contributes to frontolysis. Thus frontolysis counterbalances frontogenesis.

$$\frac{d}{dt}\left(-\frac{\partial u}{\partial y}\right) = -g\left\{\left(f - \frac{\partial u}{\partial y}\right)\frac{\partial}{\partial p}(\rho w) + \frac{\partial u}{\partial p}\frac{\partial}{\partial p}(\rho w)\right\} - \frac{df}{dt}$$

$+12 \quad = - \quad 2\times10^{-5} \quad + \quad 13 \quad + \quad 1 \quad (\text{sec}^{-1}/12\text{hr})$

The tilting term is the most important in producing positive vorticity changes. The convergence term gives small contribution. The Rossby term through small, contributes positive changes.

- Differences in the Methodology used :
  - Newton used (x,y,z,t) coordinate system while Reed and Sanders used (x,y,p,t).
  - Newton found direct circulation in studying frontolysis. Results were valid for a single point. Reed and Sanders found indirect circulation for frontogenesis. They evaluate divergence and vertical motion on the isentropic surface. how?
  - Reed and Sanders used the graphical methods developed by V. Bjerknes to obtain the values, while Newton calculated the numerical values for the four simultaneous equations using the "method of least squares". actually solution unstable unless $\frac{\partial\theta}{\partial y}$ was specified giving 4 equations in 3 unknowns
  - Newton dealt with four parameters : $\frac{\partial\theta}{\partial y}$, $\frac{\partial\theta}{\partial x}$, $-\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial z}$, while Reed and Sanders used only two parameters $-\frac{\partial\theta}{\partial y}$ and $-\frac{\partial u}{\partial y}$.

(c) Discuss concisely the important differences in concepts of frontogenesis and cyclogenesis that resulted from such studies as these versus the classical concepts of the "Norwegian School".

Discussion>>

- The basic differences lie in the way of thinking about atmosphere. Norwegian thought two-dimensionally, while the next group tried to think three-dimensionally.
- According to the classical Norwegian School concepts, frontogenesis takes place in a narrow region of transition that separates air masses of different temperatures, namely the polar and tropical air masses. Frontogenesis and cyclogenesis start at the surface and extend into the upper troposphere. The density and wind are considered as of zero-order discontinuity in the surface of polar front. The polar front separates the polar and tropical air masses which have different temperatures. The Norwegian concept is based on this idea that all moving extratropical cyclones are therefore formed in this manner. They considered the tropopause as a material surface which separated the stratospheric and tropospheric air.
- Studies conducted by Reed and Sanders and Newton of upper level fronts (Type II fronts) showed that higher tropospheric fronts can develop independently of surface fronts. These upper level fronts were considered to separate stratospheric air from tropospheric air rather than as a confluence between polar from tropical air masses. Upper fronts were not required to extend to lower troposphere (i.e., the surface) but could develop independently of lower tropospheric fronts. Fronts can exist in

kinematic sense because of spatially differential advection of temperatures and wind resulting from horizontal and vertical velocity shears which are associated with baroclinic instability (baroclinic waves). In the absence of diabatic process, horizontal potential temperature gradients are modified following the trajectory by horizontal confluence and convergence along with tilting of vertical gradients of potential temperature into horizontal plane. Reed and Sanders showed that the vertical shear terms of vorticity were responsible for the increase of cyclonic vorticity in the frontal zone during frontogenesis. On the other hand Newton showed that the tilting term of vorticity is primarily responsible for vorticity changes in mid-troposphere and upper level frontal zones. Thus the Norwegians failed to include the contribution by vorticity in the frontogenesis process. Frontal zones were no longer considered as zero order discontinuities in density and wind field but first order discontinuity. Newton, Reed and Sanders also talked about indirect circulation in the cold air side and direct circulation in the warm air side of the frontal zone generating by conversion of available potential energy to kinetic energy.

III. The vertical cross-section below shows an idealized analysis of wind speed and potential temperature in the vicinity of a strong front. For simplicity of calculation, assume that the winds blow along s directed out of the cross-section and n and k are unit vectors directed as shown.

(a) At point 0 on the 500 mb surface, estimate the usual finite difference approximations to the following quantities:

$$\zeta_p = -\left(\frac{\partial V}{\partial n}\right)_p \ , \quad \zeta_\theta = -\left(\frac{\partial V}{\partial n}\right)_\theta \ , \quad \sigma = -\frac{\partial \theta}{\partial p}$$

Solution>>

- Based on the given vertical cross-section, the wind speeds are read as ;

  $V_L = 43$m/s, $V_R = 23$ m/s.

  where subscripts L and R represent left and right side of the region respectively.
  Here, $\Delta n = 300$ km.

  $$\zeta_p = -\left(\frac{\partial V}{\partial n}\right)_p \cong -\frac{(V_R - V_L)}{\Delta n} = -\frac{(23-43)m/\sec}{3\times 10^5 m}$$

  $$\therefore \zeta_p = 6.7\times 10^{-5}\sec^{-1}$$

- From the cross-section, the wind speeds on isentropic surface are read as ;

  $V_L = 35$m/s, $V_R = 29$ m/s

  Hence,

  $$\zeta_\theta = -\left(\frac{\partial V}{\partial n}\right)_\theta \cong -\frac{(V_R - V_L)}{\Delta n} = -\frac{(29-35)m/\sec}{3\times 10^5 m}$$

  $$\therefore \zeta_\theta = 2.0\times 10^{-5}\sec^{-1}$$

- The θ_U and θ_L are read from the given cross-section as followings ;

  $\theta_U = 305^\circ K$, $\theta_L = 293^\circ K$

  where subscripts U and L represent upper and lower side of the region respectively.
  Here $\Delta p = 50$mb ($\Delta z = 1$ km).

  So,

  $$\sigma = -\frac{\partial \theta}{\partial p} \cong -\frac{(\theta_U - \theta_L)}{\Delta p} = -\frac{(305-293)^\circ K}{-50mb}$$

  $$\therefore \sigma = 0.24^\circ\ Kmb^{-1}$$

(b) Reed states that

$$\text{if } P = \sigma(\zeta_\theta + f) \text{ and } P' = \sigma(\zeta_p + f) \text{ then } P = P' + \mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times \nabla_p \theta\right).$$

From this show that $\zeta_\theta - \zeta_p = -\frac{1}{\sigma}\frac{\partial V}{\partial p}\left(\frac{\partial \theta}{\partial n}\right)$

and evaluate the right-hand-side. How does this compare with the value of $(\zeta_\theta - \zeta_p)$ as obtained in (a) above?

Solution >>

- Proof that $\zeta_\theta - \zeta_p = -\frac{1}{\sigma}\frac{\partial V}{\partial p}\left(\frac{\partial \theta}{\partial n}\right)$

 Stating from $P = P' + \mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla_p\theta\right)$, i.e., $P - P' = +\mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla_p\theta\right)$

 Substituting $P = \sigma(\zeta_\theta + f)$ and $P' = \sigma(\zeta_p + f)$ into above equation,

 we have

 $$\sigma(\zeta_\theta + f) - \sigma(\zeta_p + f) = \mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla_p\theta\right).$$

 Rewriting the equation,

 $$\zeta_\theta - \zeta_p = \frac{1}{\sigma}\mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla_p\theta\right).$$

 Let's apply the coordinate system $(\mathbf{s},\mathbf{n},\mathbf{k})$ to above equation. Then $\nabla_p\theta$ can be expressed like,

 $$\nabla_p\theta = \frac{\partial \theta}{\partial s}\mathbf{s} + \left(-\frac{\partial \theta}{\partial n}\right)\mathbf{n} + \frac{\partial \theta}{\partial p}\mathbf{k}$$

 where θ decreases with $\mathbf{n}$ (so, the "negative sign" remains.)

 In the cross-section, there is no variation of θ along the $\mathbf{s}$. Hence, $\frac{\partial \theta}{\partial s} = 0$.

 Substituting above expression, we obtain

 $$\frac{1}{\sigma}\mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla_p\theta\right) = \frac{1}{\sigma}\mathbf{k}\cdot\left(-\frac{\partial V}{\partial p}\frac{\partial \theta}{\partial n}\mathbf{k} - \frac{\partial V_k}{\partial p}\frac{\partial \theta}{\partial n}\mathbf{s} + \cdots\right)$$

 $$= -\frac{\partial V}{\partial p}\left(\frac{\partial \theta}{\partial n}\right)$$

 $$\therefore \zeta_\theta - \zeta_p = -\frac{1}{\sigma}\frac{\partial V}{\partial p}\left(\frac{\partial \theta}{\partial n}\right)$$

- From the vertical cross-section, the following quantities are read ;

 $$V_U = 40.5 m/s,\ V_L = 24.5 m/s,\ \theta_L = 305^\circ K,\ \theta_R = 294^\circ K$$

 So,

$$\frac{\partial V}{\partial p} \cong \frac{V_U - V_L}{\Delta p} = \frac{(40.5 - 24.5)m/\sec}{-50mb} = -0.32m\sec^{-1} mb^{-1}$$

$$\frac{\partial \theta}{\partial n} \cong \frac{\theta_R - \theta_L}{\Delta n} = \frac{(294 - 305)^\circ K}{3 \times 10^5 m} \approx -3.7 \times 10^{-5^\circ} Km^{-1}$$

Therefore,

$$\zeta_\theta - \zeta_p = -\frac{1}{\sigma}\frac{\partial V}{\partial p}\left(\frac{\partial \theta}{\partial n}\right)$$

$$= -\frac{1}{0.24^\circ Kmb^{-1}}(-0.32m\sec^{-1} mb^{-1})(-3.7 \times 10^{-5^\circ} Km^{-1})$$

$$= -4.9 \times 10^{-5} \sec^{-1} \approx -5.0 \times 10^{-5} \sec^{-1}$$

Meanwhile, from (a)

$$\zeta_\theta - \zeta_p = (2.0 - 6.7) \times 10^{-5} \sec^{-1} = -4.7 \times 10^{-5} \sec^{-1}$$

$$\approx -5.0 \times 10^{-5} \sec^{-1}$$

The result values of $(\zeta_\theta - \zeta_p)$ calculated in part (a) and part (b) are almost the same.

(c) Estimate the horizontal component of vorticity $\zeta_n \approx \frac{\partial V}{\partial z}$ and with the use of ζ_p from (a) estimate the direction and magnitude of the three dimensional vorticity component in the plane of the cross-section.

Solution>>

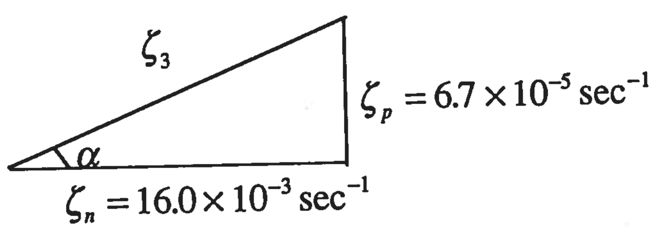
- From the vertical cross-section, the wind speeds are read as ;

  $V_U = 40.5m/s$, $V_L = 24.5m/s$

  So, the horizontal component of vorticity,

  $$\zeta_n \approx \frac{\partial V}{\partial z} = \frac{V_U - V_L}{\Delta z} = \frac{(40.5 - 24.5)m/\sec}{1 \times 10^3 m} = 16.0 \times 10^{-3} \sec^{-1}$$

- Let ζ_3 be the 3-dimensional vorticity component in the plane of the cross-section.



Based on the above figure, we have $\zeta_3^2 = \zeta_n^2 + \zeta_p^2$. So, the magnitude of ζ_3 is

$$\zeta_3 = \sqrt{(16.0 \times 10^{-3})^2 + (6.7 \times 10^{-5})^2} \sec^{-1} \approx 16.0 \times 10^{-3} \sec^{-1}$$

The direction of ζ_3 is computed as follows ;

$$\tan\alpha = \frac{\zeta_p}{\zeta_n} = \frac{6.7\times10^{-5}\,\text{sec}^{-1}}{16.0\times10^{-3}\,\text{sec}^{-1}} \approx 4.2\times10^{-3}$$

Therefore, $\alpha = \tan^{-1}(4.2\times10^{-3}) \approx 0.24^\circ$

This means ζ_3 (the three dimensional vorticity component) is pointing toward the cold air with angle of 0.24° with respect to $\mathbf{n}$.

(d) The slope of the upper (warm) boundary of the frontal zone may be estimated from :

$$\frac{\Delta z}{\Delta n} = \frac{\left(\frac{\partial\theta}{\partial n}\right)' - \frac{\partial\theta}{\partial n}}{\frac{\partial\theta}{\partial z} - \left(\frac{\partial\theta}{\partial z}\right)'} \quad \text{or} \quad = \frac{\left(\frac{\partial V}{\partial n}\right)' - \frac{\partial V}{\partial n}}{\frac{\partial V}{\partial z} - \left(\frac{\partial V}{\partial z}\right)'}$$

Where primed terms are inside front. With the use of either of these expressions, calculate the slope of this boundary and defend or deny the statement that : "the three dimensional vorticity vector component in the plane of the cress section is approximately parallel to the frontal boundary and points toward the colder air."

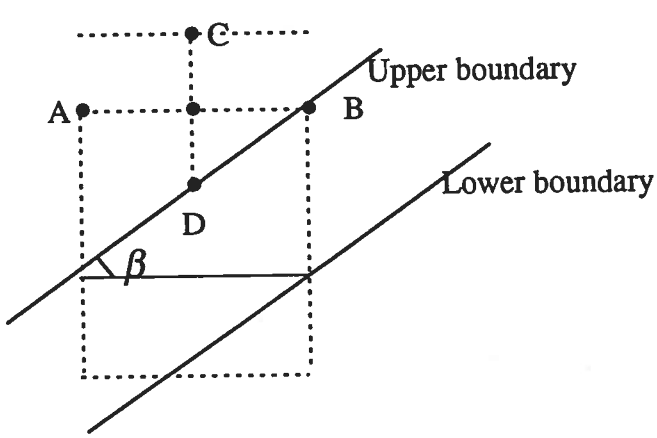
Solution>>

- The latter expression is used to estimate the slope.

From (a), $\left(\frac{\partial V}{\partial n}\right)' = -6.7\times10^{-5}\,\text{sec}^{-1}$

From (c), $\left(\frac{\partial V}{\partial z}\right)' = 16.0\times10^{-3}\,\text{sec}^{-1}$

Using the following diagram,



The following wind speeds are read from the vertical cross-section given.

$V_A = 50.0 m/\sec$, $V_B = 37.5 m/\sec$,

$V_C = 47.5 m/\sec$, $V_D = 40.5 m/\sec$

Let's assume $\overline{CD}$ to be 1 km(Δz). Then, the followings(outside front) can be computed.

$$\frac{\partial V}{\partial n} = \frac{V_B - V_A}{\Delta n} = \frac{(37.5 - 50.0)m/\sec}{3\times 10^5 m} \approx -4.2\times 10^{-5} \sec^{-1}$$

$$\frac{\partial V}{\partial z} = \frac{V_C - V_D}{\Delta z} = \frac{(47.5 - 40.5)m/\sec}{10^3 m} = 7.0\times 10^{-3} \sec^{-1}$$

So,

$$\frac{\Delta z}{\Delta n} = \frac{\left(\frac{\partial V}{\partial n}\right)' - \frac{\partial V}{\partial n}}{\frac{\partial V}{\partial z} - \left(\frac{\partial V}{\partial z}\right)'} = \frac{[-6.7 - (-4.2)]\times 10^{-5} \sec^{-1}}{(7.0 - 16.0)\times 10^{-3} \sec^{-1}}$$

$$\approx 2.8\times 10^{-3}$$

Therefore, the slope of the upper boundary of the frontal zone(β) is $\beta \approx 0.16^\circ$.

- Comment on the statement.

The slope of ζ_3 (α) was $0.24^\circ \approx 0.2^\circ$ in part (c) and the slope of the upper boundary of the front (β) was $0.16^\circ \approx 0.2^\circ$ from the above calculation. This means that we can consider the angles to be almost the same in both cases. Hence, we can defend the statement that "The three dimensional vorticity vector component in the plane of the cress section is approximately parallel to the frontal boundary and points toward the colder air."

IV. Staley showed that if atmospheric motion is not assumed to be adiabatic and frictionless then the general form of Ertel's potential vorticity theorem becomes :

$$(1)\quad \frac{d}{dt}\left(\alpha\nabla\theta\cdot\underline{\eta}\right)=\alpha\underline{\eta}\cdot\nabla\left(\frac{d\theta}{dt}\right)+\alpha\nabla\theta\cdot\left(\nabla\times\underline{F}\right)$$

in which all quantities and operations are three dimensional. He further showed if only the essentially important components normal to isentropic surfaces are retained that (1) may be transformed

$$(2)\quad \frac{dP}{dt}\equiv\frac{d}{dt}\left[\sigma(\zeta_\theta+f)\right]=(\zeta_\theta+f)\left(-\frac{\partial}{\partial p}\frac{d\theta}{dt}\right)+\sigma\left(\nabla\times F_n\right)$$

(a) Discuss concisely the physical processes that may contribute to the terms on the right-hand-side of (2) to produce changes in P.

Discussion>>

- In equation (2), $\zeta_\theta+f$ means absolute vorticity on θ surface and σ static stability i.e., $(-\frac{\partial\theta}{\partial p})$. The left hand side , $\frac{d}{dt}\left[\sigma(\zeta_\theta+f)\right]$, is the rate of change of potential absolute vorticity with time following the air parcel in 3-dimensions.

  The term on right hand side, $\frac{\partial}{\partial p}\left(\frac{d\theta}{dt}\right)$, is the vertical gradient of the diabatic heating such as radiative cooling, condensation , evaporation, divergence of vertical turbulent heat fluxes and so on. effect of $\nabla\theta$?

  The term, $\sigma\left(\nabla\times F_n\right)$, means generation of potential vorticity due to frictional torque. $\nabla\times F_n$ is the component normal to isentropic surface of the curl of the frictional force with component along $\nabla\theta$. effect of σ?

In synoptic situations involving well-defined Type II frontal zones and tropopauses, Staley attempted direct evaluation of the changes in P as represented by the left-hand-side of (2), and to interpret his results in terms of the physical processes represented by the terms on the right-hand-side.

(b) Discuss the synoptic-scale analytical procedures employed by Staley in evaluation the changes in P and summarize the principal results of his efforts as to the magnitude and distribution of the changes in P in relation to frontal zones and tropopauses. Briefly indicate those aspects of Staley's procedures which could contribute to significant errors in his evaluations of the changes in P.

Discussion >>

- Staley evaluated individual potential vorticity change by subtracting the initial value from the final value along the isentropic trajectory. These quantity were obtained from wind and stability analyses on the isentropic charts. The mean isentropic potential spacing?

vorticity was taken as the average at the four surrounding grid point. For the evaluation of trajectories required are isentropic charts upon which isotachs and streamlines were constructed. To obtain trajectories for twelve hours parcels of air were moved for six hours along streamlines valid at initial time and for six hours along streamlines for final time.

- Staley found that positive changes of isentropic potential vorticity occur both in upper troposphere and lower stratosphere on the cold-air side of the frontal zone. The values of individual potential vorticity increase just below the tropopause. The negative changes occur in the frontal zone and around the positive area. Individual potential vorticity changes increase with southward descending air in the lower stratosphere. However, these changes decrease as the air enters the troposphere near the frontal zone. The potential vorticity changes have positive values in the tropopause funnel. Typical values of potential vorticity changes in 12 hours is about 3×10^{-3} deg cm gm^{-1} and is found in lower stratosphere and frontal zones.
- In evaluating potential vorticity changes, Staley made adiabatic assumption which can contribute significant errors. As the gradient of diabatic heating contribute to change of potential vorticity, it is unlikely that the parcel will move adiabatically. Trajectories may have errors because the pressure system can move very fast so that deriving trajectories from isentropic streamlines in twelve hours apart can be difficult. Errors could arise also because errors made in observation of data(all instruments are assuming "hydrostatic approximation").

(c) Summarize concisely Staley's conclusions as to the probable relative importance of the physical processes in producing changes in P.

Summary>>

- Staley concluded that radiative cooling, condensation or evaporation and divergence of vertical turbulence of heat fluxes which are processes of diabatic heating can affect stability. If stability is less conservative than vorticity, then vertical gradient of temperature change is more important than the frictional torque. On the contrary if vorticity is less conservative than stability the frictional torque effect is more important than the vertical gradient of temperature change.

was it?

(d) Shapiro (1976 and 1978) also considered the question of non-conservative changes of P in relation to tropopause "folding"; compare his results with those of Staley and comment briefly on agreements and/or disagreements in their results as to location, magnitude and mechanisms of $\frac{dP}{dt}$.

Solution>>

- Shapiro found that the organized patterns of the vertical eddy heat flux produce high maximum positive values of potential vorticity. This is in agreement with Staley's in the non-conservative properties of potential vorticity.
- Staley's result about location and magnitude was summarized in part (b). Meanwhile, Shapiro found that anomalously high values of potential vorticity are shown to

coincide with the mesoscale cyclonic shear zone. The high values of P within an upper-level frontal zone were shown to result from shearing vorticity in the mesoscale high potential vorticity region of the stratosphere which is transported downward into the tropospheric frontal zone and becomes transformed into curvature vorticity with little change in thermal stability. ?

- There are differences in mechanisms. Staley felt that ~~latent heat~~ processes is mostly responsible for the diabatic requirements involved in $\frac{dP}{dt}$. Shapiro felt that mesoscale CAT can produce vertical divergence of eddy heat fluxes that will change P. Staley also found $\frac{\partial \theta}{\partial t}$ to be order of magnitude larger.

Shapiro measured $\overline{w'\theta'}$

V. All parts of this question refer to your various analyses of 15Z March 28, 1956.

(a) p_θ and T_θ are uniquely related on isentropic surfaces.

What are the appropriate values of T_θ for

p_θ=290K=850mb and 700mb; p_θ=320K=400mb and 300mb.

Solution>>

- method 1) Based on the Skew-T log-p diagram, T_θ can easily be obtained by the given θ. In the diagram, If the air parcel was raised adiabatically from 1000mb to the given pressure level, then T_θ is the temperature value at that level.
- method 2) Poisson's equation can be used to calculate T_θ, i.e.,

$$T = \theta\left(\frac{p}{p_0}\right)^{R/c_p}$$

where, p_0=1000mb, $R = 287\ \mathrm{JK^{-1}kg^{-1}}$, and $c_p = 1004\ \mathrm{JK^{-1}kg^{-1}}$

For $\theta = 290^\circ K$

$$T_\theta\,(\text{at 850mb}) \approx \underline{276.8^\circ K} = 3.8^\circ C$$

$$T_\theta\,(\text{at 700mb}) \approx \underline{261.9^\circ K} = -11.1^\circ C$$

For $\theta = 320^\circ K$

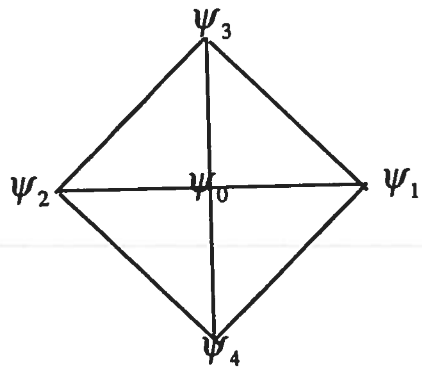
$$T_\theta\,(\text{at 400mb}) \approx \underline{246.2^\circ K} = -26.8^\circ C$$

$$T_\theta\,(\text{at 300mb}) \approx \underline{226.8^\circ K} = -46.2^\circ C$$

(b) Geostrophic vorticity on isentropic surfaces $(\zeta_g)_\theta = \frac{1}{f}\nabla^2\psi$. With the usual "5-point" finite difference approximation $\nabla^2\psi = \frac{1}{d^2}\sum_1^4(\psi_i - \psi_o)$, calculate $(\zeta_g)_\theta$ at stations 340 and 456 with d=4 deg latitude($d^2 = 2\times 10^{11}\mathrm{m}^2$) and $f = 10^{-4}\mathrm{s}^{-1}$. From the soundings at 340 and 456 calculate appropriate values of $\sigma = -\frac{\partial\theta}{\partial p}$ and with the use of the geostrophic vorticity values estimate the Potential Vorticity at these stations. Comment on the reasons for differences in the values.

Solution>>

- Using the "ζ-point" finite approximation, i.e., $\nabla^2\psi = \frac{1}{d^2}\sum_1^4(\psi_i - \psi_o)$



- At station 340(LITTLE ROCK), $\psi_o = 317090 m^2 \sec^{-2}$

  ψ's are read from $\theta = 320^\circ K$ chart as followings:

  $\psi_1 = 317450 m^2 \sec^{-2}$
  $\psi_2 = 316600 m^2 \sec^{-2}$
  $\psi_3 = 314780 m^2 \sec^{-2}$
  $\psi_4 = 318100 m^2 \sec^{-2}$

Hence,

$$\nabla^2\psi = \frac{1}{d^2}(\psi_1 + \psi_2 + \psi_3 + \psi_4 - 4\psi_o)$$
$$= \frac{1}{2\times10^{11} m^2}(317450 + 316600 + 314780 + 318100$$
$$- 4\times317090) m^2 \sec^{-2}$$
$$= -7.15\times10^{-9} \sec^{-2}$$

So,

$$(\zeta_g)_\theta = \frac{1}{f}\nabla^2\psi \approx \underline{-7.2\times10^{-5}} \sec^{-1}$$

- At station 456(TOPEKA), $\psi_o = 313730 m^2 \sec^{-2}$

  ψ's are read from $\theta = 320^\circ K$ chart as followings:

  $\psi_1 = 314900 m^2 \sec^{-2}$
  $\psi_2 = 313800 m^2 \sec^{-2}$
  $\psi_3 = 312300 m^2 \sec^{-2}$
  $\psi_4 = 316600 m^2 \sec^{-2}$

$$\nabla^2\psi = \frac{1}{d^2}(\psi_1+\psi_2+\psi_3+\psi_4-4\psi_o)$$
$$= \frac{1}{2\times10^{11}m^2}(314900+313800+312300+316600$$
$$-4\times313730)m^2\sec^{-2}$$
$$=1.34\times10^{-8}\sec^{-2}$$

So,

$$(\zeta_g)_\theta = \frac{1}{f}\nabla^2\psi \approx \underline{1.3\times10^{-4}}\sec^{-1}$$

- At station 340

$$\theta\,(\text{at 400mb}) = 43^\circ C = 316^\circ K$$
$$\theta\,(\text{at 300mb}) = 47^\circ C = 320^\circ K$$
$$\sigma = -\frac{\partial\theta}{\partial p} = -\frac{(320-316)^\circ K}{(300-400)mb} = 0.04^\circ Kmb^{-1}$$

- At station 456

$$\theta\,(\text{at 400mb}) = 29^\circ C = 302^\circ K$$
$$\theta\,(\text{at 300mb}) = 52^\circ C = 325^\circ K$$
$$\sigma = -\frac{\partial\theta}{\partial p} = -\frac{(325-302)^\circ K}{(300-400)mb} = 0.23^\circ Kmb^{-1}$$

- Since potential absolute vorticity(PAV) = $\sigma\left[(\zeta_g)_\theta + f\right]$

  - At station 340,

$$PAV = 0.04Kmb^{-1}\times(-7.2\times10^{-5}+10^{-4})\sec^{-1}$$
$$= \underline{1.12\times10^{-6}Kmb^{-1}}\sec^{-1}$$

  - At station 456,

$$PAV = 0.23Kmb^{-1}\times(1.3\times10^{-4}+10^{-4})\sec^{-1}$$
$$= \underline{5.29\times10^{-5}Kmb^{-1}}\sec^{-1}$$

- Comment on the reasons for the differences in the values :

  Looking at the 320K streamfunction chart, the station 456 is located on the cyclonic side of vorticity ,while the station 340 is not. That is, more vorticity exists at station 456.

  Also, on the vertical cross section, the station 456 (more stable) is located in the stratosphere at 300mb level, while the station 340, at the same level, is in the troposphere and outside the frontal zone. Therefore the PVA at station 456 is larger than that of the station 340.

(c) The vertical shear of the geostrophic wind is given by

(1) $$\frac{\partial V_g}{\partial z} = -\frac{g}{f\theta}\left(\frac{\partial \theta}{\partial n}\right)_p$$

Show that if $S = \frac{\partial \theta}{\partial z}$ this may be written as

(2) $$\frac{\partial V_g}{\partial z} = \frac{g}{f\theta} S \tan\alpha \quad \text{where} \left(\frac{\Delta z}{\Delta n}\right)_\theta = \tan\alpha \equiv \frac{-\frac{\partial \theta}{\partial n}}{\frac{\partial \theta}{\partial z}}$$

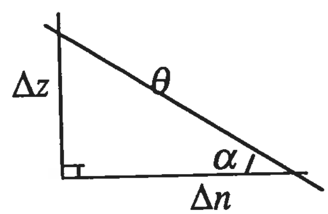
The isentropic surface $\theta = 31^\circ C$ lies in a stable layer that extends from 340 to 445 to 456. At 445 this layer is mostly contained between 4 and 5 km through which the reported wind shear is $\frac{\partial V}{\partial z} = 20 ms^{-1} km^{-1} = 2\times10^{-2} s^{-1}$. Using values of S for $\theta = 31^\circ C$ at 445 and the mean slope of $\theta = 31^\circ C$ from 340 to 456 compare the geostrophic wind shear as given by (2) with the reported wind shear at 445. Comment on possible reasons for differences in the values.

Solution>>

- derivation from equation (1) to (2)

In the θ surface, we can write

$$(\Delta\theta)_\theta = \frac{\partial \theta}{\partial n}\Delta n + \frac{\partial \theta}{\partial z}\Delta z.$$



Following the θ surface $(\Delta\theta)_\theta = 0$, so $\frac{\partial \theta}{\partial n}\Delta n + \frac{\partial \theta}{\partial z}\Delta z = 0$, i.e.,

$$\left(\frac{\Delta z}{\Delta n}\right)_\theta = \frac{-\frac{\partial \theta}{\partial n}}{\frac{\partial \theta}{\partial z}} = \tan\alpha$$

Rewriting the above equation as, $\frac{\partial \theta}{\partial n} = -\frac{\partial \theta}{\partial z}\tan\alpha = -S\tan\alpha$

If $\left(\frac{\partial \theta}{\partial n}\right)_p = \frac{\partial \theta}{\partial n}$ and substituting $\frac{\partial \theta}{\partial n} = -S\tan\alpha$, the equation (1), the vertical shear of the geostrophic wind , can be written as

$$\frac{\partial V_g}{\partial z} = \frac{-g}{f\theta}(-S\tan\alpha) = \frac{g}{f\theta} S\tan\alpha$$

- Calculation of $\frac{\partial V_g}{\partial z}$ at station 445.

At station 340, the pressure at $\theta = 31^\circ C$ is 645mb ,i.e., $z \approx 3.6km$

At station 456, the pressure at $\theta = 31^\circ C$ is 385mb ,i.e., $z \approx 7.4km$

So, $\Delta z = 7.4 - 3.6 \approx 3.8km$

Based on the chart, the distance(Δn) between stations 340 and 456 is approximately 5° latitude($\approx 550km$)

Using above values,

$$\tan\alpha = \frac{\Delta z}{\Delta n} = \frac{3.8km}{550km} = 6.9 \times 10^{-3}$$

The slope of $\theta = 31^\circ C$ passes the station 445 which is between stations 340 and 456 at level about 550mb. Taking $\Delta p = 50mb(\approx 1km)$ with 550mb as middle level, we can select 525mb and 575mb as reference level for θ.

At 525mb(station 445), $\theta_{525mb} = 33^\circ C = 306^\circ K$

At 575mb(station 445), $\theta_{575mb} = 26^\circ C = 299^\circ K$

So we have, $S = \frac{\partial \theta}{\partial z} = \frac{\Delta\theta}{\Delta z} = \frac{(306-299)^\circ K}{10^3 m} = 7 \times 10^{-3\circ} Km^{-1}$

Then the geostrophic wind shear at station 445 can be computed;

$$\frac{\partial V_g}{\partial z} = \frac{g}{f\theta} S\tan\alpha = \frac{9.8}{10^{-4} \times 304} \times 7 \times 10^{-3} \times 6.9 \times 10^{-3}$$

$$\approx 1.56 \times 10^{-2}\ \sec^{-1}$$

- Comment on possible reasons for differences in the values.

$$\frac{\partial V}{\partial z} = 2 \times 10^{-2}\ \sec^{-1} \quad ; \quad \frac{\partial V_g}{\partial z} \approx 1.56 \times 10^{-2}\ \sec^{-1}$$

The possible reasons for the difference could be due to

* the ageostrophic components in (real) atmosphere. cyclonic curvature if $V \sim V_{grad}$ $\frac{\partial V_G}{\partial z} < \frac{\partial V_g}{\partial z}$

* the geostrophic assumption used in the calculation of $\frac{\partial V_g}{\partial z}$.

* the round off errors(but not significant).

* the error in the reported $\frac{\partial V}{\partial z}$ and so on.

(d) If pressure $p = p(x, y, \theta, t)$ then

$$\omega \equiv \frac{dp}{dt} = \left.\frac{\partial p}{\partial t}\right)_\theta + V\left.\frac{\partial p}{\partial s}\right)_\theta + \frac{d\theta}{dt}\frac{\partial p}{\partial \theta}$$

Assume the flow is isentropic and use this expression to estimate $\frac{dp}{dt}$ at station 562 North Platte, Nebraska on $\theta = 320^\circ K$. The table below gives observed values of $p_{\theta=320^\circ K}$ at this station at the times. (* Express ω in mb/12hr)

$$@\ 562,\ p_{\theta=320^\circ K} = \begin{cases} 336\text{mb} \ @\ 28/03\text{Z} \\ 296\text{mb} \ @\ 28/15\text{Z} \\ 270\text{mb} \ @\ 29/03\text{Z} \\ 254\text{mb} \ @\ 29/15\text{Z} \end{cases}$$

Solution>>

According to the isentropic assumption, $\frac{d\theta}{dt} = 0.$

So, $\omega = \left.\frac{\partial p}{\partial t}\right)_\theta + V\left.\frac{\partial p}{\partial s}\right)_\theta$ ✓

Using the above table, $\frac{\partial p}{\partial t}$ can be calculated as follows;

for 28/03Z to 28/15z

$$\left.\frac{\partial p}{\partial t}\right)_\theta = \frac{(296-336)mb}{12hr} = -40\frac{mb}{12hr}$$

for 28/15Z to 29/03Z

$$\left.\frac{\partial p}{\partial t}\right)_\theta = \frac{(270-296)mb}{12hr} = -26\frac{mb}{12hr}$$

use these for centered difference @ 15Z $= -33\frac{mb}{12hr}$

for 29/03Z to 29/15Z

$$\left.\frac{\partial p}{\partial t}\right)_\theta = \frac{(254-270)mb}{12hr} = -16\frac{mb}{12hr}$$

So, the mean value of the $\left.\frac{\partial p}{\partial t}\right)_\theta$ is $-27.3\frac{mb}{12hr}$

reasonable

At station 562, roughly $V \approx V_g \cong 30m/\sec$ is obtained from the isotach and the 320K streamfunction charts.

To estimate $\frac{\partial p}{\partial s}$, the 320K streamfunction and isobaric topography charts can be used. Δp (by topography chart) along Δs (by streamfunction chart) was found to be approximately 25mb. Isangle

So, $\dfrac{\partial p}{\partial s} \cong \dfrac{\Delta p}{\Delta s} = \dfrac{(310-285)mb}{550\times10^{3}m} \approx 4.5\times10^{-5}\,mb/m$?

Hence,

$$V\frac{\partial p}{\partial s} = 30\times4.5\times10^{-5} = 1.35\times10^{-3}\,mb\sec^{-1}$$

$$\approx 58.3\frac{mb}{12hr}$$

Finally, adding two above results,

$$\omega = -27.3\frac{mb}{12hr} + 58.3\frac{mb}{12hr} = 31.0\frac{mb}{12hr} \checkmark$$ ie sinking motion

== **END**