5 Summary and Conclusions

This concluding chapter begins by briefly summarising the main themes from the literature review, and the reasoning that led to the hypothesis that:

> *The low-level wind maxima observed in tropical cyclones are supergradient and steady-state, and are produced by inward advection of absolute angular momentum. The inflow at the height of the jet is maintained against gradient adjustment by a combination of advection and turbulent transport to be determined.*

Conclusions from the analytical and numerical modelling work are then brought together, and predictions from this part of the work compared with the results of the observational analyses.

5.1 The Hypothesis Revisited

Observational studies[1], summarised in Chapter 1, show that the maximum boundary-layer wind speeds in tropical cyclones are most often found on the right (left) hand side of the storm in the Northern (Southern) Hemisphere. There was some suggestion from these analyses that this maximum rotates with height through the right-forward quadrant to be nearer the front of the storm at the surface. The storm-relative radial flow generally consists of inflow in the right-rear quadrant and outflow at the left-front, or in stationary coordinates inflow from the right-front and outflow to the left-rear. Thus the inflow angle for the earth-relative flow is a maximum in the right-rear quadrant of the storm, and a minimum (or even outflow) to the left-front. The horizontal convergence at the surface is typically largest ahead of the storm, near the RMW, while

[1]For brevity, references are not given in this brief recapitulation of the literature review. They may be found in sections 1.2 and 1.3.

a region of horizontal divergence may extend from the centre of the storm into the left-rear in the upper part of the boundary-layer. Although the above describes the usual situation, other distributions of wind are possible, for example in sheared storms.

Observed vertical profiles show that a boundary-layer wind speed maximum is a common feature, with the reported height varying between 60 m and 1.5 km. Where several profiles at different radii in the one storm are available, they show that the height of the maximum tends to decrease toward the storm centre. The ratio of wind speed aloft to that at 10 m was found to vary from about 0.55 to, or even slightly exceeding, 1. Evidence was discussed that this ratio varies with stability, with the lowest (highest) values being observed when the sea was cooler (warmer) than the air.

One multilevel study had sufficient data to analyse both the symmetric and asymmetric flow components at several heights within the boundary layer. This study showed that the depth of the symmetric component decreased rapidly towards the centre of the storm, while the asymmetric decays more slowly with height.

The simplest theoretical model of the atmospheric boundary layer which includes the effects of the earth’s rotation is the Ekman spiral. Correct prediction of the near-surface wind speed and cross-isobar angle requires that the surface stress be correctly modelled. This may be achieved by either the use of a semi-slip boundary condition, or a two-layer model in which the turbulent diffusivity is constant in the upper layer but decreases toward the surface in the lower layer. The more usual use of a no-slip condition overestimates the stress, giving surface winds that are too weak and directed too sharply across the isobars. Regardless of surface boundary condition, the frictionally-forced updraft is proportional to the curl of the surface stress.

Modifications of the Ekman boundary-layer model to the case of tropical cyclone-like vortices produce markedly different predictions to the straight, geostrophic flow case. Firstly, the updraft depends strongly on the surface boundary condition, with a semi-slip condition correctly organising the maximum vertical motion near the RMW, while the no-slip condition gives a quite uniform updraft across the eye. Secondly, the boundary-layer depth decreases towards the centre of the storm, although this effect could be counteracted by an increase in the turbulent diffusivity. Several studies showed weakly supergradient winds in the upper boundary layer, although it is difficult to attach much credence to the early numerical models due to the very large values of horizontal diffusion used. Weak outflow was often found above this maximum.

A two-dimensional depth-averaged "slab" model of the boundary layer of a translating cyclone was used to predict that the strongest winds (which were supergradient) lay directly ahead of the storm, and somewhat inside of the gradient RMW. They were nearly co-located with the maximum frictionally forced updraft.

A one-dimensional profile model, which has been frequently used for estimating surface winds from aircraft measurements, was shown to produce lower values of the surface-aircraft wind speed ratio at high winds, and in stable conditions. The former variation arises in the model because of the increase in surface roughness of the ocean with wind speed.

It is clear from the above discussion that the tropical cyclone boundary layer has a complex three-dimensional structure. A rough calculation was presented, using the thermal wind equation, which suggested that the upper boundary-layer wind speed maximum may be supergradient. It was hypothesised that this could be produced by inwards advection of angular momentum. However, if the maximum is steady and supergradient, the radial momentum balance is important, and the question of how inflow is maintained in the presence of supergradient winds must also be addressed.

These considerations led to the central hypothesis, restated at the beginning of this chapter. A scale analysis showed that the radial and azimuthal advection of inflow in a moving storm are of similar magnitude near the RMW, so the motion-induced asymmetry may well play an important role in the jet structure. Moreover, as the three-dimensional structure of the asymmetry is not well known, determining it would be a major focus of the study.

5.2 Theory, Modelling and Observations

Two models were developed to explore the above ideas, a linear analytical model, and a high-resolution numerical model. The former used a constant drag coefficient and turbulent diffusivity, while the latter incorporated sophisticated parameterizations of the relevant physical processes. In each, the focus was on diagnosing the boundary-layer flow as the response to some known gradient-level forcing. Thus the influence that changes in the boundary-layer structure may have on the cyclone as a whole was ignored. Although such feedbacks clearly exist, the intent was rather to explore just one side of what is undoubtedly a two-way interaction. Similarly, there was no attempt to resolve the effects of convection on the boundary layer, concentrating instead on the larger scales. Transient features that may arise due to instabilities in the prescribed vortex flow were also excluded. Again, while these are known to be important in the real atmosphere, the concern here was with determining the steady, frictionally forced flow beneath an idealised, translating tropical cyclone.

The solution to the analytical model bears some resemblance to the Ekman boundary-layer model. However, it has three component: a symmetric one due to the cyclone, and two asymmetric ones resulting from the interaction of the moving cyclone with the underlying surface. Each has a different depth scale, which vary from that of the classical Ekman solution. There is also an asymmetry between the radial and azimuthal components of the flow not present in the classic solution, which makes the radial velocity relatively stronger than the azimuthal, in all three components. The symmetric component is an improvement of the symmetric vortex models of Rosenthal (1962) and Eliassen and Lystad (1977), while the asymmetric solution is believed to be new.

It was shown that strong inwards advection of absolute angular momentum was necessary to produce the jet. In the linear model, the required inflow was maintained against gradient adjustment by vertical diffusion, and the wind maximum was found to be a few percent supergradient in a stationary cyclone. It was argued that the forcing due to vertical advection should be of similar magnitude, and further strengthen the supergradient jet. This was confirmed using an extension of the linear model with a crude representation of the vertical advection, in which the vertical velocity was prescribed and assumed to be constant with height. It was speculated that the outer side of a rain-band may be a preferred location for jet formation, since here there is stronger inflow and angular momentum gradient, and a stronger updraft.

The numerical model was used to extend this linear analysis to include the full nonlinear terms (including the vertical advection), along with more realistic representations of the surface fluxes and turbulent diffusion. It confirmed that a supergradient jet could be produced in an axisymmetric storm by strong inwards advection of angular momentum, with the inflow maintained against gradient adjustment by upwards diffusion and advection. The main conclusion from this comparison of models is that including the nonlinear terms and particularly the vertical advection of radial wind provided enhanced inflow forcing and allows a jet that is several times more supergradient than in the linear model. In particular, the wind maxima were found here to be between 10% and 25% supergradient near the RMW of a stationary cyclone, with the jet being more supergradient in a more intense system and in a storm with a peaked radial wind profile, but less supergradient in the outer part of the storm.

All the storms analysed as case-studies possessed significant asymmetries, due to either motion, proximity to land, or both. However, the analysis showed that the azimuthal mean is identical to a stationary storm in the linear case, and very similar in the numerical model. Thus analysing azimuthal-mean gradient balance is a suitable test for the above statements. Such an analysis was carried out using dropsonde data in Hurricanes Georges and Mitch, in two ways. First, a pressure profile was fit to the pressure observations, and the gradient wind calculated from the fitted pressures compared to the storm-relative azimuthal wind component. Secondly, a wind profile was fit to the storm-relative azimuthal wind observations, and the gradient wind equation radially integrated to give a gradient pressure profile, which was compared to the pressure observations. The two approaches produced results which were consistent with each other. Hurricane Georges showed no evidence of supergradient flow near the eyewall. This contradicted the theory, and it was speculated may have been a consequence of Georges being in the early stages of an eyewall replacement cycle. Hurricane Mitch, in contrast to Georges, had azimuthal-mean supergradient flow from 400 m to 2 km height. At its peak, near 800 m, the wind was about 10 m s^{-1}, or 15%, supergradient. A Monte-Carlo technique was used to derive confidence intervals on the fitted curves, and show that the differences were statistically significant to at least the 95% level.

These analyses required accurate knowledge of the cyclone track at all levels, and a technique was developed to objectively derive this from dropsonde pressure observations. It was also found that the inward displacement of the dropsonde as it fell produced a systematic bias in the hydrostatic integration which produced pressure-height data, which led to incorrect results in the gradient balance analysis. Techniques were developed to correct for this bias, and the hydrostatic integrations redone. Finally, it was shown in the case of Georges that small changes in the cyclone structure between the two observational periods led to an incorrect analysis, and an underestimate of the pressure gradient, if data from the two periods were combined.

In summary, the prediction of markedly supergradient flow in the upper boundary-layer near the eyewall is confirmed in one of the two cases for which it could be tested. Unfortunately, there was not sufficient data available to test the prediction of less marked azimuthal-mean supergradient flow at larger radii.

The jet height was predicted by the linear model to scale as $\delta_0 = (2K/I)^{1/2}$ and is thus defined by the turbulent diffusivity and inertial stability only. The effect of the inclusion of vertical advection with w constant with height was to modify the governing depth scale. In particular, the oscillation-depth scale modestly increased, while the decay-depth scale increased in an updraft and decreased in a downdraft. The scales were equal only when there was no vertical motion. It was argued that because the height scale δ_0 also defines the height at which the frictionally induced updraft becomes fully established, the introduction of a more realistic representation of the vertical advection to the linear model would not bring any new height scales, but rather modify those already applying. This was confirmed using the numerical model, which it was shown predicted a jet height in close agreement with that obtained from the linear model in the inertially stable case.

Quantitative comparisons of the jet height with observations were impractical, due to the difficulty of measuring the turbulent diffusivity. However, the qualitative result, that the jet height should decrease towards the centre of the cyclone, with the largest changes near the RMW, was confirmed by calculating mean winds over a series of concentric annuli for Hurricanes Georges and Mitch. The changes were particularly marked in Mitch, with the jet height being about 800 m in the 25 – 40 km annulus, 300 m in the 15 – 25 km annulus, and 200 m in the 0 – 15 km range. Outflow was also apparent above the maximum, in further agreement with the modelling work.

The spatial distribution of the jet in the axisymmetric storm was found from the modelling to depend upon the "peakiness" of the radial gradient level wind profile. A compact storm with a relatively rapid decrease in wind speed outside the radius of maximum wind tended to produce a strong jet confined to the immediate vicinity of the eyewall, while a more inertially stable radial profile resulted in a more widely distributed, but less intense, jet. The difference was explained in terms of the different angular momentum profiles of the two storms, and the consequently differing abilities of the two storms to generate significant horizontal advection of angular momentum.

The Ekman spiral, at least in its original form, is nowadays generally regarded as a fairly poor model of the atmospheric boundary layer, yet the linear model, which is closely related, was here proposed as being appropriate in tropical cyclones[2]. However, several of the factors which commonly disturb the classical Ekman spiral will apply to a much lesser degree in the tropical cyclone boundary layer, and so the model is not thereby invalidated. The first of these factors, the nonslip boundary condition, was here replaced by one of several possible semi-slip conditions.

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[2]There is good evidence that combining a stress-dominated layer near the surface, in which pressure gradient and Coriolis terms are negligible so a logarithmic layer is obtained, with an Ekman outer layer, can give quite a good representation of the steady neutral barotropic atmospheric boundary-layer (Hess and Garratt 2002, Garratt and Hess 2002). However this case is relatively rare.

Second is the role of buoyancy in generating turbulence. In the strongly sheared environment of the tropical cyclone boundary layer, turbulence would be expected to be dominantly shear-generated. This would lead to a relatively simple turbulent diffusivity structure, not subject to large diurnal variations. In the normal atmospheric boundary layer, the time scale $1/f$ for the establishment of an Ekman spiral is similar to the time over which diurnally induced variations in diffusivity occur. Hence it is perhaps hardly surprising that it is rarely observed over the land. Indeed, it is worth noting that Taylor (1915), in his comparison of aircraft data to an Ekman spiral (with a semi-slip boundary condition), restricted attention to the strong wind case for precisely the reason that there the turbulent diffusivity would be less affected by diurnal changes. In a tropical cyclone, on the other hand, significant diurnal changes in turbulent diffusivity do not occur, and the time scale $1/I$ for boundary-layer adjustment is much shorter, so the boundary-layer winds are much more likely to be in equilibrium with the diffusivity.

Another factor that can eliminate or even reverse the turning of the winds in the boundary layer is baroclinicity. This would be less important in a tropical cyclone, as the near-surface temperature gradients are weak (except near the eye) and tend to be aligned perpendicular to the flow. Near the eye, the thermal shear would normally be directed against the gradient wind, and this will tend to sharpen the maximum in the upper boundary layer. Moreover, the altered scaling which results in a markedly shallower boundary layer here than in the classical Ekman case, also reduces the extent to which temperature gradients can contribute to significant wind change across the boundary layer.

A final factor which, in contrast to the others, does apply in the tropical cyclone boundary layer, is the hydrodynamic instability of the Ekman spiral. For instance, the numerical studies of Faller and Kaylor (1966) and Lilly (1966), and the analytical work of Brown (1970, 1972a, 1972b), show that the classical Ekman spiral is unstable and breaks down into longitudinal rolls, aligned at approximately 14º to 17º to the geostrophic flow. Longitudinal rolls are well known to occur in the atmospheric boundary layer, and recently some evidence of their occurrence in the tropical cyclone boundary layer has appeared (Wurman and Winslow, 1998).

Is the jet, then, nothing more than the weakly supergradient flow found near the top of the Ekman boundary layer? The answer is essentially yes; albeit with the complication of three separate components in a moving storm, and the crucial role of vertical advection in strengthening the jet. The major role of upwards advection was shown using both the extension of the linear model and the numerical model, and causes the supergradient component to be several times stronger than in its absence. This seems to be peculiar to intense vortices and does not occur in more normally considered cases, because the rapid, almost step-like increase in inertial stability near the radius of maximum winds produces an updraft which is much stronger than would be expected from the classical Ekman theory, in which the updraft is proportional to the curl of the surface stress. The answer to the question is thus qualified, by adding that nonlinearities significantly modify the Ekman profiles, giving markedly stronger supergradient flow in the upper part of the spiral.

It was further shown that the distribution of vertical velocity outside the core region may not follow the predictions of the classical Ekman theory, as surface divergence may prevail even in the presence of cyclonic relative vorticity, provided the inertial stability is weak. Within the eye, the updraft is approximately proportional to radius, in agreement with the results of Eliassen (1971) and Eliassen and Lystad (1977).

The asymmetric components introduce a wave number one asymmetry to the vertical motion, which is superimposed on the updraft due to the symmetric component. The updraft is greatly strengthened in the right-forward (left-forward) quadrant, while weak subsidence occurs to the left-rear (right-rear), in the Northern (Southern) Hemisphere. This is consistent with observed convective asymmetries in the eyewall of a moving tropical cyclone.

For a moving storm, it was found using both models that the supergradient jet was generally located in the left-forward (right-forward) quadrant of the storm in the Northern (Southern) Hemisphere, away from the strongest earth-relative near-surface winds in the right-forward (left-forward) quadrant. The jet was substantially more supergradient than in the stationary case. In the linear model, the majority of the asymmetric flow was shown to be contained in the deeper of the two asymmetric components, with the shallower one being much weaker. These asymmetric components were interpreted as frictionally stalled inertia waves, where the decay and rotation depth-scales adjust so as to provide precisely enough retardation to bring the wave to a halt. The asymmetric part of the flow in the numerical model was found to decay more slowly with height than the symmetric, in agreement with the linear results.

Severe Tropical Cyclone Vance made landfall almost directly over a boundary-layer wind profiler, with nearby tower-mounted sonic anemometers and pressure measurements. Data were available until the storm was about 120 km away. The radial pressure profile was analysed, and used, together with the observed motion, to force the numerical model. The simulation predicted that, in the region in which data were available, the low-level jet was about 15% supergradient and at a height of 1.5 km. This was in excellent agreement with the observations. It was found that improved agreement between model and observations in the lower-level winds could be obtained by increasing the Charnock coefficient from 0.011 to 0.1, which was justified on the basis of the shallow water and short fetch. In particular, the near-surface winds were too strong and the inflow too weak, when the smaller value was used.

The jet asymmetry near the RMW was studied in Hurricanes Georges. The jet was most marked and lowest to the left-rear of the storm, and nearly absent to the right, in good qualitative agreement with the theory. A quantitative comparison was made by forcing the numerical model with the fitted pressure profile from the gradient balance analysis, and the observed motion. The agreement between model and observations for both the radial and azimuthal wind components was remarkably strong. While there was some significant differences, they are less than might have been expected given the strong turbulence, convective nature of the eyewall, and large radial gradients.

The wind asymmetry in Hurricane Mitch was located almost 180^{o} in azimuth out of phase from where motion would have placed it. However, the asymmetry had the same structure as the frictionally stalled inertia wave: radial and azimuthal components were in quadrature with maximum inflow upstream of the strongest winds, and it rotated anticyclonically with increasing height. It was suggested that the frictional asymmetry forcing this was supplied not by motion, but was rather due to the proximity to land. Simulation of Mitch with the numerical model modified to include a region of rough land in the surface boundary condition showed good agreement between model and observations in both wind components. Moreover, the frictionally forced updraft in the model had a maximum near the upstream end of the eyewall convection asymmetry apparent on the radar imagery, so it appears that the surface frictional asymmetry was also organising the eyewall convection.

The surface wind factor (SWF), or ratio of the surface wind speed to the gradient wind speed, is a widely used parameter. In a stationary storm, this was shown by both models to increase from approximately 0.7 at large radii, to 0.9 or more at and inside of the RMW. A similar trend was found in the observational analysis by Mitsuta et al. (1988). For a moving storm, there is additionally a left-to-right gradient in both models, with higher values on the left (right) side of the storm in the Northern (Southern) Hemisphere; that is, the side with the weaker surface winds. The use of a universal constant for surface wind reduction is thus not supported by either model. Equally, the results invalidate the use of one-dimensional profile models, since the strong contribution of horizontal advection to the momentum budgets means that the assumption of one-dimensionality in profile models is incorrect. In particular, the prediction of one such model, that the SWF should decrease at high wind speeds due to the increasing roughness of the sea surface, is directly contradicted by the result here that the SWF increases towards the RMW of the storm. Observational studies have found wide variation in the surface wind factor, from approximately 0.55 to 1. Powell and Black (1990) have shown that differences in the static stability can explain some of this observed variability. However, this effect is not present here and so these dynamical factors seem to be a further significant contributor. It was further shown that caution may be necessary in choosing a level for comparison in calculating these from observed winds, as the asymmetric component can still be large as high as 2 km above the surface in the nonlinear model.

Objective analyses of the SWF in Hurricane Georges showed complete agreement with the above pattern. The increase towards the centre, the left-right asymmetry and the typical magnitudes in the analyses were all in excellent agreement with the modelling. Similar analyses in Hurricane Mitch displayed the increase towards the RMW, but had a wave-number two asymmetry. Analysis of previously published data in Hurricanes Hugo and Andrew confirmed the increase to the centre, and the left-right asymmetry. However, some caution is needed in the interpretation of these latter cases since landfall would also have contributed to the observed asymmetry.

Previous approaches at modelling the tropical cyclone boundary-layer can be conveniently organised into a hierarchy. The simplest of all is the use of an empirical constant SWF to estimate the near-surface wind from a gradient or aircraft-measured wind. One-dimensional column models, such as the DMRP model reviewed in Chapter 1, can include stability and surface-roughness effects, but implicitly assume horizontal homogeneity. A second class of one-dimensional models are depth-averaged and of an axisymmetric storm (e.g. Smith 2002), and do not resolve the vertical structure or motion asymmetry. The next level in complexity is provided by the momentum integral model of Smith (1968) and its successors (Leslie and Smith 1970, Bode and Smith 1975), which assume an Ekman-like vertical profile, but derive some of the governing parameters. Fully two-dimensional models fall into two categories. Most assume an axisymmetric storm and resolve the radius-height structure, with examples including Rosenthal (1962), Anthes (1971), Kuo (1971, 1982), Eliassen and Lystad (1977) and Montgomery et al. (2001). The alternative two-dimensional approach, of modelling the depth-averaged horizontal structure and hence obtaining the motion-induced asymmetry, but not the vertical structure, was taken by Shapiro (1983). The models described in this thesis thus can be regarded as sitting at the head of this hierarchy, being (to my knowledge) the only three-dimensional idealised models of the flow in the boundary-layer of a translating tropical cyclone. While the three-dimensional structure of the tropical cyclone boundary-layer has been modelled previously, this has been at relatively coarse resolution as the lower part of the domain of models of the entire tropical cyclone, and the physical processes have not been analysed in the detail that the models described here have allowed.

5.3 Closing Remarks

Tropical cyclones represent risks to people and property on several fronts. Of the most significant hazards, damage from wind, storm surge and ocean waves directly involve the boundary-layer, and only fresh-water flooding is less related. Thus improved understanding of the boundary-layer can be expected to lead to improved management of the risk. In principal, this should apply both in the short term, through better forecasts and warnings, and over the longer term, through improved risk estimation, engineering design and forward planning.

The formulation of the models presented here is conceptually quite simple, and the linear model in particular is based on a very abbreviated subset of the physics applying in a real tropical cyclone. Yet the solutions to the models displayed a relatively rich variety of behaviour, and an interesting range of sensitivities to various aspects of the storm structure. Of these, the existence of supergradient flow in the upper boundary-layer, and the spatial variability in the surface wind factor, would seem to be particularly important to improved risk management.

Until recently, the boundary-layer has arguably been the least well-observed part of the tropical cyclone. While aircraft observations have made an enormous contribution to our knowledge in the rest of the storm, safety considerations severely limit their availability too close to the surface. Remote sensing instruments such as Doppler radars can be adversely affected by sea clutter, and the inherent volume-averaging makes interpretation less easy in the relatively strong vertical gradients. The recent advent of new technology, including portable radars and wind profiles, but most particularly the GPS dropsonde, promises to lead to a revolution in our understanding of the tropical cyclone boundary-layer. The analyses of Hurricanes Georges and Mitch in this thesis showed a highly encouraging level of agreement with the analytical and numerical modelling work. However, these analyses have not exploited the full potential of this new data, and it can be expected that much will also be learnt about the boundary-layer thermodynamics, turbulence and air-sea exchanges from them.

These closing remarks began with a relatively narrow focus, on the importance of the boundary-layer winds to tropical cyclone risk, and the potential for improvements to risk management from the results in this thesis. Moreover, the focus of this thesis has been that the boundary-layer flow is the frictional response to a forcing supplied by the rest of the cyclone. It is appropriate to finish with a much broader view, and recall that the warm, moist air which fuels the cyclone, passes through the boundary-layer. Cyclone intensity is known to be related to boundary-layer equivalent potential temperature, but the processes that determine the latter are not fully understood. Likewise, questions remain over the magnitude of the momentum transfer to the ocean, and little is known about the fine-scale structure of the boundary-layer winds.

The advent of the high-quality data from the GPS dropsonde encouraged the modelling work in this thesis, by showing that the few previous observations of low-level jets actually represented the norm. Improvements in models rely on the availability of data for verification and for clues as to which processes are important, while models can in turn suggest future observational strategies and indicate shortcomings in the data. The availability of boundary-layer data has led to improvements in our ability to model the boundary-layer of tropical cyclones, but also indicated areas where the models need further development and testing. Further progress will require that data acquisition and modelling development go hand-in-hand.