Wind Energy Input into Ekman Motions and Vertical Viscosity: A Spectral View

Shane Elipot   Sarah Gille   selipot@ucsd.edu   sgille@ucsd.edu
Scripps Institution of Oceanography, University of California, San Diego, La Jolla, CA

1. What's in this poster?

- Drifter trajectories, altimeter-derived geostrophic velocities and wind stress reanalyses are used to infer the rate at which wind energy is input into Ekman motions, with an emphasis on the Southern Ocean (box 2).
- The real part of the cross-spectrum between drifter ageostrophic velocities $\mathbf{u}_{\text{ag}} = \mathbf{u}_{\text{drifter}} - \mathbf{u}_{\text{geostrophic}}$ and wind stresses $\boldsymbol{\tau}$ is interpreted as a measure of the wind energy input through the Ekman momentum balance (box 3).
- The wind energy input into Ekman motions is a function of frequency and geographical location (box 4).
- The transfer function is used to estimate vertical viscosity κ in the Ekman layer (box 5).
- After the Southern Ocean, we are working towards a global estimate of this energy input (box 6).

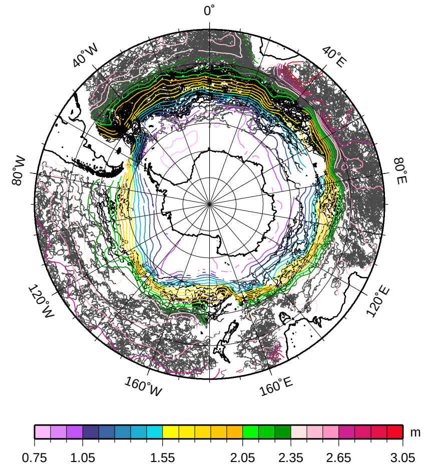
2. Which data?

Along 40-day segments with 20-day overlap of Global Drifter Program trajectories we consider:

$\mathbf{u}_{\text{drifter}}$ at 6-hour intervals.

$\mathbf{u}_{\text{geostrophic}}$ derived from AVISO gridded sea level anomalies at 7-day intervals augmented by GRACE dynamic topography.

$\boldsymbol{\tau}$ from ECMWF ERA-40 reanalyses wind stresses at 6-hour intervals.



Drifter trajectory segments used for estimates in the Southern Ocean and dynamic height contours from Gouretski and Jancke (1998). Segments in the Antarctic Circumpolar Current are drawn with a darker shade.

3. An interpretation of the cross-spectrum

The linearized horizontal momentum balance,

$$\frac{\partial \mathbf{u}(t,z)}{\partial t} + if\mathbf{u}(t,z) = \frac{1}{\rho}\frac{\partial \boldsymbol{\tau}(t,z)}{\partial z} = \frac{\partial}{\partial z}\left(\kappa(z)\frac{\partial \mathbf{u}}{\partial z}\right), \qquad \frac{\boldsymbol{\tau}(z)}{\rho} = \kappa(z)\frac{\partial \mathbf{u}(t,z)}{\partial z}$$

$$,\times \int_{-\infty}^{+\infty} e^{-i2\pi\nu t}dt, \times \mathbf{U}^*, \times \int_{-\infty}^{0} dz, \langle\cdot\rangle/\mathcal{T},$$

gives a spectral energy equation:

$$i(2\pi\nu + f)E + D = S_{\mathbf{u}\boldsymbol{\tau}} = C_{\mathbf{u}\boldsymbol{\tau}} - i\,Q_{\mathbf{u}\boldsymbol{\tau}}$$

where

$E = \rho\int_{-\infty}^{0}\frac{\left\langle|\mathbf{U}|^2\right\rangle}{\mathcal{T}}dz$ is the Ekman power

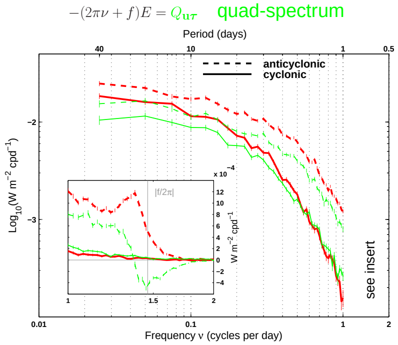
$D = \int_{-\infty}^{0}\rho\kappa\frac{\left\langle\left|\frac{\partial \mathbf{U}}{\partial z}\right|^2\right\rangle}{\mathcal{T}}dz$ is the dissipated power

$S_{\mathbf{u}\boldsymbol{\tau}} = \left\langle \mathbf{T}_k\,\mathbf{U}^*\Big|_{z=0}\right\rangle/\mathcal{T}$ is the cross-spectrum

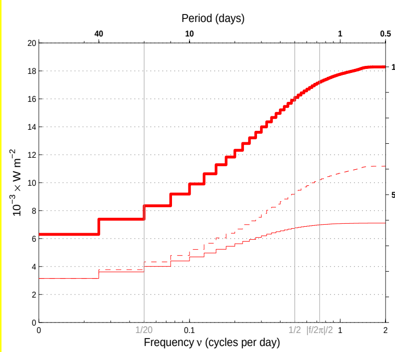
In terms of the real and imaginary parts

$D = C_{\mathbf{u}\boldsymbol{\tau}}$ co-spectrum

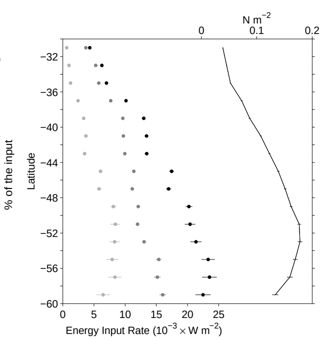
$-(2\pi\nu + f)E = Q_{\mathbf{u}\boldsymbol{\tau}}$ quad-spectrum



4. A frequency dependent energy input



Cumulative integration of the co-spectra in the Antarctic Circumpolar Current for anticyclonic frequencies (dashed lines), cyclonic frequencies (thin solid line) and the sum of anticyclonic and cyclonic frequencies (solid heavy line). 64% of the total wind energy input rate is due to time-varying motions; of these, 58% is due to anticyclonic motions.



Energy input rate across the Southern Ocean: contribution from the mean (light gray dots), contribution from the time-varying terms (gray dots) and total (black dots). Upper axis: mean of the wind stresses interpolated on the drifter positions.

5. Vertical viscosity estimates

The transfer function $\mathbf{H}$, a function of frequency ν and depth z, is the ratio of the cross-spectrum and the auto-spectrum of the wind stress. It was obtained from data and fitted to theoretical models to estimate the vertical viscosity.

The transfer function for a homogeneous ocean of infinite depth with constant eddy viscosity is ($z < 0$):

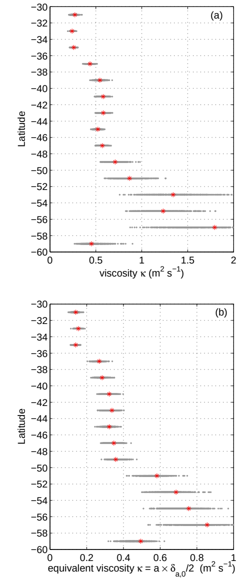
$$\mathbf{H}_1(\nu,z) = \frac{1}{\rho\kappa}\frac{e^{(1+i)z/\delta_\nu}}{\sqrt{i\left(\frac{2\pi\nu+f}{\kappa}\right)}},$$

$$\delta_\nu = \sqrt{\frac{2\kappa}{2\pi\nu+f}},$$

whereas when the eddy viscosity increases linearly with depth as $\kappa = az$ ($z > 0$), the transfer function for an homogeneous ocean of infinite depth is

$$\mathbf{H}_2(\nu,z) = \frac{2}{\rho a}K_0\left(2\sqrt{\frac{iz}{\delta_a}}\right),$$

$$\delta_a = \frac{a}{2\pi\nu+f}.$$



(a) κ estimates across the Southern Ocean by fitting to $\mathbf{H}_1$. Gray dots indicate all values found by bootstrapping. (b) depth-equivalent viscosity $\kappa = a \times \delta_{a,0}/2$ by fitting to $\mathbf{H}_2$.

6. Towards a global estimate

The global estimates of wind energy input rate give a global integral of 2.97±0.41 TW, comparable with the theoretical estimate of about 2.4 TW by Wang and Huang (2004). This is to be compared to the energy input rate into inertial motions (0.5-0.7 TW, Alford (2003), Watanabe and Hibiya (2002)) and to the wind energy input into geostrophic motion (about 0.8 TW, Wunsch (1998)).