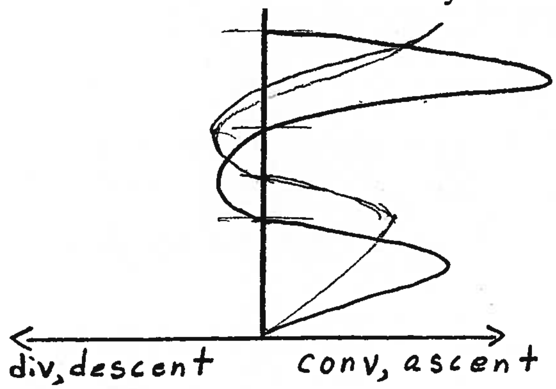
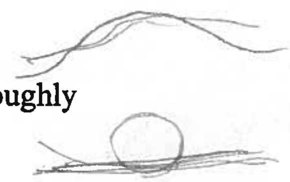
FUELBERG

1. The following questions require only brief answers. Pick any four of the five questions.

a) The graph below is a plot of horizontal divergence as a function of height. **Superimpose** the profile of vertical motion (w). Make sure that the altitudes of maximum and minimum vertical motion are clearly indicated. Assume $w=0$ at surface.



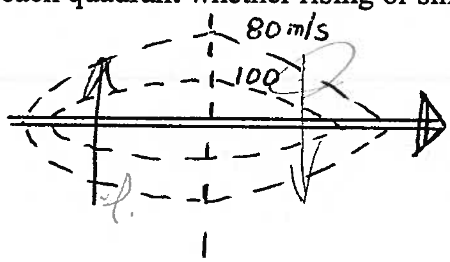
b) How does the intensity of a warm core anticyclone vary with altitude? Thoroughly **explain** your answer.



c) Is cold or warm temperature advection occurring when winds veer with increasing altitude? **Sketch** a vector diagram showing this situation. **Explain** your answer thoroughly, making use of the thermal wind concept.

U_s $\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j}$

d) The diagram below shows a typical jet streak at 300 hPa. In each of the four quadrants, **indicate** whether there is horizontal divergence or convergence at 300 hPa. Also **indicate** in each quadrant whether rising or sinking air is occurring in the middle troposphere.

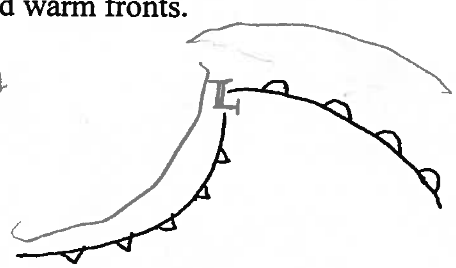


e) The following diagram shows a frontal system at sea level. **Superimpose** typical frontal positions at 850 hPa. Assuming typical frontal slopes, indicate on your diagram the **horizontal distance** (in km) between the surface and 850 positions for both the cold and warm fronts.

1.6k

1500 m

1500 km



1:200

1:50

1500
200
300000

KRISH

2. This question is on the response of the atmospheric circulation from a prescribed (symmetric) heat source at the equator. Starting from the linearized shallow water equations with this forcing, how does one obtain this solution? What does this solution look like for the following: wind field over the lower troposphere, pressure field, vertical velocity field and hadley cell. Where are the Kelvin waves and Rossby gravity waves in these solutions?

3. This question pertains to hurricane formation, structure, dynamics:

a) What are some of the important major environmental factors during the genesis of hurricanes?
b) Sketch a typical vertical cross section across a hurricane of the temperature, anamoly and equivalent potential temperature. Discuss any salient features in these cross sections.
c) How would you use the angular momentum principle to address the intensity of a hurricane?

LASEUR

Ph. D. Question- MET5510C-Mid latitude Synoptics

(45-60 minutes)

4. Combination of the three-dimensional vorticity equation, the continuity equation, and the three-dimensional frontogenesis equation yields the following potential vorticity theorem:

$$\frac{d}{dt}\left(\alpha\nabla\theta\cdot\underline{\eta}\right) = \alpha\,\underline{\eta}\cdot\nabla\left(\frac{d\theta}{dt}\right) + \alpha\nabla\theta\cdot(\nabla\times\underline{F}) \qquad (1)$$

in which
α = specific volume
θ = potential temperature
$\underline{\eta} = (\nabla_3\times V) + \Omega$ = 3-dim vorticity
$\underline{F}$ = frictional force/mass

(a) Show how (1) may be transformed to the following more useful form:

$$\frac{d}{dt}\left[\left(-\frac{\partial\theta}{\partial p}\right)(\zeta_\theta+f)\right] = (\zeta_\theta+f)\left(-\frac{\partial}{\partial p}\frac{d\theta}{dt}\right) - \frac{\partial\theta}{\partial p}\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)_\theta \qquad (2)$$

The term in brackets on the LHS of (2) is known as the potential absolute vorticity;
(b) Explain clearly how this quantity may be evaluated from synoptic analyses. What have studies of P shown as to its variations in magnitude in the troposphere and lower stratosphere?

The two terms on the RHS of (2) represent physical processes that may produce changes in P following the same volume of air;
(c) Give a clear physical interpretation of these terms and discuss the physical processes in the atmosphere which can produce significant changes in P. Under what circumstances is P conserved, i.e., $\frac{dP}{dt}$ =0?

DAY I

October 18, 1999

KWANG

5a Consider a differential equation

$$\mathcal{L}T(x) + A + BT(x) = f(x),$$

where A and B are parameters. The linear differential operator, $\mathcal{L}$, has eigenvectors and eigenvalues such that

$$\mathcal{L}\psi_n(x) = \lambda_n \psi_n(x),$$

where $\psi_n(x)$ and λ_n are the nth eigenvector and the corresponding eigenvalue, respectively. Further, the eigenfunctions satisfy the following orthogonality condition:

$$\int_R \psi_n(x)\psi_m(x)\,dx = \delta_{nm} \qquad \text{and} \qquad \int_R \psi_n(x)\,dx = \delta_{n0}.$$

Obtain the solution T_n for arbitrary n, where

$$T(x) = \sum_n T_n \psi_n(x). \longrightarrow T_n = \frac{f_n - A\delta_{n0}}{\lambda_n + B}$$

Klono

b. What is sub-scale parameterization? Discuss it in detail by adopting an example in your own field.

c. Let us consider a simple stochastic rain model

$$\frac{\partial p(\mathbf{r},t)}{\partial t} + Bp(\mathbf{r},t) - D\nabla^2 p(\mathbf{r},t) = F(\mathbf{r},t), \qquad -D\frac{\delta p}{L^2} = FL^2$$

where $p(\mathbf{r},t)$ is the rain field, B and D are model parameters, and $F(\mathbf{r},t)$ is a white, rotationally-invariant (i.e., spatially white) noise forcing. Such a model is often used to make inferences on the space-time statistics of the rain field.

(a) Find the inherent space and time scales of the rain field resulting from the model above.

(b) How long will the rain field predictable from a given initial condition?

Scale ana

Stroge

√ 6 **Physical Question (PhD Prelim – Ruscher) – 1.5 hour**

Klone Kronecker

Derive the turbulent kinetic energy equation starting with the equation of motion (written as a local tendency of the zonal wind). Give a brief interpretation for each term. Show how the Richardson number can be derived from terms of this equation, and give an interpretation for laminar, turbulent, and transitional flows in terms of the Richardson number. If you can think of a more convenient form of the Richardson number which does not include turbulent fluxes, provide further information.

flux

$$g - \frac{\theta_v'}{\overline{\theta_v}} g$$

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\delta_{i3} g + f\varepsilon_{ij3} u_j - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}$$

$2u_i'$

DAY 2

Krishnamurti 5541 (one hour test question)

1. The following expression appears as a nonlinear term in the divergence equation of the shallow water system of equations:

$$\frac{1}{\cos^2\theta}\left[\frac{\partial V(\nabla^2\psi+f)}{\partial\lambda}-\cos\theta\frac{\partial U(\nabla^2\psi+f)}{\partial\theta}\right]-\nabla^2\left(\frac{U^2+V^2}{2\cos^2\theta}\right)$$

Here, U and V are the Robert function pseudo velocitites

θ is latitude, λ is longitude

ψ is streamfunction on a sphere

f is the coriolis parameter

You are given U,V and ψ on a spherical grid θ,λ. Show sequentially how you would use the spectral transform method to evaluate this nonlinear term and obtain the final result on a spherical transform grid.

O' BRIEN

SUBJ: Exam question for Ph.D. Exam 1999

2. EOF Analysis (O'Brien, 1 hour)

You have sea-surface data from the North Pacific on a 2° x 2° grid, monthly for 40 years. You want to look for "prominent" standing modes so you choose EOF (or PCC analysis which is the same thing.) Let us call your datum

$\{T_{s,t}\}$ s=1(1) S and t=1(1)480 and S is about 15,000.

$\frac{12}{480}$ 40

Answer the following questions:

A. Will you do any "preprocessing" of the data before trying to find the EOF's? Yes.

B. What matrix will you compute to help find the EOF's? Covariance matrix.

C. How will you find the eigenvalues? $|C-\lambda I|=0$

D. After the computer gives you the eigenvalues, will you do any processing? ordering

E. How do you find the eigenvectors? Same routine

E. In your choice of "matrix," will the eigenvectors be the spatial or temporal loadings?

G. How do you find the other set of eigenvectors? projecting

H. Some of the EOF's may be significant. How will you determine which EOF's are not useful or significant?

GCM

instead of the original data GC

C

DAY 2

$1.01 a_1 + a_2 = 1$
$-)\ 1 a_1 + a_2 = 2$
$0.01 a_1 = -1$
$a_1 = 0,\ a_2 = 1$
$a_1 = -\frac{2}{0.01}$ $a = -$
$a_1 = -0$

$0.99 a_1 + a_2 = 1$
$-)\ 1 a_1 + a_2 = 1$
$a_1 = 0,\ a_2 = 1$

$1 - 0.01 = 0.99$

3. **Time series: least squares and EOF analysis** (Ahlquist, 1 hour)

Suppose you want to use ensemble forecasting to make a 24-hour forecast of quantity $q(t)$ for day t. Let $f(t,i)$ denote the i-th of N 24-hour forecasts that all apply to day t. We seek an ensemble forecast for $q(t)$ of the form

$$f(t) = \sum_{i=1}^{N} a_i f(t,i) \tag{1}$$

To allow for bias correction, we can take the N-th forecast to be $f(t,N) = 1$.

(a) Minimize the squared error, $E = \sum_t (q(t) - f(t))^2$, to derive the equations that must be solved to determine $a_1, \ldots, a_N$ based on an old set of forecasts and analyses for $q(t)$. This is often called the training data set.

(b) A common difficulty with the traditional least squares approach is that some of the predictors, $f(t,i)$, may be strongly correlated with each other. In that case, the equations you derived in part (a) are ill-conditioned, meaning that a small change in a term in the equations for $a_1, \ldots, a_N$ can make a big difference in the solution. For example, consider this example where $N = 2$.

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\begin{pmatrix} 1+\epsilon & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2}$$

Solve for a_1 and a_2 when $\epsilon = 0.01$. Solve again for a_1 and a_2 when $\epsilon = -0.01$. (Do this by hand. You don't need a calculator.)

$\frac{1}{\epsilon}\begin{pmatrix} 1 & -1 \\ -1 & 1+\epsilon \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $-1+1+\epsilon$

$\frac{1}{\epsilon}\begin{pmatrix} 0 \\ \epsilon \end{pmatrix}$

$\therefore a_1 = 0$

(c) One way around the ill-conditioning difficulty is to perform an empirical orthogonal function (EOF) analysis on the training data set. Let $e_j(i)$ denote the i-th element of the j-th EOF. Then, using the training data set again, we compute the j-th principal component (PC),

$$p_j(t) = \sum_{i=1}^{N} f(t,i) e_j(i) \Bigg/ \sum_{i=1}^{N} e_j^2(i) \,. \tag{3}$$

Similar to but instead of (1), we write our forecast as

$$f(t) = \sum_{j=1}^{n} b_j p_j(t). \tag{4}$$

eigenvecte

where we choose $n < N$ if the EOF analysis shows that forecasts are strongly correlated. Using the same equations that you derived for part (a) and the fact that the PCs are orthogonal, i.e., $\sum_t p_j(t) p_k(t) = 0$ if $j \neq k$, show that the equations for b_j do not suffer from ill-conditioning. In fact, the formula for b_k is completely independent of the formula for any other b_j.

(d) What information is contained in the EOFs, $e_j(i)$, for this application?

DAY 2

√ 4. **Synoptic Question (General – Ruscher) – one hour**

easy

Four methods for obtaining large scale vertical motion estimates in meteorology include the kinematic method, adiabatic method, quasi-geostrophic method, and a direct method using remote sensors.

Discuss each of these methods, including quantitative details and derivations for the first two listed. For the QG method, the equation below may be used as a starting point, but it requires some further explanation. For the latter method, discuss at least one type of remote sensing instrument and how it may be utilized to obtain such estimates. For each method you should comment on its utility and any assumptions needed.

$$\left(\nabla^2 + \frac{f_o^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \cdot \vec{\nabla}\left(\frac{1}{f_0}\nabla^2\Phi + f_0\right)\right] + \frac{1}{\sigma}\nabla^2\left[\vec{V}_g \cdot \vec{\nabla}\left(-\frac{\partial\Phi}{\partial p}\right)\right]$$

ZOU

√5. a. How many existing data analysis/assimilation methods do you know? Can you comment on their features including advantages and disadvantages?

b. Is there a direct way to incorporate observed rainfall data over regions where the assimilation model initialized with the guess field does not produced rain in 4D-Var? If yes, how?

Xiaolei

Ruby

Comprehensive Exam Question for Mr. Shin

30 minutes

6 a. The Bonssinesq approximation is often used for describing compressible fluid flows. If a layer of compressible fluid of depth d is heated from below so that its potential temperature at the bottom is higher by ∇T than at the top of the layer, write the governing equations in the Boussinesq appoximation. Indicate the limit in which these equations are valid.

T

T+ΔT

30 minutes

b: If a linear stability analysis of a problem, such as the one described above, yields degenerate eigenfunctions (that is, many eigensolutions exist for the same eigenvalue), outline a procedure by which you might resolve this degeneracy. (Hint: in the situation described above, infinitely many eigensolutions exist at a critical value of ∇T, but at nearby values of ∇T, it can be shown that only one solution is stable. Indicate how you would proceed to analyze such a situation.)

P!