Laseur

① Gradient wind eq. (a) derive eq for V_g (b) discuss four solutions for V_g. What are possible? (c) for anticyclonic curvature gradient wind eq implies weak winds in center of H. why? (d) differentiate V_g by $\frac{\partial}{\partial z}$. For $\frac{\partial k_T}{\partial z} \sim 0$ what can you say about $\frac{\partial V_g}{\partial z}$ + $\frac{\partial V_{grad}}{\partial z}$ for normal L + H flow?

② Potential vorticity (a) derive from vector eq. (b) discuss how to compute PV, general characteristics in troposphere + stratosphere. (c) discuss PV changes, PV conserved (d) compare/contrast ζ_θ + ζ_p

③ Norwegian school (a) summarize ideas of Norwegian school. why successful? (b) Subsequent work has led to modification to NS. discuss 2 major modifications to NS.

④ Moist static energy (a) define + describe. does it include all forms of energy? (b) vertical profile of h in undisturbed tropics + in disturbed tropics. (c) from difference in two profiles in (b) discuss physical processes leading to difference. what is the scale of these processes. (d) what tropical phenomena satisfy (c). what does this imply for tropical NWP?

⑤ From eq. of motion (a) make geostrophic assumption. Why physically do we omit? (b) rewrite eq. of motion using V_g. diagram of forces. (c) in light of (b) how are $\frac{dV}{dt}$ + V related, i.e. in terms of magnitude + direction (d) make gradient wind assumption, sketch balance of forces. Show V_{geo}, V_{grad}, V for cyclonic flow

⑥ How do we determine ($\frac{\Phi}{g_0}$) geopotential heights from radiosonde observation? discuss physical assumptions, importance of baseline observation, impact random/systematic errors on T, p, T_d + Φ.

⑦ Take Q_1 + Q_2 budget eqs. (a) if vertical advection dominates sketch Q_1 + Q_2 profiles (b) what other physical process are important? In this framework how do Q_1 + Q_2 relate to (c) outline assumptions made by Arakawa to incorporate physical process in (a) + (b). What is neglected? large scale atmosphere (d) Discuss deficiencies in Arakawa approach.
(for (c) + (d), repeat for Kuo scheme + contrast w/ AS scheme.

⑧ In local cylindrical coord., sketch + discuss structure of mature hurricane. Include anomalies of T, φ, |V| from average tropical conditions. ~~Include~~ Discuss physical processes responsible for creation + maintenance of this structure.

⑨ Given the observation of strong vort. max + gradients in around L, vort min + weak gradients in around H, discuss using the vorticity eq these observations.

MET? 5510 : LaSeur (?)

• The "Norwegian School" (NS) is generally credited with the first major successful application of the scientific method to the problem of the relationships between extra-tropical cyclones and fronts. Their empirical observational and analytical work plus a few theoretical concepts resulted in a quite comprehensive framework between fronts and the formation, structure and life-cycle of cyclones.

A) Summarize concisely the main aspects of the "Norwegian School" concepts as to those relationships, and discuss briefly why they were successful where previous work was not.

B) Subsequent studies since WWII has led to major modifications of several of the Norwegian concepts. Discuss at least two aspects for which this work has produced such major modification to Norwegian ideas about fronts and cyclone formation, structure and life history. Where appropriate make reference to specific work that led to these modifications.

Sol)

A) Main aspects of NS : The NS made advances on two fronts : descriptive discoveries/advances and new concepts/theories.

Descriptive Discoveries/Advances :

① Lines of confluence : Streamline analysis → discovery of lines of confluence which are observed to correspond with areas of precip.

- Steering line : Observed to extend from center of cyclone, where the tangent of this line pointed to directed of propagation (hence, "steering"). They hypothesized the line extended over the cold air → Steering surface.
- Squall line : The observed another line extending behind the steering line, associated with a more narrow band of intense precip. This line was believed to slope back into cold air → Squall surface.

② Thermal distribution : lines of confluence were observed to correspond to the thermal pattern → warm front (steering line) + cold front (squall line). Note that these lines of confluence were observed to be ahead of the actual thermal frontal boundaries.

③ Weather : They described that at the warm front, warm moist air rises over the cooler air ahead of it → broad area of rain, whereas at the cold front, cold air undercuts warm moist air → rapid ascent producing a narrow band of precip. more intense than observed at warm front.

④ Vertical motion : Precip patterns were observed to be associated with vert. motion, with ascending motion in warm air and descending motion in cold air → thermally direct circulation.

⑤ Life cycle : Cyclones were observed to form along smooth polar front boundary. Small disturbance on this boundary grew to form cyclones, with a low pressure center at the sfc. As the cyclone grows, the cold air eventually "catches up" and undercuts the warm front, forming an occlusion. They observed the appearance of the occlusion to to mark the beginning of cyclone decay.

New Concepts/theories

① Polar front : Cyclones form along polar front seperating cold air to north from warm air to south (in N.H.). Moreover the temperature contrast is concentrated in narrow layer, which on the larger scale can be considered as a zero-order discontinuity in temp, moisture, and pressure gradient.

② Cyclogenesis : They used theories of others to form their theory of cyclogenesis.

- Margules : Demonstrated that a cyclonic shear zone between air masses could be at equilibrium, and
- Helmholtz : provided a theoretical basis how perturbations along this discontinuity could become stable and grow
- Streamline analysis by NS supported theory that cyclones form along frontal zones.

③ Cyclone families : This concept resulted from their observation that cyclones that crossed Europe tended to come in succession, and that properties of individual cyclones tended to be a function of their place in the series. They proposed that a cyclone forms on the polar front, and that successive cyclones form along the trailing cold front of the previous cyclone.

④ General circulation : They proposed that cyclones provide for meridional energy exchange between warm, tropical air masses and cold, polar air masses

⑤ Energy exchange : Source of energy for cyclones was conversion of potential energy → kinetic energy. This is accomplished by direct circulation (warm air rising, cold air sinking). This energy conversion → increase in winds and deepening of depression. Occlusion cuts off source of potential energy → loss of energy through friction and pumping cold air upwards.

Why was NS successful while others were not?

① Scientific method : First to approach meteorology in this way. First they attempted to adequately describe the process, and then to understand it.

② Adequate obs. network : In describing the atmosphere, they saw need to form a network dense enough to determine the spatial structure, and measurements frequent enough to detect the temporal changes. They then had the where-with-all and resources to actually build such a network.

③ Better Analysis methods : They used better methods to extract information from their network, such as streamline analysis of wind fields. They also incorporate analysis of upper clouds to infer properties of upper atmosphere such as wind speed and stability (no in-situ measurement available), sometimes inferred to as indirect aerology.

④ Conceptual model : Used ideas of Margules and Helmholtz to provide a physical basis to their model of cyclones.

⑤ Prediction : Carried scientific method to next logical step, prediction.

B) Modifications to NS concepts.

Examples to modification of NS concepts.

① 2-D vs 3-D : The NS rooted their explanation of frontogenesis + cyclogenesis in a simple 2-D horizontal framework, in which they extended features measured at sfc. to tropopause. In the 1950's, Newton, and Reed + Sanders (among others) introduced the concept of type II fronts, which addressed the upper-level front through stratospheric injection, or "tropopause folding."

② Exchange across fronts : NS treated fronts as material sfcs. → no air exchange across fronts. Later studies revealed small-scale eddy diffusion and turbulent fluxes can lead to significant air exchange across fronts.

③ Circulations : NS considered only direct circulations. For example, Newton showed that near the delta region of the jet stream, div. and deformation were important to cyclogenesis. It is not surprising the NS didn't deal with this, as the jet stream hat not yet been discovered!

6

note

Discontinuity surfaces and concepts of fronts

I. Theoretical properties of discontinuity surfaces; based upon ideal, incompressible, immiscible fluids.

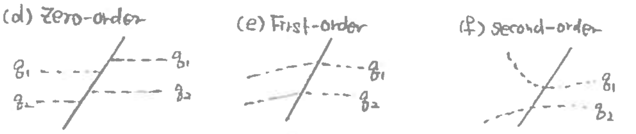
(a) discontinuity sfc is a material sfc. : ie no mixing across sfc.

(b) pressure is always itself continuous across sfc. i.e no infinite pressure forces (dynamic boundary condition)

(c) motion ⊥ to discontinuity sfc is continuous across sfc. (kinematic boundary condition)

Discontinuity may be of different order, ie

(d) Zero-order (e) First-order (f) second-order



ie, quantity itself is discontinuous

i.e 1st order derivative (slope) are discontinuous

ie 2nd order derivatives (curvature) are discontinuous

II. "Norwegian School" frontal concepts

adapted properties in I to atmosphere on synoptic scale and considered approx. to to valid on that scale;

(a) Fronts approx. as zero-order discontinuity in ρ, T, or θ

(b) I(a), I(b) + I(c) are valid on that scale

It follows that

(c) pressure (geopotential) is discontinuous of first-order, ie. isobar kink at fronts

(d) If $V \simeq V_g$, V_n is continuous, but V_t is discontinuous of zero-order, i.e cyclonic shear across front

Approach of NS dominated by kinematic processes of quasi-horizontal motion of air-masses, especially, significant vertical motion on synoptic scale associated with lifting of warm air by frontal sfc; ie assoc. of fronts + precipitation.

III. Modifications of II from studies since WWII

(a) Atmospheric fronts are transition zones bounded by two 1st order discon in ρ, T, θ

(b) Such frontal zone boundaries cannot be treated as material sfcs, small sc eddy diffusion, conduction + turbulent fluxes allow significant exchange of air across boundaries.

It follows that:

(c) discontinuity in pressure (geopotential) is of 2nd order

(d) " in $V \simeq V_g$ is of 1st order

(e) " in divergence, vorticity is zero-order

Also has been established that frontal zones importantly related to dynamic processes (accelerations), with associated divergence + vertical motion

Major discoveries of NS :

Cyclone contains two principal lines of confluence or thermal boundary lines distinguished by thermal properties, which are warm front and cold fronts. These lines meet at low pressure center and mark the boundary of a region of warm air. The warm moist air flows up over the cold air, forming a system of clouds and rain. In the rear the cold air flows underneath the warm air. Great breadth of the zones of clouds and precip in front of the warm front.

As the cyclone develops, the cold air in the rear pushes forwarding displacing the warm air upward and narrowing the tongue of warm air at ground. This transport of P.E. stored in the initial system into K.E. Finally the cyclone consists entirely of cold air. this is the occlusion process. At this stage there is no P.E that can be directly transformed into K.E. As a result, the cyclone decays.

NS was able to describe the observed structure of fronts + cyclones and their evolution in time. The basic fundamentals of the theory of front were definitely established. The existence of fronts, frontal characteristic of a cyclone, energy conversion in cyclones and the cloud and precip system associated with them were all verified.

A number of abstract previous ideas were thus explained in a physically obvious way on the basis of sound dynamic + energetic reasoning.

The reasons for quick success

① Very dense system of observing stations organized in Norway during that period.

② Their approach was scientific. They introduced new ideas and at the same time incorporated and substantiated many of the results of previous theories.

③ They developed methods of displaying these significant phenomena on surface weather maps by introducing new graphical representation

Summary

① Relating baroclinic instability + cyclogenesis based on zero-order discontinuity in ρ, T, θ at surface. This discontinuity leads to instability on synoptic scale which intensify so long as warm air rise and cold air sink until full occlusion of the cyclone.

② The concepts of the life-cycle of cyclones and cyclone families based on the conversion of P.E. to K.E. This requires the existence of a strong surface front for cyclogenesis. Cyclone is a stage in the life cycle of cyclones which were subsequently seen as never developing along an extended polar front.

③ Cyclones must be regarded as a part of the general circulation. Cyclone links the interchange of air between the polar region and the equatorial zone

After WWII modifications to NS concepts

a) Fronts are transition zones with 1st order discontinuity of T+ρ

b) Frontal surface are not treated as a material surface

→ Small-scale eddy diffusion + turbulent fluxes allow significant exchange of air across the boundaries

c) Discontinuity of pressure is of second order

d) " " divergence + vorticity is of zero order

e) Frontal zone is related to dynamic process

The important models of fronts after WWII are given below

Palmer (1949):

- The folded tropopause is replaced by a broken region, separating the tropospheric frontal layer + the tropopauses overlying the tropical + polar air masses.
- The tropospheric frontal zone separates polar from tropical air and is discontinued in the upper troposphere where the horizontal temperature gradient vanishes.
- A jet core at the level of tropical tropopause situated above the position of the frontal layer in the midtroposphere.

Bergen (1952) ⟶ look at other notes.

Reed + Danielsen (1959) ↗

Summary of NS

① Fronts are zero-order discontinuity in ρ, T, θ

② Discontinuity surface is a material surface, no mixing across the surface.

③ Pressure is first order discontinuous, so isobars kink at front.

④ If $V \sim V_g$ along the front wind or cross-front pressure gradient is discontinuity of first order

⑤ Precip. patterns are related to upward motion arising from the relative motion of air w.r.t. inclined frontal surface

The contributions of the Norwegian School

5510

MET? : LaSeur (75 minutes, 1991)

• Beginning with the work of the "Norwegian School", some 75 years ago, Meteorology has been dominated by studies of the intimate relationships between fronts and mid-latitude cyclones, both descriptively and theoretically.

Using the format of a lecture presentation appropriate for Sr. level students in Meteorology, prepare a summary disc. of these interrelationships between fronts and cyclones which includes:

1. A brief summary of the "Norwegian School" points of view.
2. A disc. of the evolutionary changes in the "Norwegian School" concept that have resulted from work since World War II.
3. A disc. of aspects of these interrelationships that need further investigation at the present time.

(Sol + note from Russ)

ⓐ Major descriptive discoveries and advances made by the NS include the following:

① fronts

- Every moving cyclone has two lines of confluence distinguished by characteristic thermal properties; the steering (warm) + squall (cold) lines. The steering line was called a warm front. The squall line was called a cold front. These names emphasized that the lines of confluence in the wind were more appropriately thermal boundaries. The warm front entered the cyclone from the front and appeared to indicate the direction of cyclone propagation. The cold front entered in the rear of the cyclone. The fronts joined in the center of the cyclone at the northernmost tip of the tongue of warm air. Warm + cold air flowing into the cyclone are separated by these boundaries.
- Warm moist air flows up over the cold air separated by the warm front. Extensive clouds + widespread, uniform precip. in ahead of the warm front. Peak precip intensity occurs just prior to frontal passage.
- To the rear of the cyclone, cold air flows underneath the warm air. Rising motion in the warm air is more violent + irregular at the cold front and precip is far less extensive. In both cases (warm + cold fronts) the precip falls on the cold side of the front.
- Precip patterns are associated w/ vertical motion fields. There is ascending motion in the warm air + descending motion in the cold air (direct circulation). The main area of rising air is along the warm front. (?)
- Fronts separate polar + tropical air throughout the depth of the troposphere. Frontal surfaces slope towards the cold air. The warm front has a very small slope (~1 : 100) which could be reasonably calculated from Margules' slope formula.

② Life cycle

- The initial formation of a cyclone was given by two oppositely directed currents of air of different temperatures separated by a straight line. (Thus, we see that in the NS fronts are a pre-requisite to cyclone development unlike the baroclinic instability ideas of Charney, Eady + others)
- Warm air bulges towards the cold air with a low pressure center formi at the top of the projecting tongue of warm air. The warm tongue is identified to the warm sector of the cyclone. Ascending air is found along the warm front. Cold air descends behind the cold front.
- Further growth of the cyclone leads to cold air behind the cold front to reach + undercut the warm air along the warm front. This is the process of occlusion and makes the decay stage of the wave cyclone and its associated fronts.

③ Cyclone families + the polar fronts.

A series of cyclones form along the polar front which separates polar and tropical air. The formation of local cyclones and their associated fronts play a role in the general circulation by transporting warm tropical air poleward + cool polar air equatorward. Wave cyclones were a physical process by which the tropics could get rid of surplus energy + the polar regions could make up for a deficit of energy

⇒ implications for global energy balance of the atmosphere.

ⓑ Why was the NS so successful given the slow progress in the prior 50 to 100 years?

① They followed the classical scientific method. Availability of data from a dense network of sfc stations allowed them to observe the features they described in their polar front cyclone model. They used better analys methods to extract more meaningful information from the data. Wind fields were analyzed using streamlines. Confluence/diffluence of streamlines were emphasized. They felt that given a description of the current state of the atmosphere it would be possible to predict its future evolution.

② They applied ideas developed by Helmholtz + Margules to provide a physical basis for their model. Their synthesis of ideas from the 19th + 20th centuries helped in the development of their conceptual model. From Helmholtz came the idea of shear instability. Given a line of discontinuity in the thermal and wind fields, shear instability suggested that eventually a perturbation would develop. This perturbatio was the wave cyclone. With warm air moving poleward and cool air equatorward the ideas of Margules described how P.E. was converte to K.E. His slope eq. showed how a sloping frontal sfc could exist.

③ The model described a close relationship between observable cloud patterns + low level frontal boundaries. It provided a physical basis for diagnosing regions of rising motion and hence precip. Given a network of observing stations their ideas could be easily applied to the forecasting problem. Their observational emphasis provided a basis for theoretical studies of cyclogenesis.

ⓒ New concepts from NS

① Cyclones have a definite <u>life cycle</u>, in which the wave cyclone is just one stage in the life history of cyclones + cyclone families. Prior to cyclogenesis there must be a pre-existing front (line of discontinuity). The shear instability ideas of Helmholtz suggest that eventually a wave will develop along this line of discontinuity. Margules ideas of conversion from P.E to K.E describe how the thermally direct circulations in growing waves furnish energy to the wave.

② New cyclones form at the <u>triple point</u> intersection of the cold, warm + occluded fronts

③ After occlusion the cyclone <u>decays</u>, the idea of rainfall diminishes + the intersection of the frontal boundaries is no longer near the center of the storm.

④ <u>Fronts</u> are considered to by <u>surfaces</u> w/ <u>zero order discontinuities</u> in temperature, density, and along-front wind (or, equally, cross front pressure gradient). Fronts are viewed as <u>material sfc</u> across which large scale exchanges of air are not possible. Fronts are formed primarily as a result of <u>2D horizontal motions</u>. (deformation + convergence)

⑤ <u>Source of energy for general atmospheric circulation</u> is the meridional thermal gradient between the tropics + polar regions. Wave cyclones form a link between the two areas and allow for conversion of APE into KE. They noted that as the cyclone develops, warm tropical air rises and cool polar air descends. This converts APE → KE. The winds in the cyclone accelerate and the central pressure drops. Once the occlusion stage is reached there is no longer a store of PE available for transfer to KE. The cyclone weakens.

ⓓ Physical processes used to describe structure + behavior of frontal cyclones

- Helmholtz idea of shear instability for the development of wave cyclones along a frontal boundary.
- Margules idea of APE → KE conversion and his ideas on frontal slope to explain how thermally direct circulations in a growing wave cyclone lead to intensification and to explain why frontal sfc slope
- Considered 2D horizontal deformation + convergence to concentrate sfc temperatures ⟹ 2D frontogenesis
- The idea of QG dynamics are implied in their linking of sfc effects to features aloft ⟹ patterns of vertical motion
- Poleward heat flux for global atmospheric circulation is accounted for (in part) by polar front wave theory.

ⓔ <u>Deficiencies</u> of NS model

① Assume polar front is prerequisite to cyclogenesis. This is not necessarily true. Baroclinicity is required

② One of the main components of the model, the warm front, is weak and limited in horizontal extent for mature and dying cyclones.

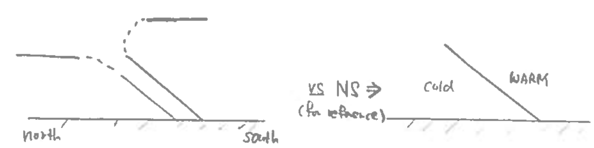
③ The formation of occluded front by collision of cold + warm fronts is not documented. Occlusions are new fronts that form northward from the junction of cold and warm fronts. Instant occlusion is that in which comma cloud features join w/ open waves to produce mature occluded systems.

④ The existence of cyclogenesis on the cold air side of major frontal bands was not included.

⑤ They lack a description of upper tropospheric structure as well as their interaction w/ low level disturbances.

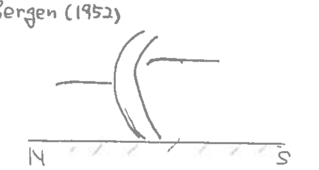
ⓕ <u>Modifications</u> to NS concepts after WWII

- Changes to the idea of fronts
  - (1) fronts are transition zones w/ 1st order discontinuities in T and ρ
  - (2) frontal surface is not a material sfc
    ↳ small scale eddy diffusion + turbulent fluxes allow significant exchange of air across frontal boundaries
  - (3) discontinuity in pressure is second order, along-front wind is 1st order discontinuity
  - (4) discontinuity in $\nabla \cdot \mathbf{V}$ + ζ are zero order
  - (5) frontal zone is related to dynamic processes
- Palmer + Nagler (1949)



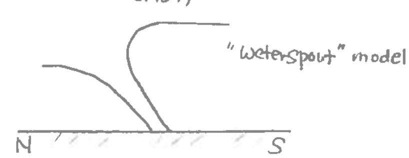
- The folded tropopause is replaced by a break region separating the tropospheric frontal layer and the tropopauses overlying the tropical + polar air masses.
- The tropospheric frontal zone separates polar from tropical air and is discontinued in the upper troposphere where the horizontal temp gradient vanishes.
- A jet core at the level of the tropical tropopause is situated above the position of the frontal layer in the mid troposphere.

- Bergen (1952)



- The frontal zone is defined by strong cyclonic shear
- The tropospheric frontal layer is continued upward in the region of the tropopause break in the vicinity of max wind.

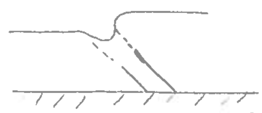
- Reed + Danielsen (1959)



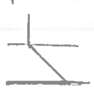
- The polar and tropical tropopauses are respectively joined w/ the lower (cold) and upper (warm) boundary of the tropospheric frontal layer.
- The cyclonic shear zone in the stratosphere has a scale of hundreds of km.
- There is stratospheric-tropospheric exchange through the hypothesis that upper level frontal systems result from tropopause folding in which upper and mid tropospheric subsidence transport lower stratospheric air downward into the troposphere.

- upper level fronts are developed through the effect of tilting due to differential subsidence associated w/ tropopause folding.
- upper level fronts separate stratospheric air from tropospheric air rather than separating polar air from tropical air.
- upper level fronts are not required to extend to the surface, but could arise independently of low level fronts and frontogenesis processes.

• Bjerknes + Palmer (1937) ← prior to WWII!



- vertical lapse rate of temp + overlaying isotherms of stratosphere



- surface front extends from sfc to upper troposphere
- polar air is separated from tropical air at all longitudes in troposphere
- a folded tropopause occurs in the upper frontal zone. the exchange of stratospheric air and tropospheric air can occur in this folding region.

④ Future work

① observations documenting the evolution of fronts throughout the life cycle of baroclinic waves.

② resolution of the relative importance of 2- and 3-D dynamical processes in upper level frontogenesis and consideration of the potential importance of dynamical processes excluded by the geostrophic momentum approx.

③ investigation of the interaction between upper level frontogenesis and baroclinic wave amplification and their effect on low level cyclogenesis.

④ Consideration of the relationship between upper level frontal circulations + low level circulations.

⑤ Investigation of the interaction between mesoscale convective systems and upper level frontal systems.

⑥ the formation of occlusion fronts and the warm core cyclone (explosive cyclogenesis)

Gradient wind.

MET5510C: LaSeur (45-60 minutes) ***

- Neglecting viscosity, the equation of motion for the (horizontal) wind may be written:

(1) $\dot{\mathbf{V}} + f\mathbf{k} \times \mathbf{V} = -\nabla\phi$

in which $\mathbf{V}$ = wind vector, $\dot{\mathbf{V}} \equiv d\mathbf{V}/dt$ is the acceleration of $\mathbf{V}$ and ϕ is the geopotential, $\mathbf{k}$ the vertical unit vector and f is the Coriolis parameter.

(A) What form does (1) take when the geostrophic assumption is made, i.e., what is neglected physically?

(B) With the result in A, show that (1) may be re-written as:

(2) $\dot{\mathbf{V}} = -f\mathbf{k}(\mathbf{V} - \mathbf{V}_g)$

And sketch a vector diagram of the relationship of $\mathbf{V}$, $\mathbf{V}_g$ and $\dot{\mathbf{V}}$.

(C) Discuss briefly the implications of B as to the direction and magnitude of $\dot{\mathbf{V}}$ with respect to $\mathbf{V}$.

(D) What approximation as to $\dot{\mathbf{V}}$ is made when the gradient wind assumption is used? Give any form of the gradient wind equation that follows from this assumption, and sketch a vector diagram illustrating $\mathbf{V}_g$, $\mathbf{V}_G$ and $\dot{\mathbf{V}}$ for cyclonically curved flows.

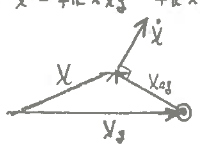
Sol) (A) For geostrophic assumption, we neglect accel. of wind, $\frac{d\mathbf{V}}{dt}$

$\rightarrow \boxed{f\mathbf{k} \times \mathbf{V}_g = -\nabla\phi}$, where Coliolis is balanced by pressure gradient.

(B) From (A) + eq (1), substitute for $\nabla\phi$

$\rightarrow \dot{\mathbf{V}} + f\mathbf{k} \times \mathbf{V} = f\mathbf{k} \times \mathbf{V}_g = -\nabla\phi$ solving for $\dot{\mathbf{V}}$

$\rightarrow \dot{\mathbf{V}} = f\mathbf{k} \times \mathbf{V}_g - f\mathbf{k} \times \mathbf{V} = -f\mathbf{k} \times (\mathbf{V} - \mathbf{V}_g) = -f\mathbf{k} \times \mathbf{V}_{ag}$



So the accel. is orthogonal to the ageostrophic wind component

(C) Magnitude : We see that there must be an ageostrophic component for there to be an acceleration. Hence the larger the magnitude of $\mathbf{V}_{ag}$, the larger $\dot{\mathbf{V}}$.

Direction : The direction of the accel. ($\dot{\mathbf{V}}$) is to the right of $\mathbf{V}_{ag}$

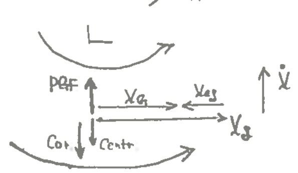
(D) For gradient wind, we assume a 3-way balance between (1) Coriolis, (2) pressure gradient, and (3) Centripetal forces.

$\rightarrow K_T V_G^2 + f V_G = f V_g$, $K_T = \frac{1}{R_T}$

divide by $fV_G \rightarrow \frac{1}{f} K_T V_G + 1 = \frac{V_g}{V_G}$

As for normal cyclonic $K_T > 0 \rightarrow V_g > V_G$

Hence a vector diagram may appear as follows.



Hence for cyclonic flow in gradient balance, the flow is subgeostrophic and accel. towards lower pressure.

Gradient wind

MET5510C: LaSeur (45-60 minutes) **

- To a good approximation the gradient wind V_G may be related to the geostrophic wind V_g by the expression:

$$(1) \qquad K_T V_G^2 + fV_G = fV_g$$

in which $K_T \equiv 1/R_T$ is the curvature of the air trajectory (the reciprocal of the radius of curvature of the trajectory), and f is the Coriolis parameter.

a. Show that one (convenient) solution of (1) may be obtained in the form:

$$(2) \qquad V_G = \frac{2V_g}{1 \pm \left(1 + \dfrac{4K_T V_g}{f}\right)^{1/2}}$$

Hint: Write (1) as a quadratic in $1/V_G$, solve, and invert.

b. Since K_T may be > or < 0 and there is a choice of $\pm(\ )^{1/2}$, (2) contains four possible cases. Discuss concisely those cases which are physically realistic in the atmosphere.

c. For anticyclonic curvature, (2) may be interpreted as implying the existence of a limiting value of K_T. Why is this so, and what is the limiting value of K_T for $V_g = 50$ m/s and $f = 10^{-4}\,\mathrm{s}^{-1}$?

d. Show that if (1) is differentiated with respect to z that

$$(3) \qquad \frac{\partial V_G}{\partial z} = \left(\frac{V_G}{2V_g - V_G}\right)\left(\frac{\partial V_g}{\partial z} - \frac{V_G^2}{f}\frac{\partial K_T}{\partial z}\right)$$

(or an equivalent expression)

e. Given that $\partial K_T/\partial z \sim 0$, consider the expected relationship of $\partial V_G/\partial z$ and $\partial V_g/\partial z$ for normal cyclonic and anticyclonic gradient winds.

f. If $\partial K_T/\partial z \neq 0$ consider the circumstances in which $\partial V_G/\partial z$ may be opposite in sign of $\partial V_g/\partial z$ for normal cyclonic and anticyclonic gradient winds.

Sol)

(A) Divide eq (1) by $fV_G^2 \rightarrow -\frac{K_T}{f} - \frac{1}{V_G} + V_g\left(\frac{1}{V_G}\right)^2 = 0$

For $ax^2+bx+c=0$, quadratic formula $\rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\rightarrow \frac{1}{V_G} = \frac{1 \pm \sqrt{1+4V_g K_T/f}}{2V_g}$

$\rightarrow V_G = \frac{2V_g}{1 \pm \sqrt{1+4V_g K_T/f}}$

(B) Case #1 ; $K_T > 0$, $+(\cdots)^{1/2} \Rightarrow$ normal cyclonic flow → flow around normal L

$f>0, K_T>0, V_g>0 \longrightarrow$ denom. >0

$\rightarrow \boxed{V_g > V_G}$

Case #2 : $K_T < 0$, $+(\cdots)^{1/2} \Rightarrow$ normal anticyclonic flow → flow around normal H

$f>0, K_T<0, V_g>0 \longrightarrow 4V_g K_T/f \geq -1$ for real root

$\longrightarrow$ denominator is between 1 + 2.

$\rightarrow \boxed{V_g < V_G < 2V_g}$

Case #3 : $K_T < 0$, $-(\cdots)^{1/2} \Rightarrow$ anomalous anticyclone flow

$f>0$, $K_T<0$, $V_g>0 \longrightarrow$ denom. $<1 \longrightarrow \boxed{V_G > 2V_g}$

→ flow around anomalous H → super geostrophic flow. This is physically realistic though not usually observed

Case #4 : $K_T > 0$, $-(\cdots)^{1/2} \Rightarrow$ anomalous cyclonic flow

$f>0$, $K_T>0$, $V_g>0 \rightarrow$ denom. $<0 \Rightarrow V_G$ is negative! Not realistic (as in the natural coord.) The exception is when balance is cyclostrophic around L (e.g., dustdevils, tornadoes)

(C) This comes from the fact that for the solution of eq.(2) to be real.

$1 + 4V_g K_T/f \geq 0 \longrightarrow K_T \geq -f/4V_g$

So for $\begin{cases} V_g = 50\,\mathrm{m/s} \\ f = 10^{-4}\,\mathrm{s}^{-1} \end{cases} \Rightarrow K_T \geq \frac{-10^{-4}}{4 \cdot 50} = -5 \times 10^{-7}\,\mathrm{m}^{-1}$, or $R_T \leq -2 \times 10^6$ m or -2000 km

Hence for anticyclonic flow, the trajectory radius $\geq$ 2000km

This does not apply for cyclonic flow, as the radical in the denom. of eq (2) is real regardless of K_T (as $K_T > 0$ for cyclonic)

(D) $\frac{\partial}{\partial z}(1) \rightarrow$

$$V_G^2 \frac{\partial K_T}{\partial z} + 2V_G K_T \frac{\partial V_G}{\partial z} + f\frac{\partial V_G}{\partial z} = \frac{\partial K_T}{\partial z}V_G^2 + (2V_G K_T + f)\frac{\partial V_G}{\partial z} = f\frac{\partial V_g}{\partial z}$$

Divided by $f \rightarrow \frac{\partial V_G}{\partial z}\underbrace{\left(\frac{2V_G K_T}{f} + 1\right)}_{(*)} = \frac{\partial V_g}{\partial z} - \frac{1}{f}\frac{\partial K_T}{\partial z}V_G^2$

From eq (1), $K_T V_G^2 + fV_G = fV_g$, divide by $fV_G \rightarrow \frac{K_T V_G}{f} + 1 = \frac{V_g}{V_G}$

$\rightarrow \frac{K_T V_G}{f} = \frac{V_g}{V_G} - 1 = \frac{V_g - V_G}{V_G} \rightarrow \frac{2K_T V_G}{f} + 1 = 2\frac{V_g - V_G}{V_G} + \frac{V_G}{V_G} = \frac{2V_g - V_G}{V_G}$

Substitute for (*) above, divide by on both sides

$$\rightarrow \frac{\partial V_G}{\partial z} = \left(\frac{V_G}{2V_g - V_G}\right)\left(\frac{\partial V_g}{\partial z} - \frac{V_G^2}{f}\frac{\partial K_T}{\partial z}\right)$$

(E)

$\frac{\partial K_T}{\partial z} \sim 0 \rightarrow$ (3) becomes $\frac{\partial V_G}{\partial z} \simeq \frac{V_G}{2V_g - V_G}\frac{\partial V_g}{\partial z}$

Recall for normal cyclonic, $K_T > 0$, $V_g > V_G \Rightarrow \frac{V_G}{2V_g - V_G} < 1$

Hence $\frac{\partial V_G}{\partial z} < \frac{\partial V_g}{\partial z}$. Also note that $\frac{\partial V_G}{\partial z}$ + $\frac{\partial V_g}{\partial z}$ have same sign.

Recall for normal anticyclonic, $K_T < 0$, $V_g < V_G < 2V_g \Rightarrow \frac{V_G}{2V_g - V_G} > 1$

Hence $\frac{\partial V_G}{\partial z} > \frac{\partial V_g}{\partial z}$ (again, same sign)

(F) For normal cyclonic, we already shown that $0 < \frac{V_G}{2V_g - V_G} < 1$

Hence from eq (3), we get opposite signs when $\frac{V_G^2}{f}\frac{\partial K_T}{\partial z} > \frac{\partial V_g}{\partial z}$, or

$\boxed{\frac{\partial K_T}{\partial z} > \frac{f}{V_G^2}\frac{\partial V_g}{\partial z}}$ So for $\frac{\partial V_g}{\partial z} > 0$, $f > 0$,

condition is for increasing cyclonic curvature with height

For normal anticyclonic, we have already shown that $\frac{V_G}{2V_g - V_G} > 1$

As for cyclonic, we get opposite signs when $\frac{\partial K_T}{\partial z} > \frac{f}{V_G^2}\frac{\partial V_g}{\partial z}$.

However, we recall that $K_T < 0$ for anticyclonic flow.

So for $\frac{\partial V_g}{\partial z} > 0$ (usual case) + $f > 0$ $\Rightarrow \frac{\partial K_T}{\partial z} > 0$

But as $K_T < 0$, condition is for decreasing anticyclonic curvature with height.

Vort. distribution

5510
MET? : LaSeur (?) *

- Surface weather maps typically show cyclones associated with concentrated maxima of absolute vorticity which have large values (two to four times f) and large gradients of vorticity; and anticyclones associated with minima of absolute vorticity that typically occupy a considerably greater area and with much weaker gradients.

With the use of the appropriate form of the vorticity equation at the earth's surface (consider it flat), discuss why the above observed relationships between absolute vorticity, cyclones and anticyclones are to be expected.

Sol)

(A) Abs. vort. gradient (note: $\eta = \zeta + f$)

Consider vort. eq. in isobaric coords:

$$\underbrace{\frac{\partial}{\partial t}\eta = -\mathbf{V}\cdot\nabla\eta}_{\text{horiz. adv.}} \underbrace{- \omega\frac{\partial}{\partial p}\eta}_{\text{vert. adv.}} \underbrace{- \eta\nabla\cdot\mathbf{V}}_{\text{DIV}} \underbrace{+ \mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla\omega\right)}_{\text{tilting/twisting}}$$

For synoptic scales in mid-lats, we can neglect the tilting/twisting term to a fairly good approx. Also we can combine the advection terms and time deriv. → total derivative of η. Hence the above eq. becomes

$$\frac{d}{dt}\eta \simeq -\eta(\nabla\cdot\mathbf{V}) \longrightarrow \frac{1}{\eta}\frac{d\eta}{dt} = \frac{d}{dt}[\ln\eta] = -\nabla\cdot\mathbf{V}$$

Integrate over time ($t_o \to t$, $\Delta t = t - t_o$); assume div. changes little over small Δt

$$\to \int_{t_o}^{t} d\ln\eta = -\int_{t_o}^{t}\nabla\cdot\mathbf{V}\,dt \longrightarrow \ln\left(\frac{\eta_t}{\eta_o}\right) \simeq -(\nabla\cdot\mathbf{V})\Delta t$$

$$\to \eta_t = \eta_o \exp[-(\nabla\cdot\mathbf{V})\Delta t] \qquad \begin{cases} -\text{DIV} < 0 = \text{DIV} \\ -\text{DIV} > 0 = \text{CONV} \end{cases}$$

At sfc, there is convergence (−DIV) for cyclones, while there is divergence for anticyclones. Hence from the above approx. for vort. change, η grows exponentially for cyclones while there is exponential decay of η for anticyclones. Hence large magnitudes + gradients of η can build for cyclones, while gradients of η are prevented from building to large values for anticyclones. (Note: in the real atmosphere, this growth/decay is not truly exponential, as other factors neglected in this simple representation tend to partially counter the growth/decay)

(B) Horiz. scale/area: Recall the following form of the gradient wind eq. $K_T V_G^2 + fV_G = fV_g$, $K_T = 1/R_T$, R_T = Radius (traj.), G − gradient, g − geostrophic

If you divide by V_G^2, you can solve for $\frac{1}{V_G}$ using the quadratic form. Inverting solution,

$$\to V_G = \frac{2V_g}{1 \pm \left(1 + \frac{4K_T V_g}{f}\right)^{1/2}}$$

For real solutions, $\left(1 + \frac{4K_T V_g}{f}\right) \geq 0$. For cyclonic flow in the N.H., $K_T > 0$, $V_g > 0$, $f > 0$. Hence this quantity will always be real and hence there is no restriction on K_T. However for anticyclonic flow in the N.H., $K_T < 0$, $V_g > 0$, $f > 0$, Hence

$$1 + \frac{4K_T V_g}{f} > 0 \longrightarrow K_T > -\frac{f}{4V_g} \longrightarrow R_T < -\frac{4V_g}{f}$$

Noting that $R_T < 0$ for anticyclonic, $|R_T| > \frac{4V_g}{f}$

Hence we see the horiz. scale of an anticyclone (crudely represented by R_T) must be larger than some minimum value, while no such restriction is required for cyclones. Note that the restriction on R_T for anticyclones allows smaller scales in higher lats. (where f becomes large)

Sol)

Consider the vorticity eq in isobaric coords

$$\frac{\partial}{\partial t}(\zeta+f) = \underbrace{-\mathbf{V}\cdot\nabla(\zeta+f)}_{\text{horiz. adv.}} \underbrace{- \omega\frac{\partial}{\partial p}(\zeta+f)}_{\text{vertical adv.}} \underbrace{- (\zeta+f)\nabla\cdot\mathbf{V}}_{\text{divergence term}} \underbrace{+ \mathbf{k}\cdot\left(\frac{\partial \mathbf{V}}{\partial p}\times\nabla\omega\right)}_{\text{twisting/tilting}}$$

f is not a function of time but we include f for reasons that will become apparent later.

At mid-lat. synoptic scales, we may rewrite the above to a good approx. as

$$\frac{d}{dt}(\zeta+f) = -(\zeta+f)\nabla\cdot\mathbf{V} \longleftarrow \text{drop twisting/tilting term}$$

or

$$\boxed{\frac{d}{dt}\ln(\zeta+f) = -\nabla\cdot\mathbf{V}}$$

Apply this near the sfc and integrate over time

$$\int_{t_o}^{t} d\ln(\zeta+f) = -\int_{t_o}^{t}\nabla\cdot\mathbf{V}\,dt$$

Suppose $t - t_o = \Delta t$ is small so that $-\int_{t_o}^{t}\nabla\cdot\mathbf{V}dt \approx -(\nabla\cdot\mathbf{V})\Delta t$

Then $\ln\left(\frac{\eta_t}{\eta_o}\right) = -(\nabla\cdot\mathbf{V})\Delta t$ $\qquad \eta_t = \zeta + f$ at time t, $\eta_o = \zeta + f$ at time 0

or

$$\eta(t) = \eta(0)\exp[-(\nabla\cdot\mathbf{V})\Delta t]$$

With convergence (common in low levels of L) we see that $\eta(t)$ can grow exponentially. (This does not happen in the real atmosphere since other effects we have omitted counter the increase in η). In contrast, in an anticyclone there is low level divergence so $\eta(t)$ exponentially decays. This prevents large gradients of η from building up near the sfc of H. The exponential growth in η near the sfc of Ls allows for strong gradients and large magnitudes of η to build up.

As to the scale of H vs L we turn to the gradient wind eq.

$$K_T V_G^2 + fV_G = fV_g$$

$V_G \equiv$ gradient wind

$V_g \equiv$ geostrophic wind

$K_T \equiv$ Curvature of trajectory of flow $\hookrightarrow \frac{1}{R_T}$

We can solve this for V_G

$$V_G = \frac{2V_g}{1 \pm \left(1 + \frac{4K_T V_g}{f}\right)^{1/2}}$$

For real solutions the term inside $(\;)^{1/2}$ must be > 0. For cyclonic flow $K_T > 0$. As $V_g > 0$ and $f > 0$ (N.H.), there is no restriction placed on the term $(\;)^{1/2}$ for real V_G. But for anticyclonic flow $K_T < 0$ as for real V_G we must have

$$0 < \frac{4|K_T|V_g}{f} < 1$$

$$\boxed{0 < |K_T| < \frac{f}{4V_g}}$$

In terms of radius of curvature ($K_T = 1/R_T$)

$$\boxed{|R_T| > \frac{4V_g}{f}}$$

Thus we see that the requirement of real solutions for gradient wind balance around an anticyclone requires that the scale of the system (as crudely represented by R_T) must exceed a lower bound. It is interesting to note that the above relationship indicates broader large scale H in equatorial latitudes and smaller scale H's towards polar regions.

There is no similar restriction on R_T for cyclonic flow in gradient balance.

Potential vort.

MET5510C (Mid latitude synoptic system): LaSeur (45-60 minutes) ***

• Combination of the three-dimensional vorticity equation, the continuity equation, and the three-dimensional frontogenesis equation yields the following potential vorticity theorem:

$$\frac{d}{dt}\left(\alpha\nabla_3\theta \bullet \underline{\eta}\right) = \alpha\underline{\eta} \bullet \nabla_3\left(\frac{d\theta}{dt}\right) + \alpha\nabla_3\theta \bullet \left(\nabla_3 \times \underline{F}\right) \qquad (1)$$

in which α = specific volume

θ = potential temperature

$\underline{\eta} = \nabla_3 \times \mathbf{V}$ = 3-dim. Vorticity

$\underline{F}$ = frictional force/mass

(a) Show how (1) may be transformed to the following more useful form:

$$\frac{dP}{dt} = \frac{d}{dt}\left[\left(-\frac{\partial\theta}{\partial p}\right)(\zeta_\theta + f)\right] = (\zeta_\theta + f)\left(-\frac{\partial}{\partial p}\frac{d\theta}{dt}\right) - \frac{\partial\theta}{\partial p}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)_\theta \qquad (2)$$

The term in brackets on the LHS of (2) is known as the potential absolute vorticity;

(b) Explain clearly how this quantity may be evaluated from synoptic analyses. What have studies of P shown as to its variations in magnitude in the troposphere and lower stratosphere?

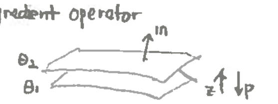
The two terms on the RHS of (2) represent physical processes that may produce changes in *P* following the same volume of air;

(c) Give a clear physical interpretation of these terms and discuss the physical processes in the atmosphere which can produce significant changes in *P*. Under what circumstances is *P* conserved, i.e., $dP/dt = 0$?

Sol)

(a)

Starting with eq (1), we will break the 3-D vectors into components.
We choose for our 3-D coord. system a 2-D sfc of constant θ, with a normal vector $\mathbf{n}$. Hence the 3-D gradient operator (∇) is



$$\boxed{\nabla_3 = \nabla_\theta + \mathbf{n}\frac{\partial}{\partial n}}$$

As $\boxed{\nabla_\theta\theta \equiv 0} \longrightarrow \boxed{\nabla\theta = \mathbf{n}\frac{\partial\theta}{\partial n}}$, $\zeta = \zeta_\theta\mathbf{n} + \zeta_H$

Recall 3-D vort. $\equiv \underset{\sim}{\eta} = (\zeta_\theta + f)\mathbf{n} + \underset{\sim}{\zeta_H}$, $\zeta_H = \zeta_{horiz}$ (orthogonal to $\zeta_\theta\mathbf{n}$)

Also, dot product of orthogonal terms = 0.

Substituting $\longrightarrow$ LHS of eq (1)

$$\frac{d}{dt}\left[\left(\alpha\frac{\partial\theta}{\partial n}\mathbf{n}\right)\cdot\left((\zeta_\theta+f)\mathbf{n} + \underset{\sim}{\zeta_H}\right)\right] = \frac{d}{dt}\left[\alpha\frac{\partial\theta}{\partial n}(\zeta_\theta+f)\right]$$

Subst. into first term of RHS $\rightarrow$ $\rightarrow$ small wrt other term $\rightarrow$ neglect.

$$\alpha\left[(\zeta_\theta+f)\mathbf{n} + \underset{\sim}{\zeta_H}\right]\cdot\left[\nabla_\theta\left(\frac{d\theta}{dt}\right) + \mathbf{n}\frac{\partial}{\partial n}\frac{d\theta}{dt}\right]$$

$$\approx \alpha(\zeta_\theta+f)\frac{\partial}{\partial n}\left(\frac{d\theta}{dt}\right)$$

Finally, subst. for 2nd term of RHS (friction term) $\longrightarrow$

$$\alpha\nabla_3\theta\cdot(\nabla_3\times\underset{\sim}{F}) = \alpha\frac{\partial\theta}{\partial n}\mathbf{n}\cdot\left[\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{n} + (\quad)\mathbf{i} + (\quad)\mathbf{j}\right]$$

$$= \alpha\frac{\partial\theta}{\partial n}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)_\theta$$

So, eq (1) now becomes

$$\frac{d}{dt}\left[\alpha\frac{\partial\theta}{\partial n}(\zeta_\theta+f)\right] = \alpha(\zeta_\theta+f)\frac{\partial}{\partial n}\left(\frac{d\theta}{dt}\right) + \alpha\frac{\partial\theta}{\partial n}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)_\theta$$

Now $\frac{\partial}{\partial n} = \frac{\partial}{\partial z}\frac{\partial z}{\partial n}$, but in the real atmosphere, $\frac{\partial z}{\partial n} \approx 1 \longrightarrow \boxed{\frac{\partial}{\partial n} \simeq \frac{\partial}{\partial z}}$

From hydrostatic eq. $\frac{\partial p}{\partial z} = -\rho g = -\frac{g}{\alpha} \longrightarrow \frac{\partial}{\partial n} \simeq \frac{\partial}{\partial z} = -\frac{g}{\alpha}\frac{\partial}{\partial p}$

Subst. this for $\frac{\partial}{\partial n}$ into above, cancel α's, divide all terms by g $\longrightarrow$

$$\frac{dP}{dt} = \frac{d}{dt}\left[-\frac{\partial\theta}{\partial p}(\zeta_\theta+f)\right] = (\zeta_\theta+f)\left(-\frac{\partial}{\partial p}\left(\frac{d\theta}{dt}\right)\right) - \frac{\partial\theta}{\partial p}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)_\theta$$

which is eq. (2)

(b)

One straightforward method to calculate the change in P following the motion was done by Staley in the 60s. He evaluated initial and final values of stability $\left(-\frac{\partial\theta}{\partial p}\right)$ and vort. $(\zeta_\theta + f)$ along an isentropic (constant θ) trajectory, then took the difference to calculate the change. Errors in this method included measurements in the basic data themselves, analysis errors due mostly to data sparseness, and errors assoc. with the assumption of an isentropic trajectory. As gradients of diabatic heating contribute to changes in P, in areas where P changes the most due to diabatic contribution, the isentropic assumption will be most in error. This calc. could then represent P of a different parcel, perhaps on a different θ sfc.

With regards to variations in P, studies have shown that largest increases in P occur in the lower stratosphere and upper troposphere on the cold-air side of the front, whereas largest decreases were located almost entirely in the troposphere and concentrated in frontal zones.

(c)

Recall eq. (2): $\frac{dP}{dt} = (\zeta_\theta + f)\left(-\frac{\partial}{\partial p}\frac{d\theta}{dt}\right) - \frac{\partial \theta}{\partial p}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)_\theta$

(A) (B)

Term (A) may be referred to as the diabatic heating term. Changes in P may be produced if there is a non-zero vertical gradient of $\frac{d\theta}{dt}$. Examples of such diabatic processes include radiation, evap. + condensation, and turbulent heat fluxes.

Term (B) may be referred to as the friction term. Hence frictional forces which lead to a non-zero curl along the direction of $\frac{\partial \theta}{\partial p}$ can produces changes in P. This is related to Reynolds stress, dependent on density, eddy viscosity, + horiz. wind shear in the vertical. ($\frac{\partial \theta}{\partial p}$ is a measure of static stability)

P is conserved (or $\frac{dP}{dt} = 0$) when (A) + (B) are both zero. These conditions are then

$$\left.\begin{array}{l} \text{(A) Adiabatic motion, } \frac{d\theta}{dt} = 0 \longrightarrow \text{(A)} = 0 \\ \text{(B) Frictionless motion, } F = 0 \longrightarrow \text{(B)} = 0 \end{array}\right\} \Rightarrow \frac{dP}{dt} = 0$$

or

curl of F = 0

Potential vort.

5510
MET? : LaSeur(?) (?)

- The concept of Potential Absolute Vorticity, P, is increasingly used in diagnostic and prognostic analysis and modelling of the atmosphere. Almost 50 years ago Ertel showed that $P \equiv (\alpha \underset{\sim}{\eta}_3 \bullet \nabla_3 \theta)$ in which $\alpha \equiv$ specific volume, $\eta_3 \equiv$ three dimensional absolute vorticity, and $\nabla_3\theta \equiv$ three dimensional gradient of potential temperature is a conserved property of adiabatic, frictionless flow.

(a) Give a physical interpretation of P.
(b) Show that P may be transformed to the more convenient form:

$$P = \alpha\eta_N \frac{\partial\theta}{\partial N} = \left(-\frac{\partial\theta}{\partial p}\right)(\zeta_\theta - f)$$

In which N is normal to isentropic surfaces, ζ_θ is relative vorticity measured on isentropic surfaces and f is the Coriolis parameter.

(Hint: Use the approximations $\frac{\partial}{\partial N} \approx \frac{\partial}{\partial z} = -g\rho\frac{\partial}{\partial p}$)

(c) Discuss the circumstances in which ζ_θ may differ significantly from ζ_p = relative vorticity measured on pressure surfaces.
(d) Discuss the circumstances in which it has been found that P is not conserved and the physical processes responsible for such non-conservation.

Sol

(a) Physical interpretation

P is the projection of abs. vort. on the gradient of potential temp.
It is called potential vort. because when the distance between two θ sfc's increase, $\frac{\partial\theta}{\partial p}$ decreases and hence η must increase proportionally for P to be conserved. So the packing of θ sfc's may be considered a reservoir of vorticity with the potential to produce vort. as the sfc's move apart.

(b)
Define coord. system of i, j, n where n is normal to θ sfc.

Hence $P = \alpha\underset{\sim}{\eta}\cdot\nabla_3\theta = \alpha(\eta_i+\eta_j+\eta_n)\cdot(\cancel{\nabla_\theta\theta}^{\,0} + \frac{\partial\theta}{\partial n})$

As $\nabla_\theta\theta = 0$, above reduces to $P = \alpha\eta_n\frac{\partial\theta}{\partial n}$

What is η_N? It is vorticity with component normal to θ sfc, so $\eta_N = \zeta_\theta + f$. Now we use approx. $\frac{\partial}{\partial n} \simeq \frac{\partial}{\partial z} = -\rho g\frac{\partial}{\partial p}$

$\rightarrow P = \alpha(\zeta_\theta+f)(-\rho g\frac{\partial\theta}{\partial p})$

$\because$ Recall $\alpha\rho = 1$

$\rightarrow \boxed{P = -g\left(\frac{\partial\theta}{\partial p}\right)(\zeta_\theta+f)}$ Ertel's Pot. Vort.

(c)
Define in terms of P

$P_\theta = -\frac{\partial\theta}{\partial p}(\zeta_\theta+f)$, $P_p = -\frac{\partial\theta}{\partial p}(\zeta_p+f)$

Reed showed that $P_\theta = P_p + \mathbb{k}\cdot\left(\frac{\partial \underset{\sim}{V}}{\partial p}\times\nabla_p\theta\right)$

Substituting and solving for $(\zeta_\theta - \zeta_p) = \left(-\frac{\partial\theta}{\partial p}\right)^{-1}\mathbb{k}\cdot\left(\frac{\partial \underset{\sim}{V}}{\partial p}\times\nabla_p\theta\right)$

So, $\zeta_\theta \neq \zeta_p$ when there is a horiz. gradient of θ on the pressure sfc, and a vertical shear in the wind (along direction $\perp$ to the θ gradient as the cross product must have a comp. in the $\mathbb{k}$ direction)

One region where ζ_θ & ζ_p would differ significantly would be near a frontal zone, as $\nabla_p\theta$ is large there. Also low static stability so $\left(-\frac{\partial\theta}{\partial p}\right)^{-1}$ will be high

(d)
$$\frac{dP}{dt} = \underset{①}{\eta\left(-\frac{\partial}{\partial p}\frac{d\theta}{dt}\right)} - \underset{②}{\frac{\partial\theta}{\partial p}(\nabla_H\times \underset{\sim}{F})}$$

P is not conserved (or $\frac{dP}{dt}\neq 0$) when either of the above terms $\neq 0$.

① Diabatic heating term : Changes in P caused when there is a vert. gradient of $\frac{d\theta}{dt}$. Examples of such diabatic processes are radiation, evap. & condensation, and turbulent heat fluxes.

② Friction term : Changes in P caused where there is a horiz. curl of the friction force. This is related to Reynold's stress, dependent on ρ, eddy viscosity, and $\frac{\partial V}{\partial p}$.

5510
MET? (Mid lat): LaSeur (1 hour)

- Staley derived an extension of Ertel's potential vorticity theorem which includes the effect of heating and friction:

$$\frac{d}{dt}\left(\alpha\nabla\theta\cdot\vec{\eta}\right)=\alpha\,\vec{\eta}\cdot\nabla\left(\frac{d\theta}{dt}\right)+\alpha\nabla\theta\cdot\left(\nabla\times\vec{F}\right) \qquad (1)$$

in which α = specific volume

$\vec{\eta}=\nabla\times\mathbf{V}+2\vec{\Omega}$ = three dimensional velocity

$\vec{F}$ = frictional force per unit mass

(a) If $\mathbf{n}$ is a unit vector normal to an isentropic surface and only diabatic and frictional effects along $\mathbf{n}$ are considered, show that (1) may be written as

$$\frac{d}{dt}\left(\sigma(\zeta_\theta+f)\right)=(\zeta_\theta+f)\left(-\frac{\partial}{\partial p}\frac{d\theta}{dt}\right)+\sigma F_n \qquad (2)$$

in which $P=\sigma(\zeta_\theta+f)$, $\sigma=-\partial\theta/\partial p$

$$\zeta_\theta=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)_\theta$$

F_n = n component of $\nabla\times\vec{F}$

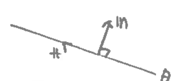
Hint: $\dfrac{\partial}{\partial n}\cong\dfrac{\partial}{\partial z}=-\rho g\dfrac{\partial}{\partial p}$

(b) Where are large values of P observed in the atmosphere? Discuss some important conclusions about atmospheric circulations that have resulted from studies of P.

(c) Discuss diabatic and frictional processes represented by the right hand side of (2) that may account for considerable non-conservation of P.

- look at the previous question

note/



$$\frac{d}{dt}(\alpha\nabla\theta\cdot\eta)=\alpha\,\eta\cdot\nabla\left(\frac{d\theta}{dt}\right)+\alpha\nabla\theta\cdot(\nabla\times F)$$

$$\frac{d}{dt}\left(\alpha\eta_n\frac{\partial\theta}{\partial n}\right)=\alpha\eta_n\frac{\partial}{\partial n}\left(\frac{d\theta}{dt}\right)+\alpha\frac{\partial\theta}{\partial n}F_n \qquad \because\left(\begin{array}{l}\nabla\theta=\frac{\partial\theta}{\partial n}\mathbf{n}\\ \eta=\eta_n\mathbf{n}+\zeta_t\mathbf{t}\end{array}\right.$$

$$\frac{d}{dt}\left[\left(-\frac{\partial\theta}{\partial p}\right)(\zeta_\theta+f)\right]=-(\zeta_\theta+f)\frac{\partial}{\partial p}\left(\frac{d\theta}{dt}\right)-\frac{\partial\theta}{\partial p}F_n \qquad \because\left(\begin{array}{l}F_n=(\nabla\times F)_n=\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)_\theta\\ \frac{\partial}{\partial n}\simeq\frac{\partial}{\partial z}\equiv-\rho g\frac{\partial}{\partial p}\\ \eta_n=\zeta_n+f\simeq\zeta_\theta+f\end{array}\right.$$

$$(\zeta_\theta+f)\equiv\eta_\theta\ ,\ \left(-\frac{\partial\theta}{\partial p}\right)\equiv\sigma$$

$$P=\sigma\eta_\theta$$

$$\frac{dP}{dt}=\eta_\theta\left[-\frac{\partial}{\partial p}\left(\frac{d\theta}{dt}\right)\right]+\sigma F_n$$

$$\frac{d\sigma}{dt}=\sigma(\nabla\cdot V)-\frac{\partial}{\partial p}\left(\frac{d\theta}{dt}\right) \qquad F_n\sim F_z=K\frac{\partial^2\zeta_\theta}{\partial z^2}$$

$$\frac{d\eta_\theta}{dt}=-\eta_\theta(\nabla_\theta\cdot V)+\frac{\partial u}{\partial\theta}\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right)+F_n$$

* look at the mid-term exam !

moist static energy

MET ~~?~~ 5534 : LaSeur (?)

- Question

(A) Write an expression for the quantity known as "moist static energy" and give a physical interpretation of the terms involved. Are any significant other forms of atmospheric energy neglected in this expression?

(B) Sketch a diagram illustrating the vertical distribution of this quantity in the average tropical troposphere with appropriate labels of units and magnitudes involved. Discuss briefly how adiabatic (moist and/or dry) processes effect this quantity.

(C) On your diagram in B, sketch a second curve illustrating how the vertical distribution of this quantity is observed to change in regions of the tropical troposphere "disturbed" on the synoptic scale by upward motion associated with lower tropospheric convergence and upper divergence. From a comparison of these curves and consideration of the physical nature of this quantity, discuss the implications as to the scale on which the upward motion must occur and the nature of the physical processes with which it is associated.

(D) What phenomena widely observed in the tropical troposphere provides a logical and rational framework for explaining the above observations? Discuss qualitatively (no mathematical development required) and physically the nature of this explanation and the impact it has on numerical weather prediction in the tropical atmosphere.

(A)

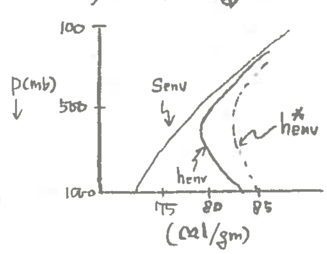
Moist static energy $\equiv h = C_pT + gz + Lq$

C_pT = enthalpy → thermal energy
gz = geopotential → potential energy
Lq = latent heat → energy from moisture processes.

Minor energy sources neglected include chemical + electrical energy.
Major energy sources neglected is kinetic energy (KE), although KE $\ll$ h.

(B)

Recall dry static energy $\equiv S = C_pT + gz$



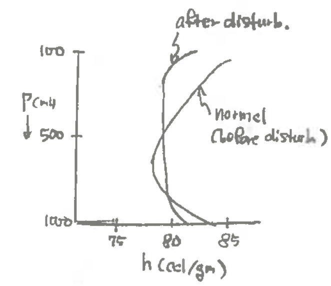
S_{env} = Senvironment
h_{env} = h "
h^*_{env} = saturation h_{env}

The distribution of h is affected by
(1) horiz. + vertical advection
(2) vertical moisture flux from eddies ($\overline{\omega' q'}$) — adiabatic
(3) vertical gradient of large scale motion → DIV — adiabatic
(4) " " " diabatic heating/cooling

Note the min of h near 600 mb. Above $\frac{\partial h}{\partial z} > 0$, and reflects the loss of h by LW radiative cooling as parcels subside between clouds. Below min h, these parcels mix with shallow clouds and hence $\frac{\partial h}{\partial z} < 0$. Note h peaks at sfc, where latent heat release from ocean sfc

(C)

After the disturbance, $\frac{\partial h}{\partial z} \sim 0$ throughout deep layer of atmosphere. This value of instability implies deep convection, which occurs on a sub-synoptic scale.



(D)

Convection provides for the upward transport of h from the sfc in the tropics. Large upward motions on the convective scale induce subsidence in the non-convective areas of the tropics → warming of the environment.
The problem is that convection is on a scale too small to adequately incorporate into numerical models. Attempts to model convection (small scale) within synopt scale model have led to the field of convective parameterization

note

We define moist static energy h as

$$h = C_pT + gz + Lq$$

Lq → latent heat, gz → geopotential energy, C_pT → enthalpy

This definition excludes the following forms of energy
① K.E. — while KE is significant in convective ↑ + ↓, it is still much smaller than h
② chemical energy
③ electrical energy
} generally those are insignificant.

Consider the following example

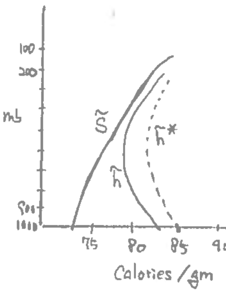
$T = 300K$, $W = 20\,m/s$, $z = 10km$, $q = 10\,g/kg = 10 \times 10^{-3}$

$$h = (1004 \tfrac{J}{Kkg} \cdot 300k) + (9.8\,m/s^2)(10 \times 10^3 m) + (2.5 \times 10^6 \tfrac{J}{kg})(10 \times 10^{-3})$$

$$\approx 4.243 \times 10^6\ J/kg$$

vs $KE = \frac{W^2}{2} = \frac{(20 m/s)^2}{2} = 200 \frac{J}{kg}$ (even if $W = 100 m/s \Rightarrow KE = 5 \times 10^3\ J/kg$)

- A vertical profile of moist static energy in the average tropical troposphere is shown below (from Yanai et al, 1973)



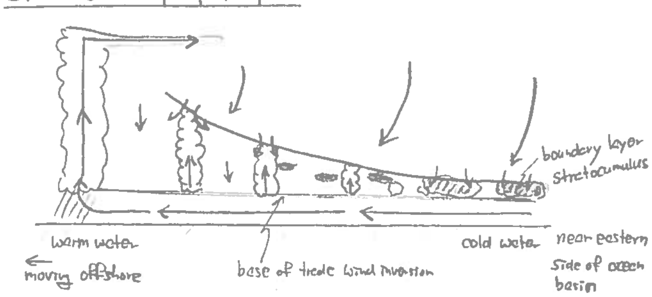
$\tilde{S}$ ≡ environmental dry static energy ($gz + C_pT$)
$\tilde{h}$ ≡ " moist static energy
$\tilde{h}^*$ ≡ " saturation moist static energy
↳ use same T structure but replace q by appropriate q_s

Below are comments by Emanuel regarding this figure. ($\tilde{h}$, $\tilde{S}$, $\tilde{h}^*$)

→ The above profiles are very similar to those for θ_e, θ, θ_e^*, respectively. The dry static stability is everywhere positive, except in the subcloud layer where it is close to neutral. The moist entropy (θ_e or $\tilde{h}$) almost always shows a min in the middle troposphere around 600mb, reflecting the subsaturation of air there. The negative gradient of entropy in the lower troposphere indicates that it is potentially unstable, but not that it is conditionally unstable. Nor does this condition imply the existence of available potential energy for convection. Air lifted adiabatically from the subcloud layer has a θ_e greater than the θ_e^* of its environment. From the definition of entropy this usually, but not always implies that it is warmer than its environment, though not necessary that it is buoyant, given the substantial contribution of condensate loading to buoyancy.

The entropy min ~600mb is an important feature of the tropics. The positive gradient in the upper troposphere reflects the loss of entropy by LW radiative cooling to space of air parcels slowly subsiding in between clouds. As these same parcels descend into the lower troposphere, they re-acquire entropy by mixing with shallow clouds, which transport the entropy up from the subcloud layer. Thus trade cumuli are important contributors to the structure of the tropical atmosphere, even in regions experiencing deep convection as well.

Structure of tropical atmosphere



Over the colder waters of the subtropics, particularly on the eastern side of major ocean basins, Sc-topped boundary layers occur. As the air flows equatorward over warmer sea surfaces, a transition occurs to trade cumulus BL. Above these BLs, the air is quite dry and has a vertical temperature structure close to that of a deep convective regime further equatorward (that is, close to moist adiabatic). This is because the dynamics of the free troposphere prevent large temp gradients from developing on isobaric surfaces. This dry air is slowly subsiding under the influence of radiative cooling.

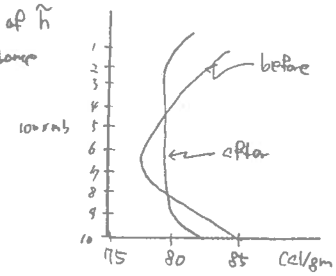
Over the warmest parts of the tropical oceans (SST, ≥ 26°C), deep convection coexists w/ a spectrum of trade cumuli. The deep convection is frequently organized into squall lines + MCCs. It is modulated on scale of 100 km or larger by tropical disturbances and occasionally tropical cyclones. Whereas the atmosphere as a whole is descending in the trade-cumuli and Sc regimes, in the warmest regions the net motion is upward. All of this upward motion is carried in the deep convective clouds while the clear air between the widely scattered Cb's is descending + cooling radiatively much as it does in the subtropics.

In contrast to other large scale parameters, the day to day variations of $\tilde{S}$ and $\tilde{h}$ are small regardless of the passage of disturbances. Main processes which affect moist static energy are described by the eq for the moist adiabatic lapse rate.

$$\frac{\partial \bar{\Gamma}_m}{\partial t} = \underbrace{-\bar{V}\cdot\nabla\bar{\Gamma}_m - \bar{\omega}\frac{\partial \bar{\Gamma}_m}{\partial p}}_{ⓐ} + \underbrace{\frac{\partial^2}{\partial p^2}(\overline{\omega' q'})}_{ⓑ} - \underbrace{\frac{\partial}{\partial p}\left(\left(\frac{P_0}{p}\right)^{R/C_p}\cdot\frac{1}{C_p}\sum_i H_i\right)}_{ⓒ} - \underbrace{\bar{\Gamma}_m\frac{\partial \bar{\omega}}{\partial p}}_{ⓓ}$$

ⓐ – horizontal + vertical advection
ⓑ – adiabatic effect due to covariance between vertical velocity eddies + eddies in moisture.
ⓒ – vertical gradient of net diabatic heating/cooling
ⓓ – adiabatic effect due to vertical gradient of large scale motion
↳ this is a divergence term since $\frac{\partial \bar{\omega}}{\partial p} = (-\nabla\cdot\bar{V})$

Before + after profiles of $\tilde{h}$
↳ post disturbance
↳ predisturbance



Dry convection which is mainly seen in the BL (up to 500m above sfc) keeps h constant because Lq does not change and T is mixed in the BL. Constant h up to 600 mb is seen only in the Sahara desert. Typically, when you have constant q and a dry adiabatic lapse rate, then you reach the LCL by ~600m above the surface. Moist adiabatic processes then take over. Convection transports the sensible + latent heat in the vertical and this releases the instability. However, large scale subsidence can suppress the instability. Therefore instability does not imply convection. The h profile is primarily governed by convection. Greatest instability implies the least convection and near zero instability implies a lot of convection. Cumulus convection causes the moist static energy profile to change. Between clouds, subsidence warms the environment.

Q_1 + Q_2 + Cumulus parameterization

MET5534 : LaSeur (1 hour) ** ※ look at Krishi study note

- Consideration of the heat and moisture budgets of larger scale (planetary or synoptic scale) tropical weather systems characterized by lower tropospheric convergence, upper tropospheric divergence and, thus, general upward vertical motion on that scale, shows that such budgets calculated from the larger scale motion alone reveal large magnitude residuals that have been termed as $Q_1 \equiv$ the "apparent heat source", and $Q_2 \equiv$ the "apparent moisture sink". Diagnostic budget calculations for such conditions typically define these as:

$$Q_1 \equiv \frac{\partial \bar{s}}{\partial t} + \nabla\cdot\overline{s\mathbf{V}} + \frac{\partial}{\partial p}\left(\overline{\omega s}\right) \approx c_p\left[\frac{\partial\bar{\theta}}{\partial t} + \nabla\cdot\overline{\theta\mathbf{V}} + \frac{\partial}{\partial p}\left(\overline{\theta\omega}\right)\right] \simeq Q_R + L(\underset{\text{condensation}}{c} - \underset{\text{evaporation}}{e}) - \frac{\partial}{\partial p}(\overline{\omega' s'})$$

$$-Q_2 \equiv L\left[\frac{\partial\bar{q}}{\partial t} + \nabla\cdot\overline{q\mathbf{V}} + \frac{\partial}{\partial p}\left(\overline{q\omega}\right)\right] \quad \text{(note minus sign)} \rightarrow Q_2 \simeq L(c-e) + L\frac{\partial}{\partial p}(\overline{\omega' q'})$$

$$\bar{s} = c_p\bar{T} + g\bar{z}$$

where s and q are dry static energy and mixing ratio, respectively.

A. Show that Q_1 and $-Q_2$ are both positive if the vertical advection terms are the dominant terms in these expressions, under the conditions given above; and sketch schematic curved of Q_1 and $-Q_2$ for the tropical troposphere for these conditions.

B. Obviously, other processes besides the large scale motion must be significant to the heat and moisture budgets in such systems. Discuss the most likely additional processes that have been hypothesized as playing that important role in determining the energy budgets in such systems.

C. Furthermore, outline briefly the major assumptions originally introduced by Arakawa (and used by many others) in an attempt to incorporate these hypothesized processes. How does this proposed methodology explain the "apparent heat source" and "apparent moisture sink" in terms of the interaction between the hypothesized processes and the larger scale motion?

D. Lastly, discuss briefly the major deficiencies of this approach that have been revealed by diagnostic studies based upon it.

(A)

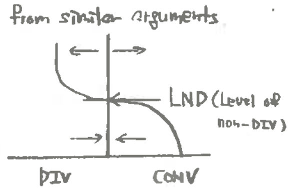
$$Q_1 = \frac{\partial\bar{s}}{\partial t} + \underbrace{\nabla\cdot\bar{s}\bar{\mathbf{V}}} + \underbrace{\frac{\partial}{\partial p}(\bar{\omega}\bar{s})}$$

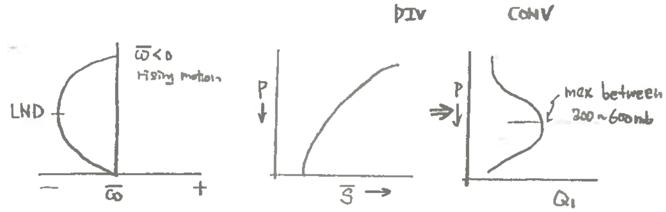
$$= \frac{\partial\bar{s}}{\partial t} + \bar{s}(\nabla\cdot\bar{\mathbf{V}}) + \bar{\mathbf{V}}\cdot\nabla\bar{s} + \bar{\omega}\frac{\partial\bar{s}}{\partial p} + \bar{s}\frac{\partial\bar{\omega}}{\partial p}$$

$$= \underbrace{\frac{\partial\bar{s}}{\partial t} + \bar{\mathbf{V}}\cdot\nabla\bar{s}}_{\text{small if vertical adv. dominates}} + \bar{s}\underbrace{\left(\nabla\cdot\bar{\mathbf{V}} + \frac{\partial\bar{\omega}}{\partial p}\right)}_{\text{mass continuity} \rightarrow 0} + \bar{\omega}\frac{\partial\bar{s}}{\partial p}$$

$\Rightarrow Q_1 \simeq \bar{\omega}\frac{\partial\bar{s}}{\partial p}$, $Q_2 \simeq -L\bar{\omega}\frac{\partial\bar{q}}{\partial p}$ from similar arguments

For low-level CONV and upper-level DIV, the CONV profile looks something like this



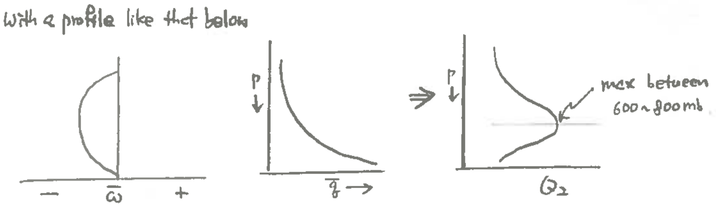


This yields a profile of $\bar{\omega}$ with max rising motion at LND.

A typical tropical profile of $\bar{s}$ is also pictured, which is similar in slope to the θ profile. This profile illustrates that P decreases, $\bar{s}$ increases $\rightarrow \frac{\partial\bar{s}}{\partial p} < 0$

Recall rising motion $\rightarrow \bar{\omega} < 0$. Hence $Q_1 \simeq \omega\frac{\partial\bar{s}}{\partial p} > 0$, with a profile pictured above.

Recall $Q_2 \simeq -L\bar{\omega}\left(\frac{\partial\bar{q}}{\partial p}\right)$ A typical profile of $\bar{q}$ is pictured below, where $\bar{q}$ decreases as P decreases $\rightarrow \frac{\partial\bar{q}}{\partial p} > 0$. As $\bar{\omega} < 0 \rightarrow Q_2 \simeq -L\bar{\omega}\left(\frac{\partial\bar{q}}{\partial p}\right) > 0$, with a profile like that below



(B)

(1) Release of latent heat from phase changes [$L(c-e)$ term]

(2) Radiative effects [Q_R term]

(3) Sub-grid or smaller scale effects [$\frac{\partial}{\partial p}(\overline{\omega' s'})$, $\frac{\partial}{\partial p}(\overline{\omega' q'})$ terms]
These include effects of mixing + entrainment.

(C)

Basic coupling of convective/synoptic scales from the AS method: ensemble on convective disturbances ($\omega < 0$) causes subsidence ($\omega > 0$) in the non-convective cloud environment, which warms and dries the environ. air. Detrainment (near cloud top) of cloud water → environ. cools and moistens environ.

Major assumptions

(1) Quasi-Equilibrium: Destability by large scale processes (low-level moisture CONV, radiative cooling of upper troposphere, sfc fluxes of sensible & latent heat, etc) balanced by stabilization of atmosphere by convection used to close the scheme.

(2) Common cloud base, with types identified by entrainment rate (λ)

(3) Single value of q_c, S_c, h_c for each cloud

(4) Steady state plume model for entrainment (Cloud entrains throughout depth, except near top where it detrains in thin layer)

(5) Total precip. fallout (→ no evaporation!)

(D) deficiencies

(1) No downdrafts! (only updrafts): Downdrafts can cool & moisten the PBL by detrainment at cloud base

(2) Single q, s, h for cloud: In reality, they have a horiz. distribution.

(3) Detrainment ONLY at cloud top: can really detrain at all levels

(4) No evaporation (immediate precip. fallout): real evap. of cloud water is an issue.

(5) Single cloud base for ensemble: real bases vary.

Note: The lack of downdrafts → overestimation of cloud mass flux
→ overestimate compensating environ subsidence
→ " warming & drying.

Cumulus Parameterization

MET5534: LaSeur (60 minutes)

• The effects of convective clouds in modifying the larger-scale environment in which they are embedded are important in the tropical atmosphere both diagnostically and prognostically. Larger-scale environmental changes are typically measured by:

$$Q_1 = \frac{\partial \bar{s}}{\partial t} + \nabla \cdot \left(\overline{s\mathbf{V}}\right) + \frac{\partial}{\partial p}\left(\overline{\omega s}\right)$$

$$Q_2 = -L\left[\frac{\partial \bar{q}}{\partial t} + \nabla \cdot \left(\overline{q\mathbf{V}}\right) + \frac{\partial}{\partial p}\left(\overline{q\omega}\right)\right] \qquad \text{(note minus sign)}$$

$\bar{s} = c_p \bar{T} + g\bar{z}$ $\qquad$ q = specific humidity

(a) Show that if the sum of the time tendency and horizontal advection on the larger-scale is small that Q_1 and Q_2 are determined by the larger-scale vertical advection and thus in regions of the usual tropical atmosphere with larger-scale upward motion, Q_1 and Q_2 are positive.

Two main approaches to "parameterizing" the effects of convective clouds have been developed by Arakawa and Kuo.

(b) Discuss the physical characteristics of the convective clouds included in the Arakawa and Kuo schemes. Emphasize the similarities and differences between the two schemes.

(c) Discuss physically the processes whereby the larger-scale environment is modified in each of these schemes. What important characteristic of actual convective clouds is neglected in these schemes?

(a) look at the previous answer!

(b) Both methods make some similar simplifying assumptions in their parameterization of convective clouds. Both seek to describe the effects of clouds with diverse properties through ensemble averaging. The ensemble is then simply described, with uniform assumptions. For example, AS uses a single value of q, s, h for the cloud, as well as uniform levels for cloud bases. Also both methods tend to create only deep, penetrative convection. There are some basic, fundamental differences though. The AS clouds are allowed to entrain environ. air throughout the depth of the column, but not allowed to heat the environment directly with detrainment of sensible & latent heat. At the top of the cloud, the mass is allowed to detrain where its temp is assumed ~ equal to the environment. The Kuo scheme handles the clouds differently, assuming parcel clouds with a lifetime τ, after which they completely detrain into environment, allowing direct heating by sensible and latent heat. Moisture and rain are also handled somewhat differently. The AS scheme assumes all non-cloud liquid H_2O immediately falls out as rain, neglecting its evaporation. The Kuo scheme allows rain and moisture storage of water vapor. Note that overestimation of this storage term weakened rainfall from the model.

(c) In both schemes, the apparent heating source (Q_1) corresponds to heating associated with sub-synoptic scale processes parameterized as an ensemble of convective clouds. However they deal with heating & moistening differently.

• Arakawa-Schubert (AS)

$$H_{env} = \sigma w_c \frac{\partial S_{env}}{\partial z} \quad (c - \text{cloud},\ env - \text{environment})$$

This heating expression appears confusing, as it looks like vertical advection on the cloud scale (w_c) of an environment quantity ($\frac{\partial S_{env}}{\partial z}$). In reality, the heating is due to the compensating environ. subsidence induced by upward cloud motion (w_c). As AS allows detrainment at cloud top where $T_c \simeq T_{env}$, there is no direct transfer of sensible & latent heat from cloud to environment. All heating comes from environ. subsidence, which also leads to drying.

• Kuo

$$H_{env} = \frac{\sigma c_p}{\tau}(T_c - T_{env}) \quad (\sigma = \text{cloud fraction},\ \tau = \text{cloud lifetime})$$

This scheme assumes clouds are pure parcel clouds, with a lifetime of τ after which the clouds fully detrain into the environment. Hence the cloud directly release sensible & latent heat to the environment. Also as clouds are assumed parcel, their temps are much warmer than the environment (as the lapse rate in cloud is allowed to warm to moist adiabatic)

• An important characteristic neglected is that of downdrafts. Downdrafts are important for many reasons, one of which is that they can cool + moisten the PBL by detrainment at cloud base, effecting stability, etc. Also lack of downdrafts tends to overestimate net upward cloud motion, and hence overestimate environ. subsidence and the warming & drying that result

MET5534: LaSeur (1 hour)

• The dominant instability characterizing the tropical atmosphere is the conditional and convective static instability for saturated adiabatic processes. As a result, a planetary- and/or synoptic-scale volume of the tropical atmosphere typically consists of a small percentage (< 5%) of active convective clouds embedded within the larger unsaturated "environment" (except for some mesoscale regions of inactive convective "debris" clouds). Since available data and other difficulties preclude explicit consideration of convection, methods have been developed whereby the cumulative statistical effects of large numbers of embedded convective clouds in modifying their environment can be estimated. Two such schemes that utilize "model" convective clouds have been developed by Kuo and Arakawa-Schubert and widely used in both diagnostic and prognostic studies of the tropical atmosphere.

(1) In a concise discussion, compare and contrast the physical properties and processes incorporated in the "model" clouds of each of these two schemes of "convective parameterization" (Little or no mathematical treatment required or expected). In particular, in your discussion explain how the hypothesized "model" clouds accomplish the transport and redistribution of heat and moisture from their principal sources in the sub-cloud boundary layer to the remainder of the tropical troposphere.

(2) Briefly discuss physical characteristics and processes in actual tropical convective clouds that are not incorporated in these schematic "model" clouds.

look at other notes ?

Note for cumulus parameterization

- Convective parameterization

Hypothesis :

— A large number of individual clouds may have significant effects on the large-scale system through the transfer of heat, moisture + momentum.

— In the total vertical column, Cumulus clouds heat and dry the atmosphere provided that precipitation reaches the ground.

Similarities

→ the cumuli modify the large-scale distribution of T + q

① through detrainment of saturated air + condensation products (evaporation, moistening) and

② by the subsidence induced by the convection

the convection tends to stabilize the atmosphere
the environment " " destabilize " "

Differences

Kuo scheme

① latent heat released through condensation changes the large-scale T + q distribution.

② Assume that convection occurs in deep layers of conditionally unstable stratification over areas of mean low-level convergence (net supply of moisture)

③ Cloud base ⟶ LCL

④ Vertical distribution of T + q in cloud is a moist adiabat

⑤ Cloud top → intersection of moist adiabat with the environment sounding

⑥ Assume that the cumulus clouds dissolve immediately by mixing with the environment air (No explicit entrainment is needed)

Arakawa–Schubert scheme

① Interaction between a cumulus ensemble and the large-scale environment

② Large-scale heat and moisture budget eqs are developed for a subcloud layer (subensemble clouds)

③ The vertical transport of heat and moisture by the cumulus ensemble continuously tends to reduce the conditional instability in the environment.

※ He assumed that

① Constant entrainment rate ($\frac{1}{m}\frac{\partial m}{\partial z} = \lambda$)

② A single detrainment layer ($h_c = h$)

③ precipitation instantly falls the moment it forms

④ Quasi-equilibrium between stabilization by convection and destabilization by the environment.

⑤ Common cloud base

- Arakawa's major assumptions

He assumes a balance between the cloud-cloud interactions which destroy the convective instability (tends to stabilize the atmosphere) and the large scale interactions which build the instability (tends to destabilize the atmosphere.

- Similarity between Arakawa + Kuo

① In both of them the cloud is treated as an entirely different physical element from the environment

② Neither of their scheme is a good representation as to what really happens in the model.

③ They tie the amount of cloud to synoptic-scale convergence

④ In both their theories cloud occur at random.

⑤ Both of them neglect the downdraft!

⑥ No detailed cloud microphysics is included

- Any convective parameterization scheme can be categorized into three different parts.

a) Dynamic control : how the environment modulates the convective cloud

b) Feed back : how convective cloud modifies the environment

c) Static control : Cloud thermodynamic properties.
→ the vertical structure of the environment
(the cloud vertical mass flux at cloud base)
(the height of cloud top)

Feature	Kuo scheme	AS scheme
Dynamic control	Convective activity is related to large-scale moisture convergence	Amount of convection is related the rate of destabilization by the environment (Quasi-equilibrium assumption)
Feedback	<Subsidence warming> Heating and moistening of environment is proportional to $T_c - T$ and $q_c - q$ (heating, moistening, rainfall rates) (Entrainment is not included explicitly)	Steady state : the latent-heat release within the clouds does not directly warm the environment but it maintains the vertical mass flux of clouds. Consequently, convection influences the environment through environmental subsidence and detrainment at the top of the updraft or the bottom of the downdraft
Static control	In cloud thermodynamic properties are represented by moist adiabat LCL – Cloud base EL – Cloud top (no buoyancy)	Constant entrainment rate with height. Cloud vertical mass flux

- AS scheme

the static control determines

① the cloud vertical mass flux at the cloud base.

② the cloud thermodynamic properties

③ the height of zero buoyancy for each subensemble.

- Cumulus parameterization

— An individual cloud does not have much influence on a large-scale system

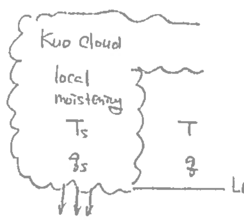
— the cumulative effect of a large number of individual clouds may have significant effect on the large-scale system through the transfer of heat (θ), moisture (q) and momentum (V)

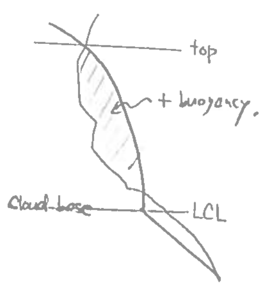
— Need to determine the statistical effects of many cumuli on large-scale systems.

- In the total vertical column, cumulus clouds heat and dry the atmosphere (stabilizes) provided that precipitation reaches the ground.
- types
  - the convective adjustment scheme
  - Kuo scheme
  - Arakawa-Schubert scheme

• Kuo method

- latent heat released through condensation changes the large-scale T + q distribution.
- He assumed that
  - Cumulus convection occurs in deep layers of conditionally unstable stratification over areas of mean low-level convergence.
  - Cloud base ⟶ LCL
  - The vertical distribution of T + q in cloud is a moist adiabat
  - Cloud top ⟶ intersection of moist adiabat with the environment sounding
- The cumulus clouds are assumed to dissolve immediately by mixing with the environment air (i.e. no explicit entrainment is needed)
- the subsidence warming is accounted for
- moistening parameter $\begin{cases} bM_t \\ (1-b)M_t \text{ : condensed and precipitates as rain} \end{cases}$
  - b = constant empirical data (Kuo)
  - $b \propto \overline{RH}$ (Anthes)





EMT

Three $\begin{cases} \text{heating} \\ \text{moistening} \\ \text{rain rate} \end{cases}$

- 2 conditions that need to be met
  - ① Atmosphere should be conditionally unstable
  - ② There must be a net supply of moisture.

• Arakawa-Schubert scheme

- a theory of interaction between a cumulus ensemble and the large-scale environment
- Large-scale heat and moisture budget equations are developed for a subcloud layer. (subensemble clouds)
  - * AS in other schemes
    - the cumuli modify the large scale T + q
      - ① through detrainment of saturated air and condensation products and
      - ② by the subsidence induced by the convection
- the partitioning of cumulus ensemble into subensemble (or cloud types, which are characterized by constant entrainment rate λ ← a unique fractional entrainment rate).
- The vertical structure of the environment (static control) determines
  - ① the cloud vertical mass flux at the cloud base
  - ② " " thermodynamic properties and
  - ③ the height of zero buoyancy.

  for each subensemble.
- The distribution of vertical mass flux at the cloud base into subensemble is determined by large-scale dynamical and physical processes together with a closure scheme.
- To relate the synoptic + cumulus scales "the cloud work function" is defined and is determined by the vertical structure of the environment.
  - The cloud work function $A(\lambda)$ represents the rate of KE generation by the buoyancy force.

$$\frac{1}{w}\frac{d}{dt}\left(\frac{w^2}{2}\right) = \frac{g(T_{vc}-\tilde{\overline{T}}_v)}{\tilde{T}_v} - F_r$$

where F_r denotes friction

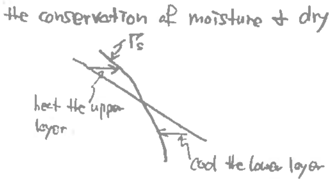
$$T_{vc} - \tilde{\overline{T}}_v = \frac{1}{c_p}(S_{vc} - \tilde{\overline{S}}_v)$$

$$\frac{d}{dt}(\widehat{KE}) = A(\lambda)\, m_B(\lambda) - \underbrace{D(\lambda)}_{\text{dissipation}}$$

  - where bar denotes the integral through cloud depth
  - The cloud work function A is an integral of the buoyancy force that governs the rate of KE generation in a subensemble.
- The vertical transport of heat and moisture by the cumulus ensemble continuously tends to reduce the conditional instability in the environment.

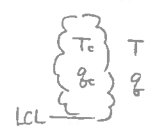
① moist convective adjustment

: based on the conservation of moisture + dry static energy



② Kuo scheme

: systems are closed by the mixing of T, q between convective clouds and environment



$$\left.\begin{aligned} \frac{dT}{dt} &\propto (T_c - T) \\ \frac{dq}{dt} &\propto (q_c - q) \end{aligned}\right\}$$ heat the layer between cloud top and base but not below the cloud base.

③ Arakawa-Schubert scheme

"balance between subsidence heating and evaporation cooling"

mass flux at cloud base for each ensemble determin

• Parameterization of cumulus

$$Q_1 - Q_R - Q_2 = -\underbrace{\frac{\partial \overline{\omega' h'}}{\partial p}}_{\text{should be parameterized.}}$$

where $\begin{cases} Q_1 = \text{apparent heat source} \\ Q_2 = \text{" moisture sink} \\ Q_R = \text{radiative heating} \\ h = \text{moist static energy.} \end{cases}$

The fact that large-scale distributions do not break down into cumulus scale suggest that there exists the interaction mechanism between synoptic scales and cumulus scales.

- Cooperative interaction idea
  - ① the clouds supply the heating to derive the vortex
  - ② The vortex maintains and organizes the cloud system by providing moisture convergence.

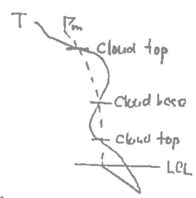
③ CISK mechanism into hurricane (in the case of gradient wind balance)

i) frictional convergence provides the moisture supply in the BL
ii) moisture + mass will be transported upward in the cumulus clouds
iii) The latent heat released in the upper cloud makes warm core to lessen the surface pressure.
iv) Increased pressure gradient increases the vorticity in the boundary layer.
v) This result increases frictional convergence + keeps amplifying
vi) It reaches the limit (establishment of moist adiabat)

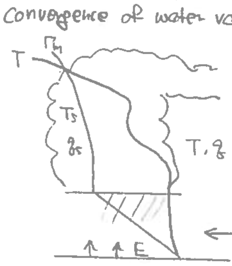
- Kuo scheme

A parameterization scheme for the statistical effects of deep cumulus convection on a tropical cyclone (?)

- latent heat released through condensation + the resulting leads to changes in the large-scale temperature and humidity distribution.
- Cumulus convection occurs where the conditionally unstable stratification exists over the area of the large-scale low-level convergence.
- Cloud base : LCL
- In the cloud layer, the vertical distribution of T_s and q_s are those of moist adiabat.
- Cloud top : EL (no buoyancy level)
- The cumulus clouds are assumed to be dissolved immediately by mixing with the environment air, imparting to it heat and moisture.



- The total rate of moisture accession is given by the convergence of moisture plus the surface evaporation
- The rate of cloud production is assumed to be proportional to the convergence of water vapor plus evaporation



← large-scale moisture flux convergence.

$$a \text{ (fractional area)} = \frac{\text{large-scale moisture flux convergence}}{\text{total moisture}}$$

total convective rainfall rate $\equiv P$

total surface evaporation $\equiv E$

moisture conservation $\left(\frac{dq}{dt}\right) = -\text{precip} + \text{Evap}$

The total large-scale moisture convergence (I)

$$= \text{fractional area} * \left\{ \begin{pmatrix} \text{moisture} \\ \text{for heating} \\ \text{all area from } T \to T_s \end{pmatrix} + \begin{pmatrix} \text{moisture} \\ \text{for saturation} \\ \text{all area from } q \to q_s \end{pmatrix} \right\}$$

(general)

MET? : LaSeur (30-45 minutes) $\frac{\Phi}{g} = Z$

• Outline clearly and concisely the procedure used to calculate the geopotential heights of pressure surfaces from a radiosonde observation. Include in your answer the basic physical approximation upon which this procedure is based, the importance of ground level "baseline" observations and a brief discussion of random and/or systematic errors in the pressure, temperature and humidity sensors of the radiosonde as they effect the accuracy of the geopotential calculations.

?