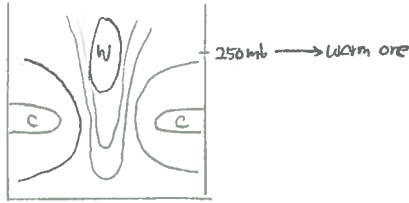


A part of falling precipitation is parameterized to evaporate in dry environment, thus providing the moistening of atmosphere due to convection. Contribution from large-scale dynamics may be positive or negative and net heating and moistening/drying depends upon the total contribution from all terms.

Hurricane

Structure of mature Hurricane

Vertical distribution of temperature anomaly



* Why does the wind not change with height?

In cylindrical coord.

$$\frac{U^2}{r} + fU = -g \frac{\partial \bar{z}}{\partial r} \quad \text{gradient}$$

$$\frac{RT_v}{p} = -g \frac{\partial \bar{z}}{\partial p} \quad \text{hydrostatic}$$

$$\frac{\partial u}{\partial r} \frac{\partial u}{\partial p} + f \frac{\partial u}{\partial p} = \frac{R \partial T_v}{p \partial r}$$

$$\text{gradient wind shear} \quad \frac{\partial u}{\partial p} \Big|_{gr} = \frac{\partial u}{\partial p} + f$$

$$\text{geostrophic wind shear from } \begin{cases} fu = -g \frac{\partial \bar{z}}{\partial r} \\ \frac{RT_v}{p} = -g \frac{\partial \bar{z}}{\partial p} \end{cases}$$

$$\frac{\partial u}{\partial p} \Big|_{geos} = \frac{R \partial T_v}{p \partial r}$$

$$\frac{\partial u}{\partial p} \Big|_{geos} = \frac{R \partial T_v}{p \partial r} \cdot \frac{p}{f}$$

If $\frac{\partial T_v}{\partial r}$ is given

$$\frac{\partial u}{\partial p} \Big|_{geos} = \frac{\partial u}{\partial p} + f \quad (= 1 + \frac{\partial u}{\partial p} \cdot \frac{p}{f})$$

∴ geostrophic wind shear is 40 times larger than gradient wind shear.

* $f \approx 0.1 \times 10^{-4} \text{ sec}^{-1}$ in hurricane

$$u \approx 30 \text{ m s}^{-1}$$

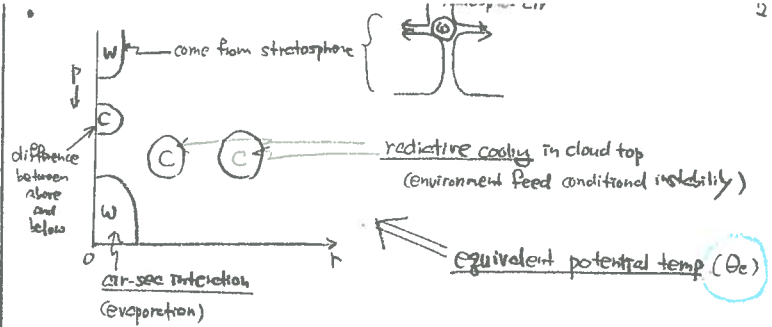
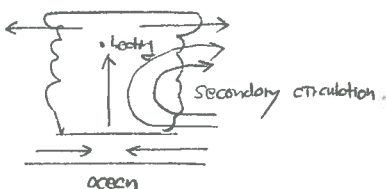
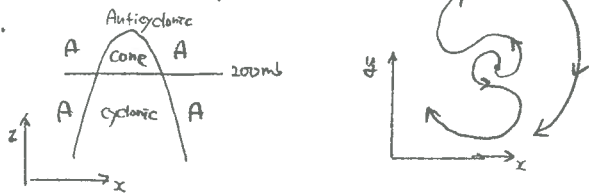
$$r \approx 20 \times 10^3 \text{ m}$$

$$\frac{\partial u}{\partial r} \approx \frac{2 \times 30}{2 \times 10^4} = 3 \times 10^{-3} \text{ s}^{-1}$$

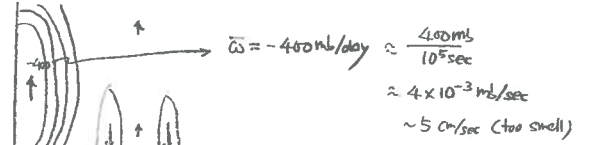
$$\frac{\partial u}{\partial r} \gg f$$

$$\frac{\partial u}{\partial r} \approx \frac{3 \times 10^{-3}}{0.1 \times 10^{-4}} \approx \frac{3}{7} \times 10^7 \approx 40$$

Inner rain area is important.



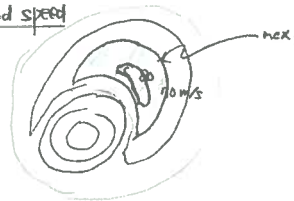
* vertical motion ($\bar{\omega}$)



→ Azimuthally Averaged

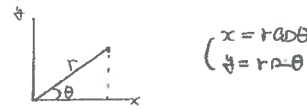
* RH

* 200mb wind speed



* Local cylindrical coordinate

From the tangent plane eqs of motion, we can transform them into cylindrical coords.



$$V_r = \frac{dr}{dt} \quad (\text{radial velocity}) ; \quad V_\theta = r \frac{d\theta}{dt} \quad (\text{tangential velocity})$$

$$u = \frac{dx}{dt} = \frac{d}{dt}(r \cos \theta) = \frac{dr}{dt} \cos \theta - r \sin \theta \frac{d\theta}{dt} = V_r \cos \theta - V_\theta \sin \theta \quad (1)$$

$$v = \frac{dy}{dt} = \frac{d}{dt}(r \sin \theta) = \frac{dr}{dt} \sin \theta + r \cos \theta \frac{d\theta}{dt} = V_r \sin \theta + V_\theta \cos \theta \quad (2)$$

$$\frac{du}{dt} = \frac{dV_r}{dt} \cos \theta - V_r \sin \theta \frac{d\theta}{dt} - \frac{dV_\theta}{dt} \sin \theta - V_\theta \cos \theta \frac{d\theta}{dt} \quad (3)$$

$$\frac{dv}{dt} = \frac{dV_r}{dt} \sin \theta + V_r \cos \theta \frac{d\theta}{dt} + \frac{dV_\theta}{dt} \cos \theta - V_\theta \sin \theta \frac{d\theta}{dt} \quad (4)$$

If we let the x axis coincide with r so $\theta = 0$,

$$\cos \theta = 1, \quad \sin \theta = 0$$

(3) + (4) reduce to

$$\frac{du}{dt} = \frac{dV_r}{dt} - V_\theta \frac{d\theta}{dt} = \frac{dV_r}{dt} - \frac{V_\theta^2}{r} \quad (5)$$

$$\frac{dv}{dt} = \frac{dV_\theta}{dt} + V_r \frac{d\theta}{dt} = \frac{dV_\theta}{dt} + \frac{V_r V_\theta}{r} \quad (6)$$

From tangent plane eqs. $\left(\begin{aligned} \frac{du}{dt} &= fu - \frac{1}{\rho} \frac{\partial p}{\partial x} \\ \frac{dv}{dt} &= -fv - \frac{1}{\rho} \frac{\partial p}{\partial y} \end{aligned} \right)$

$$\Rightarrow \frac{dV_r}{dt} - \frac{V_\theta^2}{r} = fV_\theta - \frac{1}{\rho} \frac{\partial p}{\partial r} \quad (7)$$

$$\frac{dV_\theta}{dt} + \frac{V_r V_\theta}{r} = -fV_r - \frac{1}{\rho} \frac{\partial p}{\partial \theta} \quad (8)$$

$$\text{or} \quad \frac{dV_r}{dt} - \frac{V_\theta^2}{r} - fV_\theta = -g \frac{\partial \bar{z}}{\partial r} \quad (9)$$

$$\frac{dV_\theta}{dt} + \frac{V_r V_\theta}{r} + fV_r = -g \frac{\partial \bar{z}}{\partial \theta} \quad (10)$$

* total derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V_\theta \frac{\partial}{\partial \theta} + V_r \frac{\partial}{\partial r} + \bar{\omega} \frac{\partial}{\partial p}$$

• Conservation of angular momentum.

Absolute angular momentum in cylindrical coord is defined

$$* M = V_{\theta} r + \frac{f_0 r^2}{2}$$

The first term is angular momentum about an axis, and the second term is momentum due to the earth's rotation.

$$\frac{dM}{dt} = -g \frac{\partial z}{\partial \theta} + F_{\theta} r$$

The first term is a pressure torque, and the second term is a frictional torque. If the storm is symmetric, then the pressure torque is zero.

Relating vorticity to angular momentum we obtain

$$\zeta_a = \frac{1}{r} \frac{dM}{dt}$$

The conservation of angular momentum does not imply conservation of absolute vorticity because convergence and divergence effects are not taken into account.

If we assume a symmetric storm so there are no pressure torques, and frictional torques are negligible (which is really not a valid assumption) we see that

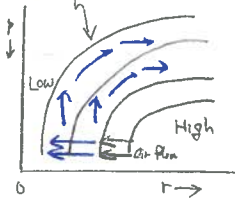
$$V_{\theta} r + \frac{f_0 r^2}{2} = C$$

For a given V_{θ} at a radius r , we can calculate the wind distribution for the entire storm. The velocity goes to ∞ at $r=0$.

Thus we see that frictional forces are negligible. These frictional torques arise from eddy motions on the cumulus scale as well as friction with the surface.

• Introduction to angular momentum

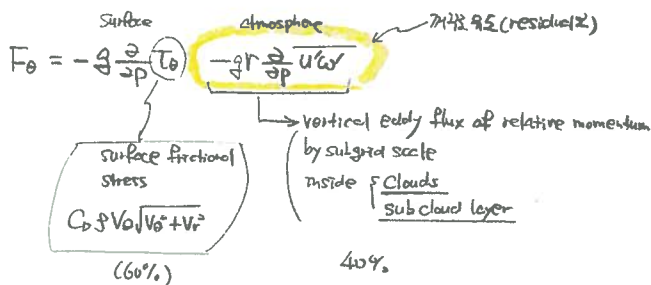
$M = ur + \frac{f_0 r^2}{2}$: angular momentum per unit mass gain where u is tangential velocity (azimuthally averaged)



$$\frac{dM}{dt} = -g \frac{\partial z}{\partial \theta} + F_{\theta} r$$

3-D change in angular momentum following a parcel

- Frictional torques (surface + cloud friction)
- Pressure torques

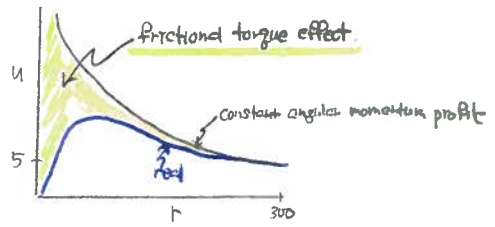


$$M = ur + \frac{f_0 r^2}{2}$$

Take a parcel of air at $r=r_1$ (300km), $u \approx 5 \text{ m s}^{-1}$

$$r=10 \text{ km} \rightarrow u=?$$

$$r=0 \rightarrow u=\infty$$



<goal> to get C_D

• Gross angular momentum budget

$$\frac{dM}{dt} = -g \frac{\partial z}{\partial \theta} + F_{\theta} r \quad \text{--- (1)}$$

∴ Derive (1)

$$M = V_{\theta} r + \frac{f_0 r^2}{2}$$

$$\frac{dM}{dt} = r \frac{dV_{\theta}}{dt} + V_{\theta} \frac{dr}{dt} + f_0 r \frac{dr}{dt} = r \frac{dV_{\theta}}{dt} + V_{\theta} V_r + f_0 r V_r$$

$$\therefore \left(\frac{dV_{\theta}}{dt} + \frac{V_r V_{\theta}}{r} + f_0 V_r \right) = -g \frac{\partial z}{r \partial \theta} + F_{\theta} \quad \leftarrow \text{eq. of motion in } \theta \text{ including friction}$$

$$= r \left[-\frac{V_r V_{\theta}}{r} - f_0 V_r - g \frac{\partial z}{r \partial \theta} + F_{\theta} \right] + V_{\theta} V_r + f_0 r V_r$$

$$= -g \frac{\partial z}{\partial \theta} + r F_{\theta}$$

advective form in cylindrical coord.

$$\frac{\partial M}{\partial t} = -V_{\theta} \frac{\partial M}{r \partial \theta} - V_r \frac{\partial M}{\partial r} - \omega \frac{\partial M}{\partial p} - g \frac{\partial z}{\partial \theta} + F_{\theta} r \quad \text{--- (2)}$$

Flux form →

$$\frac{\partial M}{\partial t} = -\frac{\partial M V_{\theta}}{r \partial \theta} - \frac{1}{r} \frac{\partial M V_r r}{\partial r} - \frac{\partial M \omega}{\partial p} - g \frac{\partial z}{\partial \theta} + F_{\theta} r \quad \text{--- (3)}$$

which uses mass continuity eq.

$$\frac{\partial V_{\theta}}{r \partial \theta} + \frac{1}{r} \frac{\partial V_r r}{\partial r} + \frac{\partial \omega}{\partial p} = 0 \quad \text{--- (4)}$$

Integrate term by term

$$-\frac{1}{g} \int_{r_0}^{r_1} \int_{\theta_0}^{\theta_1} () r dr d\theta dp$$



$$\frac{\partial \bar{M}}{\partial t} = - \frac{\partial}{\partial p} (M_a V_{r2} r_2 - M_i V_{r1} r_1) + F_{\theta} \pi (r_2^2 - r_1^2) \quad \text{--- (5)}$$

$$\checkmark F_{\theta} = -g \frac{\partial T_{\theta}}{\partial p} \quad \text{--- (6)}$$

T_{θ} is the surface wind stress

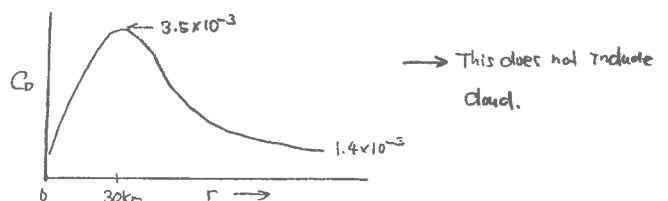
$$T_{\theta} = C_D \rho V_0 \sqrt{V_r^2 + V_{\theta}^2} \quad \text{--- (7)}$$

In equation (5) everything is known from data except F_{θ} which is deduced as a residual.

$$\bar{F}_{\theta} = -g \frac{\bar{T}_{\theta} - 0}{\Delta p} = -g \frac{\bar{T}_{\theta}}{\Delta p} \rightarrow \text{entire atmosphere}$$

hence we know \bar{T}_{θ} . In equation (7), we can now solve for C_D

Then you can plot C_D as a function of r



We can also plot T_{θ} as a function of $\sqrt{V_r^2 + V_{\theta}^2}$ i.e. the total wind speed that shows that the stress increases very rapidly at high wind speed.

How to find $\overline{M\omega}$ (r, θ, p) from angular momentum budget

$$\frac{dM}{dt} = -g \frac{\partial z}{\partial \theta} + \overline{F\theta} r$$

$$\rightarrow F\theta = -g \frac{\partial z}{\partial p} \left\{ \begin{aligned} T_{\theta} &= C_p P V_0 \sqrt{V_0^2 + V_r^2} \sin \theta \\ T_{\theta} &= -\overline{u\omega} \end{aligned} \right.$$

The same eq (3) is integrated over a much smaller mass element.

$$dm = -\frac{1}{g} \int_{r_1}^{r_2} \int_{\theta_1}^{\theta_2} r dr d\theta dp$$

Over the mass element we can obtain $\overline{F\theta} r$ as a residual after mass element integration gives us

- $\frac{\partial \overline{M}}{\partial t}$ = Tangent convergence of flux
- + Radial " " "
- + large scale vertical convergence of flux
- + $\overline{F\theta} r$

Here the only unknown is $\overline{F\theta} r$ because all else can be calculated from large scale data sets.

We can write $\overline{F\theta} r = -g \frac{\partial z}{\partial p} r$

$$= -g \frac{\partial}{\partial p} \overline{u\omega} r$$

$$= -g \frac{\partial}{\partial p} \overline{M\omega}$$

Here we have added zero which is $\frac{\partial}{\partial p} \overline{F r^2}$ (which is zero)

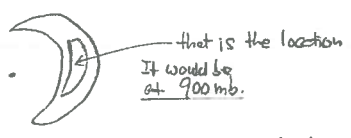
This enables us to find $\frac{\partial}{\partial p} \overline{u\omega} r$ as a residual which in turn gives us $\overline{M\omega}$ (r, θ, p) since $\overline{M\omega} = 0$ at the top of the stratosphere and is described by the similarity fluxes at the bottom of the atmosphere.

What is $-\overline{M\omega}$ physically?

It is the cloud contribution in the free atmosphere, it is the surface friction contribution at the lower boundary.

We have a forecast of hurricane intensity which looks reasonable. How do we go about diagnosing it?

1. Go to the map location of the wind maximum



2. Prepare data in storm relative coordinate.

3. Construct backward trajectories.

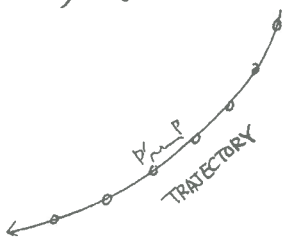
Along the trajectories, interpolate and obtain best values for

$$-g \frac{\partial z}{\partial \theta}$$

and $\overline{F\theta} r$

from your model which made the intensity forecast in the first place. You also know $\frac{dM}{dt}$ from the trajectory.

You should have a residue-free budget. You can then find why the intensity change occurred.



Thus at points P + P' along the trajectory (or relative flow) you

know $\frac{dM}{dt}$, $-g \frac{\partial z}{\partial \theta}$ and $\overline{F\theta} r$

They are to be in near balance or else you have done something wrong.

This simply says that

$$M(p') = M(p) - \left\{ \frac{g \frac{\partial z}{\partial \theta} + g \frac{\partial z}{\partial \theta} \right\} r \left\{ \text{at } p' \right.$$

$$\left. - \left\{ \frac{\overline{F\theta} r + \overline{F\theta} r \right\} r \left\{ \text{at } p \right. \right.$$

Note that

$$M(p') = (V\theta r + \frac{F\theta r^2}{2}) \text{ at } p'$$

$$M(p) = (V\theta r + \frac{F\theta r^2}{2}) \text{ at } p$$

hence the change in intensity is given by

$$\left(V\theta(p') = V\theta(p) \frac{r(p)}{r(p')} + \frac{\frac{F\theta r^2}{2} \Big|_p - \frac{F\theta r^2}{2} \Big|_{p'}}{r(p')} \right) \text{ intensity eq.}$$

$$= \frac{-g \frac{\partial z}{\partial \theta} \Big|_p + g \frac{\partial z}{\partial \theta} \Big|_{p'}}{r(p')} - \frac{F\theta r \Big|_p + F\theta r \Big|_{p'}}{r(p')}$$

Potential vorticity + Angular momentum in hurricanes (for intensity forecast)

Diabatic Potential vorticity equation

→ complete Ertel potential vorticity eq

$$\frac{d}{dt} \left(-\zeta_{\theta 0} g \frac{\partial \theta}{\partial p} \right) = \left(-\zeta_{\theta 0} g \frac{\partial \theta}{\partial p} \right) \frac{\partial}{\partial \theta} \frac{d\theta}{dt} + \left\{ \nabla \cdot \left(\frac{p \theta}{dt} \frac{\partial (\nabla \times k)}{\partial \theta} \right) \right\} g \frac{\partial \theta}{\partial p}$$

Generally, positive in N.H./negative in S.H.
 (vertical differential of diabatic heating)
 (lateral differential of diabatic heating)
 (friction term)
 (very large cross ITCZ)

where isentropic absolute vorticity is given by

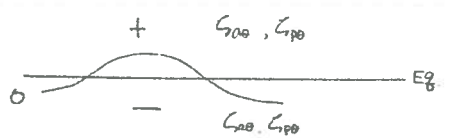
$$\zeta_{\theta 0} = \frac{\partial v}{\partial x} \Big|_0 - \frac{\partial u}{\partial y} \Big|_0 + \frac{u}{a} \tan \phi + f$$

and potential vorticity

$$\zeta_{p\theta} = -g \zeta_{\theta 0} \frac{\partial \theta}{\partial p} \quad - \frac{\partial \theta}{\partial p} \text{ is generally positive definite}$$

Main Question?
 advection

$$\left\langle \frac{PVA}{\text{each term}} \right\rangle = \text{magnitude?}$$



Order of magnitudes

- potential vorticity ($\zeta_{p\theta}$) → unit: $kg^{-1} m^2 deg s^{-1}$

$$g = 9.81 \text{ ms}^{-2}; (\zeta_{\theta 0} + f) \approx 10^{-4} s^{-1}$$

$$\frac{\partial \theta}{\partial p} \approx 5^\circ C / 100 \text{ mb} \approx 5 \times 10^{-4} \text{ kg}^{-1} \text{ deg ms}^{-2}$$

$$\zeta_{p\theta} = -g \zeta_{\theta 0} \frac{\partial \theta}{\partial p} \approx 10^{-1} \text{ kg}^{-1} \text{ m}^2 \text{ deg s}^{-1} \text{ over the tropics}$$

- Advection of PV: $-\nabla \cdot \nabla \zeta_{p\theta}$ (PVA)

$$|\nabla| \times 10 \text{ ms}^{-1}; \nabla = \frac{1}{300 \text{ km}} \approx 0.3 \times 10^{-5} \text{ m}^{-1} \text{ for large scale}$$

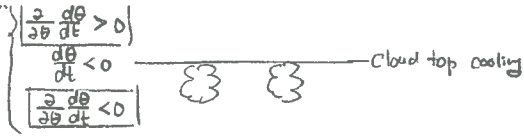
$$-\nabla \cdot \nabla \zeta_{p\theta} \approx 3 \times 10^{-12} \text{ m}^2 \text{ deg kg}^{-1} \text{ s}^{-1}$$

- differential heating along the vertical

$$\zeta_{p\theta} \frac{\partial}{\partial \theta} \frac{d\theta}{dt}$$

$$\left(\frac{d\zeta_{p\theta}}{dt} \right) = \zeta_{p\theta} \frac{\partial}{\partial \theta} \frac{d\theta}{dt}$$

in N.H. $\int > 0$ lower troposphere



$\frac{\partial \theta}{\partial t}$ is greater than zero over the lower troposphere and is less than zero above the maximum heating in the upper troposphere.

Since C_{po} is generally positive in the M.H., the effect of such a convective heating is to generate PV in the lower troposphere and to destroy PV in the upper troposphere.

Large values of $C_{po} \frac{\partial \theta}{\partial t}$ can be expected where the vertical gradient of the apparent heat source, Q_1 , is large since $Q_1 = C_p \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}} \frac{d\theta}{dt}$.

ex) ITCZ, monsoon + tropical depression, tropical waves, tropical squall + non squall systems, general monsoon rainfall systems, mid tropospheric cyclones etc

Differential lateral heating and generation of PV

Criterion $\left\{ \begin{array}{l} \text{① large lateral gradient of heating} \\ \text{② Strong wind shear} \end{array} \right.$

$\left(\nabla \theta \cdot \frac{\partial \mathbf{V} \times \mathbf{k}}{\partial t} \right) g \frac{\partial \theta}{\partial p}$ $\frac{d\theta}{dt} < 0$ } important heat

ITCZ $\frac{d\theta}{dt} \gg 0$

Fractional contribution

$-\{ \nabla \cdot (E \times k) \} g \frac{\partial \theta}{\partial p}$

$E = -g \frac{\partial T}{\partial p}$

What is the relationship between PV + AM?

$C_a = \frac{\partial v_\theta}{\partial r} - \frac{\partial v_r}{r \partial \theta} + \frac{v_\theta}{r} + f_0$ ← derive?

large shear term small large curvature term

$C_a = \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} + f_0 = \frac{1}{r} \frac{\partial (v_\theta r)}{\partial r} + f_0$

$M = v_\theta r + \frac{f_0 r^2}{2}$

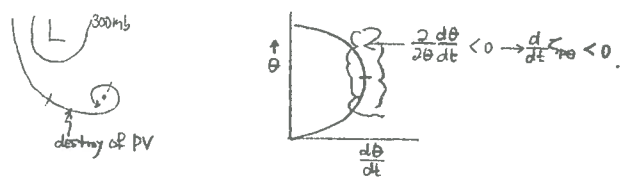
$\frac{1}{r} \frac{\partial M}{\partial r} = \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} + f_0 = \frac{1}{r} \frac{\partial (v_\theta r)}{\partial r} + f_0$

$\therefore C_a \approx \frac{1}{r} \frac{\partial M}{\partial r}$

Therefore

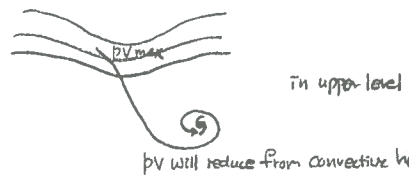
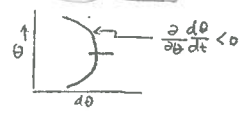
$PV = -C_a g \frac{\partial \theta}{\partial p} \approx -\frac{1}{r} \frac{\partial M}{\partial r} g \frac{\partial \theta}{\partial p}$

→ generation of PV → generation of gradient of AM



PV continue

$PV \rightarrow \frac{dC_{po}}{dt} = +C_{po} \frac{\partial \theta}{\partial \theta} \frac{d\theta}{dt} + \dots$



$C_{po} = \frac{1}{r} \frac{\partial M}{\partial r}$

$C_{po} = -g \frac{\partial \theta}{\partial p} C_{po} = -g \frac{\partial \theta}{\partial p} \frac{1}{r} \frac{\partial M}{\partial r}$

Following parcel motion, processes that reduce C_{po} will reduce $\frac{1}{r} \frac{\partial M}{\partial r}$

Hence, $\frac{\partial M}{\partial r}$ must reduce rapidly along inflowing parcels.

Outer angular momentum goes in somewhat unabated hurricane will intensify (angular momentum conservation)

Entire circulation below that region will intensify (hydrostatic)

• Lack of convection along inflow channels in low levels.



$\frac{dM}{dt} = -g \frac{\partial \theta}{\partial p} + F_0 r$

↑ Cloud vertical eddy momentum flux

$-\frac{\partial}{\partial p} M' \omega'$

* upper level development went to convection
(lower " " " " cloud free)

• We covered up to now PV + AM

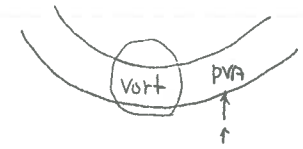
other issues ① PVA ② SST ③ concentric eye walls.

PVA (positive vorticity adv)

analogy of Q + ω - eq.

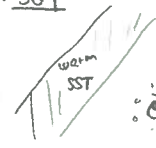
$\nabla \cdot \nabla^2 \omega + f_0 \frac{\partial \omega}{\partial p} = -\frac{\partial}{\partial p} \mathbf{V} \cdot \nabla \zeta_a - \pi \nabla^2 (\mathbf{V} \cdot \nabla T)$

Large PVA aloft } $-\frac{\partial}{\partial p} \mathbf{V} \cdot \nabla \zeta_a > 0$
small PVA below } forcing function > 0
 $\omega < 0$



→ There exist a similar eq in isentropic coord.

SST



if already warm SST exist

many clouds in warm SST region will kill hurricane

• if there is cloud around hurricane, it will grow

Formation of Hurricane

1. SST > 26°C
2. preexisting lower-troposphere cyclonic vorticity
3. Weak wind shear (vertical)
4. R H 500mb > normal.

* there is a handout for details.

Given the model output, why did the storm form?

1. Energetics

2. Kuo-Eliassen eqs. → eddy flux + differential heat

3. Angular momentum principle

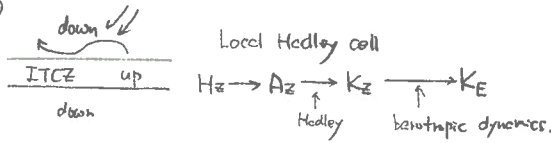
which one is more important?

- Energetics

Initial growth

vertical shear + convection } $H_E \rightarrow A_E \rightarrow K_E$

②



③ $H_E \rightarrow A_E \rightarrow K_E$

Planetary Boundary layer in the tropics

• Preamble.

Land-air, ocean-air interfaces, fluxes, how they vary with height.

Similarity theory.

Bulk-aerodynamic

→ look at RWP note

* There's a hand out → look exam questions.

Tropical cyclones

Genesis regions/conditions.

① Warm SSTs + deep oceanic mixed layer (>26°C)

ATL → easterly waves are main source, then upper level low
SH → monsoon trough

② significant values of C_a (cyclonic) in lower troposphere.

③ Weak vertical shear over disturbances. Note that shear is greater during El Niño and so fewer storms.

④ mean $w > 0$ and moist at midlevels.

Aircraft recon. Show that the following features near eye.



- max vertical velocity + radial wind
- max tangential wind
- max rain.

② extensive area of stratiform rain.

→ an order of magnitude more area covered by stratiform precip. than convective precip. but stratiform precip. only accounts for 60% of total storm precip

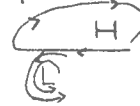
③ eyewall convection slopes outward w/ height. steeper slopes associated w/ stronger storms + smaller eyes. in the eyewall updrafts tend to follow lines of constant angular momentum that slope outward

Angular momentum

$\frac{dM}{dt} = \text{pressure torque} + \text{friction torque}$; $M = U r + \frac{r^2 \omega}{2}$

↑ relative air
↑ planetary air

→ subtropical high is a source of AM.



→ pressure + friction torques work to keep winds bounded (finite)

→ If there are many clouds in the inflow channels into a hurricane, the clouds take AM from the inflow air and therefore deplete AM.

→ The pressure gradient force is not entirely symmetric in a hurricane. when $-\frac{\partial \omega}{\partial \theta}$ is negative, it depletes AM

Scale analysis of the large-scale tropical boundary layer (PBL)

Zonal eq of motion

$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - fV = -g \frac{\partial Z}{\partial x} + F_x$ → vertical advection is assumed small.

$T + A + C = P + F$
dominant terms

non-dimensionalize →

$u = U u'$
 $\frac{\partial}{\partial x} = (U/\beta)^{-1/2} \frac{\partial}{\partial x'}$ usual beta plane approximation
 $f = \beta y$
 $\frac{\partial}{\partial t} = \omega \frac{\partial}{\partial t'}$ characteristic frequency.

$\omega U \left(\frac{\partial u'}{\partial t'} \right) + \frac{U^2}{(U/\beta)^{1/2}} \left[u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} \right] - \beta y v' = P + F$

or $\omega \left(\frac{\partial u'}{\partial t'} \right) + (U/\beta)^{1/2} \left[u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} \right] - \beta y v' = \frac{P}{U} + \frac{F}{U}$

→ three time scales

ω^{-1} , $(U/\beta)^{-1/2}$ and $(\beta y)^{-1}$

Let's consider the following three cases.

i) If $\omega < \beta y$ and $(U/\beta)^{1/2} < \beta y$, then C, P, F are dominant.

⇒ Ekman balance ($C = P + F$)

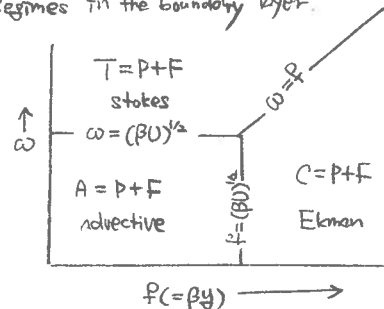
ii) $f = \beta y < (U/\beta)^{1/2}$ and $\omega < (U/\beta)^{1/2}$

⇒ advective or drift boundary layer ($A = P + F$)

iii) $\omega > \beta y$, $\omega > (U/\beta)^{1/2}$

⇒ Stokes regime ($T = P + F$)

⇒ Regimes in the boundary layer.



cf) In mid-lat. the Ekman balance is a characteristic feature in PBL.

- N.H: veering
- S.H: backing

END