An Analytic Model of the Wind and Pressure Profiles in Hurricanes

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ABSTRACT

An analytic model of the radial profiles of sea level pressure and winds in a hurricane is presented. The equations contain two parameters which may be empirically estimated from observations in a hurricane or determined climatologically to define a standard hurricane; examples are given. The model is shown to be generally superior to two other widely used models and is considered to be a valuable aid in operational forecasting, case studies and engineering work.

1. Introduction

One way of analyzing the often sparse observations near a hurricane is to use an analytic model of the sea level pressure and wind profiles, which can interpolate between observations to provide objective estimates of the maximum winds, extent of destructive winds, and other parameters. Such a model can also be used to define "standard" hurricanes for storm surge and marine and civil engineering applications.

Current models include the modified Rankine vortex (Depperman, 1947) and Schloemer's (1954) negative exponential relation. In this paper the approach adopted by Schloemer is extended to develop a universal model. Some uses and limitations of this model are described and illustrated by its application to three Australian hurricanes. Additionally, the development of a standard model, using climatological parameters, is briefly discussed. This standard model is shown to be better than the modified Rankine vortex and Schloemer's relation.

2. The model

Following Schloemer (1954) the hurricane profiles are normalized to remove variations due to differing central and ambient pressures by using the parameter

$$(p - p_c)/(p_n - p_c), \tag{1}$$

where p is the pressure at radius r, p_c the central pressure and p_n the ambient pressure (theoretically at infinite radius; however, in practice, the value of the first anticyclonically curved isobar is used).

The parametric profiles for a number of hurricanes are shown in Fig. 1. These profiles resemble a family of rectangular hyperbolas and may be approximated by

$$r^B \ln[(p_n - p_c)/(p - p_c)] = A, \tag{2}$$

where A and B are scaling parameters. On taking antilogarithms and rearranging

$$p = p_c + (p_n - p_c)\exp(-A/r^B). \tag{3}$$

Hence, using the gradient wind equations, the wind profile is

$$V_g = [AB(p_n - p_c)\exp(-A/r^B)/\rho r^B + r^2f^2/4]^{1/2} - rf/2, \tag{4}$$

where V_g is the gradient wind at radius r, f is the Coriolis parameter and ρ the air density (assumed constant at 1.15 kg m^{-3}).

In the region of maximum winds the Coriolis force is small in comparison to the pressure gradient and centrifugal forces and the air is in cyclostrophic balance. These winds are given by

$$V_c = [AB(p_n - p_c)\exp(-A/r^B)/\rho r^B]^{1/2}. \tag{5}$$

Hence, by setting $dV_c/dr = 0$, the radius of maximum winds (RMW) is

$$R_w = A^{1/B}. \tag{6}$$

The RMW is independent of the relative values of ambient and central pressure and, as expected, is defined entirely by the scaling parameters A and B. Substituting (6) back into (5) gives the maximum wind speed

$$V_m = C(p_n - p_c)^{1/2}, \tag{7}$$

where

$$C = (B/\rho e)^{1/2} \tag{8}$$

and e is the base of natural logarithms. Eq. (7),

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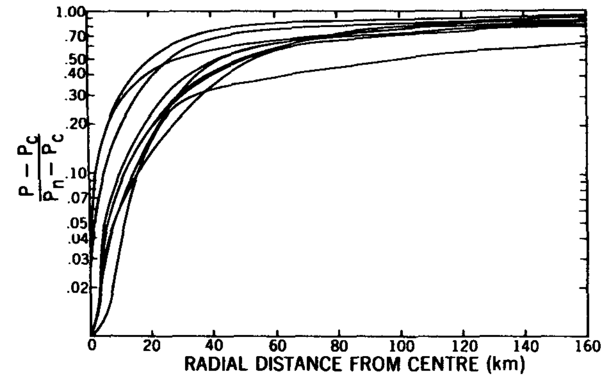

FIG. 1. Parametric log/linear pressure profiles for nine Florida hurricanes (after Schloemer, 1954).

with an empirically determined C, has been used widely for estimating the maximum winds in hurricanes (Takehashi, 1939; Myers, 1954; Kraft, 1961; Atkinson and Holliday, 1977). It is notable that the maximum wind intensity is independent of the RMW, but information on the shape of the pressure profile (through parameter B) is required.

Physically, B defines the shape of the profile and A determines its location relative to the origin. This can be seen in (6), (7) and (8). From (6), for a particular profile shape (constant B), A provides a radial scaling on the RMW. Using (7) and (8), for a constant

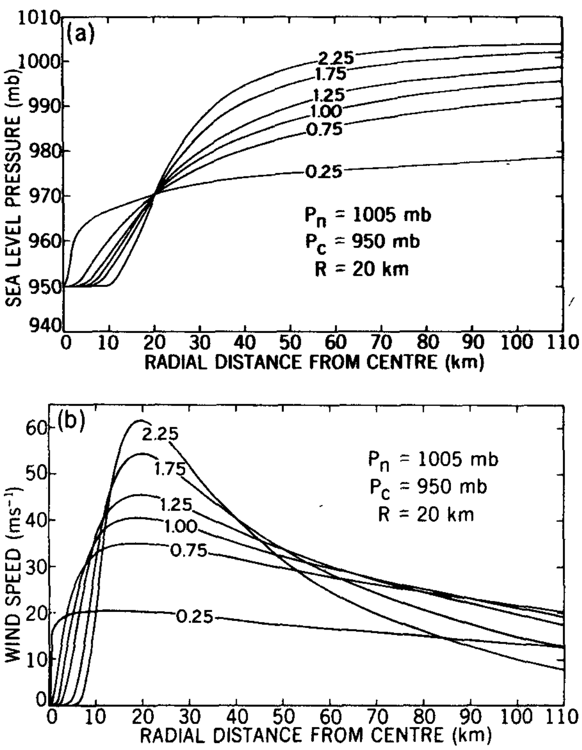

FIG. 2. The effect of varying the parameter B on (a) the sea level pressure profile and (b) the gradient wind profile.

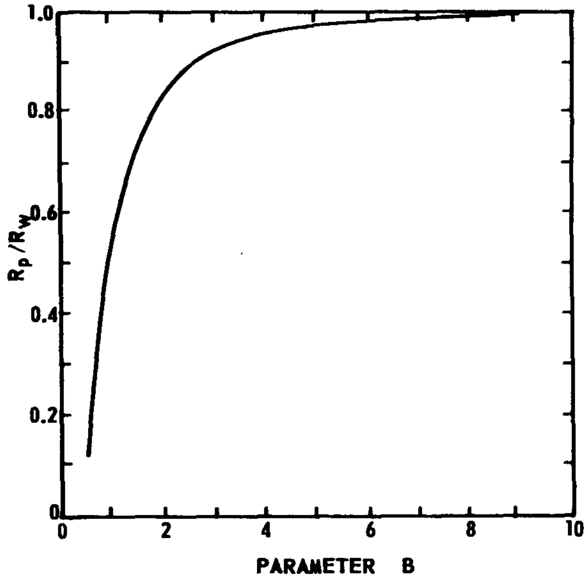

FIG. 3. The variation with B of the ratio of radius of maximum pressure gradient R_p and radius of maximum winds R_w.

pressure drop, the maximum wind speed is proportional to the square root of B, irrespective of the RMW. This is because increasing B alters the shape of the pressure profile to concentrate more of the pressure drop near the RMW. An illustration is given in Fig. 2 (note that the RMW is held constant and hence A also varies). As B increases from zero, the eye expands, the pressure drop becomes concentrated near the RMW (with a marked increase in the gradient), and the wind field adjusts to give stronger winds near the RMW and weaker winds at larger radii.

Note from Fig. 2a that the maximum pressure gradients are well separated from the RMW for small values of B. The two cannot coincide (because of the dependence of gradient or cyclostrophic winds on radius as well as pressure gradient), but such a large separation seems unrealistic and should give a lower limit to B. From (3) the radius of maximum pressure gradient (R_p) is

$$R_p = [AB/(B + 1)]^{1/B}, \tag{9}$$

and hence using (6)

$$R_p/R_w = [B/(B + 1)]^{1/B}. \tag{10}$$

The variation of R_p/R_w with B is shown in Fig. 3. An examination of a number of hurricanes for which reasonable estimates of R_p and R_w could be made (see, e.g., Myers, 1954; Graham and Hudson, 1960; Shea and Gray, 1973) indicated that these are generally nearly coincident and few, if any, hurricanes have R_p less than half R_w. Hence a lower limit of $B = 1$ seems reasonable.

As shown in Fig. 2b, increasing B increases the

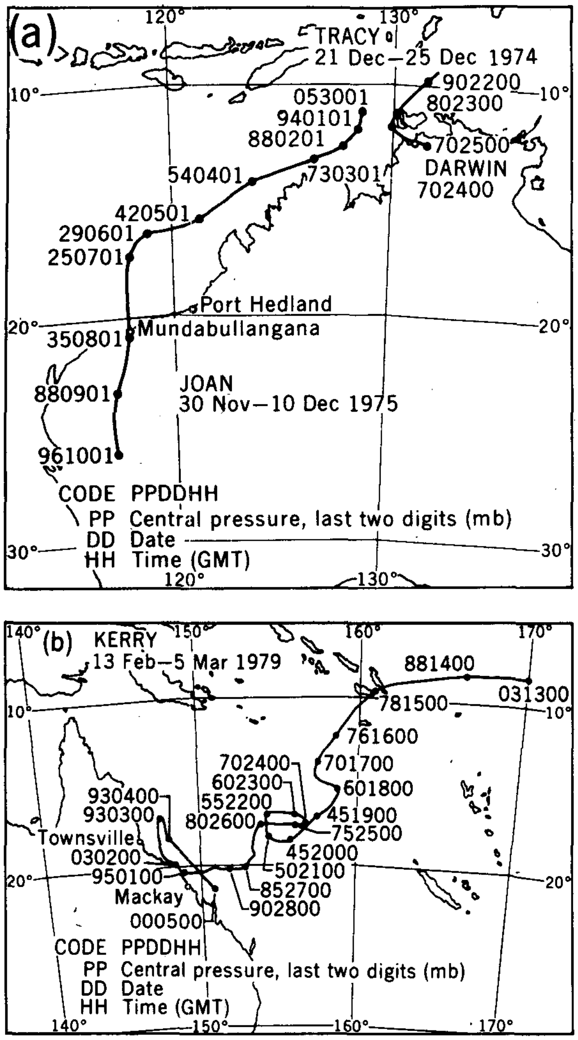

FIG. 4. Tracks of (a) Hurricanes Tracy and Joan and (b) Hurricane Kerry.

maximum winds and decreases the winds at larger radii. Low-level air flowing into the hurricane does not gain relative angular momentum and, in fact, loses cyclonic angular momentum by frictional dissipation to the surface. Hence, conservation of relative angular momentum, defined by Vr = constant, will place an upper bound on B. For the plausible ranges of central and ambient pressures and RMW's in hurricanes this upper bound is at $B = 3$. If a reduction in cyclonic angular momentum due to surface friction is incorporated, B will be lower and a realistic limit is probably around $B = 2.5$.

To summarize, the above reasoning indicates that B lies between 1 and 2.5. From (6) the observed RMW will then determine the possible values of A.

3. Applying the model

a. Direct application

No two hurricanes are exactly alike. From Fig. 1 it is obvious that, even when the variations in central and ambient pressures are removed, the pressure (and hence wind) profiles vary considerably. Hence the values of the parameters used to approximate these profiles will also vary. If there are sufficient observations within a hurricane, then it will be best to apply the model directly and optimally determine A and B. This is illustrated by application to three Australian hurricanes—Tracy, Joan and Kerry (Fig. 4).

Tracy was a small but intense hurricane which devastated Darwin over the period 24–25 December 1974 (Director of Meteorology, 1977). Using a central pressure of 950 mb, an ambient pressure of 1004 mb and a RMW of 7 km, an optimum fit, by minimizing errors at 2.5 km intervals, to the observed pressure profile was obtained from (3) with $A = 23$ and $B = 1.5$ (Fig. 5a). The derived wind profile from (4), shown in Fig. 5b, gives a good approximation to the gust envelope, except near the RMW where the smoothed pressure profile could not resolve the phenomenal peak pressure gradients. Neal (personal

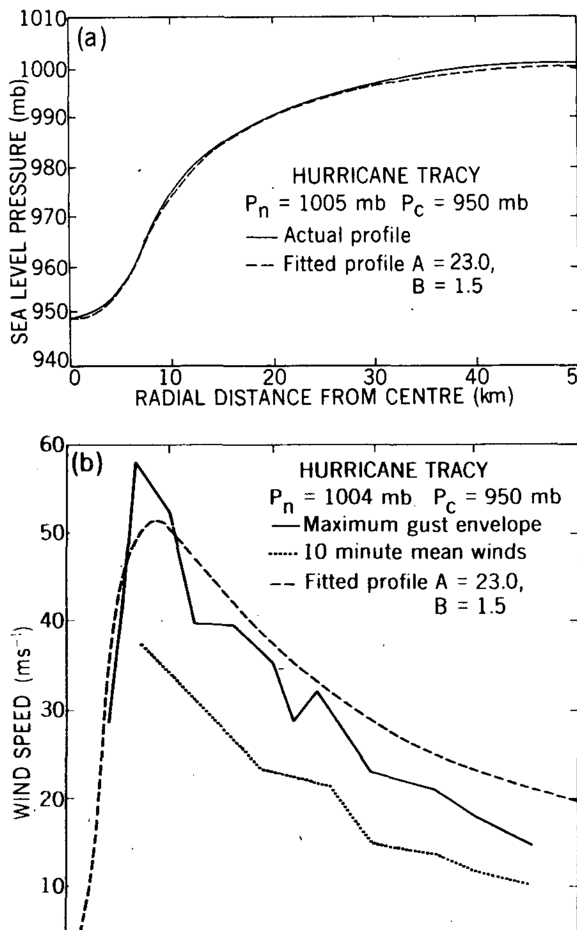

FIG. 5. Application of the model to Hurricane Tracy, 24–25 December 1974: (a) sea level pressure profiles, (b) wind profiles.

communication, 1976) estimated that pressure gradients of 5.5 mb km^{-1} were sustained over 2 km at the RMW, whereas the model equation could only resolve 4.2 mb km^{-1}.

Hurricane Joan crossed the coast of northwest Australia 50 km west-southwest of Port Hedland on 8 December 1975 (Director of Meteorology, 1979). Using an estimated central pressure of 930 mb and an ambient pressure of 1004 mb, the best fit to the Mundabullangana pressure profile was obtained from (3) with $A = 49.5$ and $B = 1.05$ (Fig. 6a). (This assumes that there was only a 5 mb filling of the cyclone as it moved overland to Mundabullangana.) No direct observation of a RMW was made, but the calculated value of 40 km is compatible with a radar eye radius of ~35 km. However, a visual extrapolation of the observed winds (Fig. 6b) to the RMW indicates that the calculated winds may have been too low there.

Kerry was the first hurricane in the Southern Hemisphere to be reconnoitred by a fully instrumented research aircraft—a WP-3D Orion from the Research Facilities Center of the U.S. National

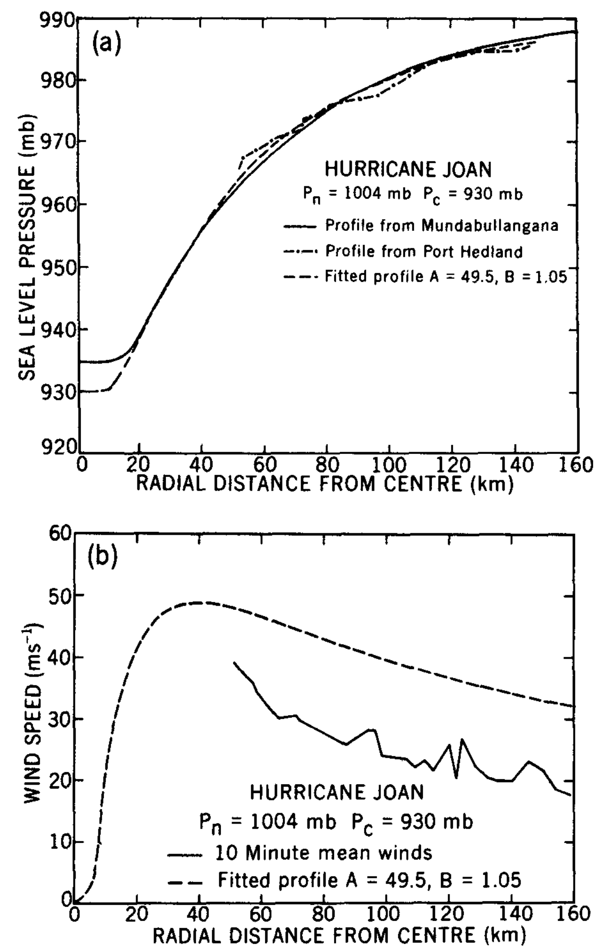

FIG. 6. Application of the model to Hurricane Joan, 8 December 1975: (a) sea level pressure profiles, (b) wind profiles.

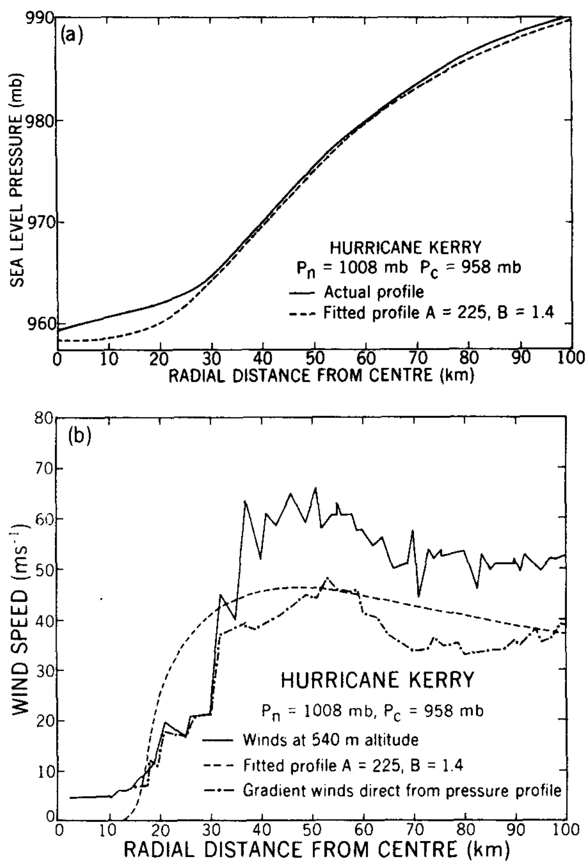

FIG. 7. Application of the model to Hurricane Kerry, 22 February 1979: (a) sea level pressure profiles, (b) wind profiles.

Oceanic and Atmospheric Administration. The wind profile was observed by the aircraft flying at a mean height of 540 m along a southwest leg out of the nearly stationary hurricane on 22 February 1979. The pressure profile was derived from the radar altitude, the density altitude, and the ambient air temperature assuming a constant lapse rate to the surface. Using a central pressure of 958 mb and an ambient pressure of 1005 mb, Eq. (3) with $A = 225$ and $B = 1.4$ gave a good approximation to the observed pressure profile (Fig. 7a). However, the resulting wind profile was a marked underestimate of the observed winds (Fig. 7b), with a calculated maximum wind of 20 m s^{-1} less than the observed. This was caused by a large supergradient component in the observed wind field. As shown in Fig. 7b, calculating the gradient winds directly from the observed pressures gave a profile similar to the model. Supergradient winds can only be accommodated if the model is applied directly to the wind field.

To summarize, pressure observations are generally more conservative and less error prone than wind observations in the hurricane circulation; hence fitting the model to pressure observations and then deriving the wind profile might be preferred to the direct use of wind observations. However, this

TABLE 1. A comparison of maximum wind speeds at various central pressures for northwest Pacific tropical cyclones. D is from Dvorak (1975), AH from Atkinson and Holliday (1975), and S from Schloemer (1954).

Central pressure (mb)	Maximum wind speed (m s^{-1}) D (1-min surface wind)	AH (1-min surface wind)	S (gradient wind)
981	33	30	30
973	40	35	34
964	46	41	38
954	53	46	42
942	59	52	47
929	69	58	51
915	72	65	55
900	80	71	59
884	88	78	63

approach will often underestimate the peak winds, as supergradient winds and very sharp pressure gradients will not be resolved. Hence, if there are good quality wind observations, these should be used directly. In any case the *highest* value of parameter B (compatible with observations) should be used. Note that all the above applications used observations within a single quadrant of the hurricane. If observations are composited from all quadrants then the asymmetry in the wind field (Shea and Gray, 1973), due to hurricane movement and other factors, must be accounted for.

b. Climatological applications

The above discussion indicates that the model can be applied realistically to individual hurricanes by varying the parameters to obtain an optimum fit to observed pressures or winds. However, in many cases observations will not be available or they may be few in number. There is also a use for a standard hurricane profile in engineering applications and storm surge modeling. In such cases climatological values of B may be obtained from hurricane observations in the region of interest.

Climatological values of B can be obtained by applying (7) and (8) to known relations for maximum winds in hurricanes. Atkinson and Holliday (1977) carefully examined many wind observations from hurricanes in the northwest Pacific and selected only those from well-exposed coastal or island stations and for which central pressures were known. They also empirically determined the exponent in (7) to get

$$V_m = 3.44(1010 - p_c)^{0.644}. \tag{11}$$

The resulting maximum winds for a selection of central pressures are given in Table 1 together with those from Dvorak (1975) for intensifying and steady-state hurricanes. The values of B resulting from applying Eqs. (7) and (8) to the data in Table 1 are shown in Fig. 8 (an ambient pressure of 1010 mb is assumed for Dvorak).

The Atkinson and Holliday, and Dvorak relations consistently give different estimates of B over the whole range of central pressures. Atkinson and Holliday derived their 1 min winds by applying corrections to observed peak gusts in hurricanes. The validity of these has been questioned by Spillane (private communication, 1976), who noted that the corrections were based largely on the results of Sissenwine *et al.* (1973) which are not applicable to the turbulence spectrum associated with hurricanes over the ocean. Perhaps this caused a consistent bias, but Dvorak's relation may also be biased.

Unfortunately, then, no precise climatological values of B can be given, unless one of the above relations is accepted as accurate. However, the inference may be made that B lies between 1.5 and 2.5; this is consistent with the theoretical limits of 1 and 2.5 described earlier.

There is a consistent increase in B with decreasing central pressure for both the Dvorak and Atkinson and Holliday relations. This means that as the central pressure decreases the cyclone wind field becomes proportionally more "peaked" and the destructive winds are confined to a smaller area around the center (Fig. 2b). Hence, either the greater pressure drop between the ambient and the central pressures is to a large extent accommodated by an increased pressure gradient near the RMW, or there

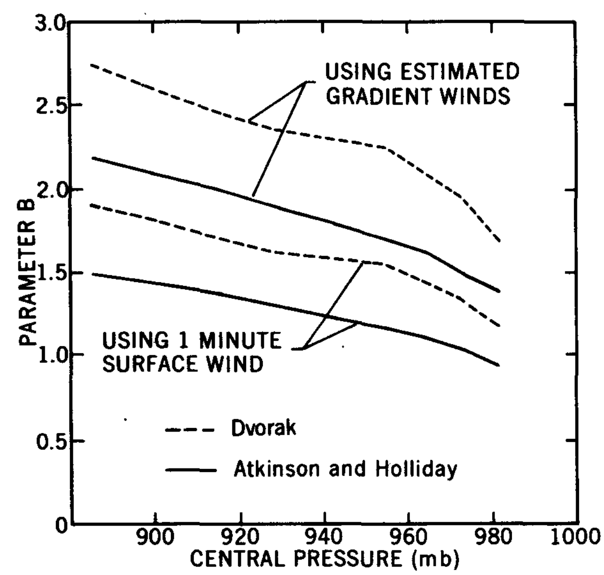

FIG. 8. Climatological values of parameter B determined from applying Eqs. (8) and (9) to the Dvorak (1975) and Atkinson and Holliday (1975) estimates of maximum winds. The lower curves are from a direct application to the 1 min surface winds; the upper curves use an estimated gradient level wind speed of 20% higher.

is an increased component of supergradient winds at the RMW as the central pressure decreases. Perhaps it is a combination of these two effects.

This correlation between B and central pressure may seem to negate an earlier statement that the RMW [see Eq. (6)] is independent of central pressure. This is not so. Gray (personal communication, 1980) has shown from many observations that the RMW is not or is only very weakly correlated to central pressure. Hence A in (6) must also vary with central pressure such that the A and B variations cancel.

4. A comparison with other relations

a. The Schloemer relation

Schloemer (1954) proposed (3) and (4) with $A = R_w$ and $B = 1$ as a universal relation. This has since been used in engineering and storm surge modeling by Myers (1954), Graham and Hudson (1960) and Marinas and Woodward (1968), and in a slightly different form by Das (1972) and Coastal Engineering Research Centre (1973). It was applied to typhoons around Taiwan and recommended to forecasters by Wang (1978).

However, the analysis of Hurricanes Tracy and Kerry and the climatological results shown in Fig. 8 and Table 1 indicate that setting $B = 1$ will markedly underestimate the maximum wind speeds of most hurricanes. This is because the model is confined to a generally incorrect profile shape. And no account can be made for the regular change in profile shape with central pressure, nor for the differences between hurricanes of similar intensity. If sufficient observations are available in the near vicinity of the hurricane, then using (3) with an optimally determined B will be superior to using a predetermined profile shape. This is illustrated in Table 2, which shows the pressure errors resulting from using Schloemer's model and fitting (3) to the nine hurricanes shown in Fig. 1. The advantage of using (3) is obvious.

TABLE 2. Mean absolute errors and maximum errors resulting from applying Schloemer's (1954) model and Eq. (3) to the nine hurricanes in Fig. 1.

	Schloemer		Eq. (3)	
Radius (km)	Mean absolute errors (mb)	Maximum error (mb)	Mean absolute errors (mb)	Maximum error (mb)
5	3	13	3	11
10	5	19	3	6
20	5	23	2	8
30	6	26	1	5
40	7	28	1	3
60	8	26	1	3
100	7	18	1	2
150	6	13	1	4
200	6	13	1	3

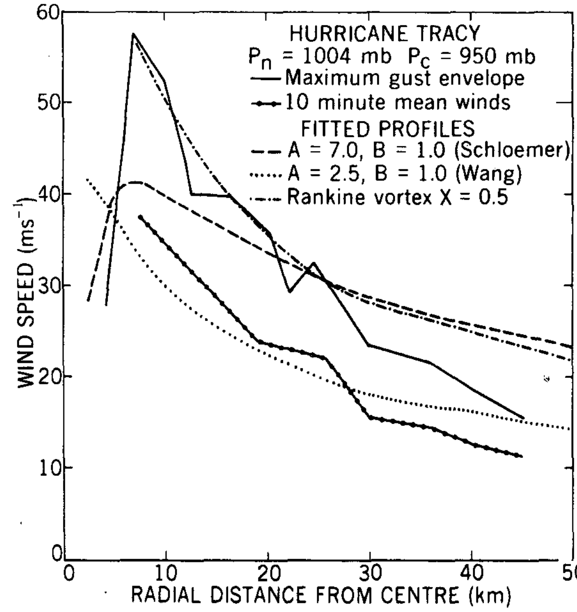

FIG. 9. Application of the models of Schloemer (1954), Wang (1978) and the modified Rankine vortex to the wind profiles of Hurricane Tracy.

An illustration of the application of Schloemer's model to Hurricane Tracy is shown in Fig. 9. Also shown is the profile resulting from using Wang's (1978) estimate of the RMW as being one-twentieth the radius of gale force winds (gale force winds extended to 50 km around Tracy). Both profiles are substantially in error.

b. The modified Rankine vortex

Depperman (1947) proposed that a hurricane could be approximated by a Rankine vortex, where

$$Vr^{-1} = \text{constant} \tag{12}$$

inside the RMW, and

$$Vr = \text{constant} \tag{13}$$

outside. That is, the hurricane is in solid body rotation inside the RMW and conserves relative angular momentum outside. However, the air spiraling inward in the boundary layer loses cyclonic relative angular momentum by frictional dissipation at the surface and (13) is modified to

$$Vr^X = D, \tag{14}$$

where $X < 1$. X and D are determined empirically from wind observations in hurricanes and X generally lies between 0.4 and 0.6 (Hughes, 1952; Riehl, 1954, 1963; Gray and Shea, 1973).

As illustrated in Fig. 9, this approach can give a good approximation to the wind profile in a hurricane. However, it requires a very accurate estimate of the RMW; even small errors in the RMW will result in large errors in estimating the maximum winds. By comparison, applying (4) or (5) directly to wind observations will give an equally good approximation to the wind profile without the need for an accurate estimate of the RMW. In fact, the RMW may be calculated directly from (6).

6. Summary and conclusions

An analytic model for the radial profiles of sea level pressure and winds in a hurricane has been presented as a generalization of the relation proposed by Schloemer (1954). The model has been applied to three Australian and nine Florida hurricanes and accurately reproduces the profiles for these hurricanes with two qualifications: if it is applied to pressure observations, then very strong pressure gradients over small distances may not be resolved, and supergradient winds cannot be accommodated. This will cause an underestimate of the maximum winds. Applying the model to wind observations will alleviate this problem, but only if the wind observations are reliable.

Physical reasoning indicates that the parameter B, which dictates the shape of the pressure or wind profile, should be between 1 and 2.5. This is supported by a climatological discussion, which constrains B to between 1.5 and 2.5. An important result of the climatologically derived profiles is that their shape varies consistently with decreasing central pressure. As the central pressure decreases, the wind profile becomes more "peaked" with proportionally higher winds at the RMW and a decreased extent of destructive winds. This may be explained by either an increased concentration of the pressure drop (from the ambient) at the RMW, by a stronger supergradient wind component, or a combination of these effects.

The model has been shown to be superior to two others—Schloemer's (1954) relation and the modified Rankine vortex. It provides more realistic profiles than Schloemer's relation which is constrained to a single profile shape of $B = 1$ and markedly underestimates the maximum winds and overestimates the radial extent of destructive winds in most hurricanes. Both the model and modified Rankine vortex give realistic profiles when applied direct to the wind field if the RMW is accurately known. If not, large errors may result in the winds from the modified Rankine vortex, but little if any difference will occur with the model; in fact, the RMW may be derived from it.

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