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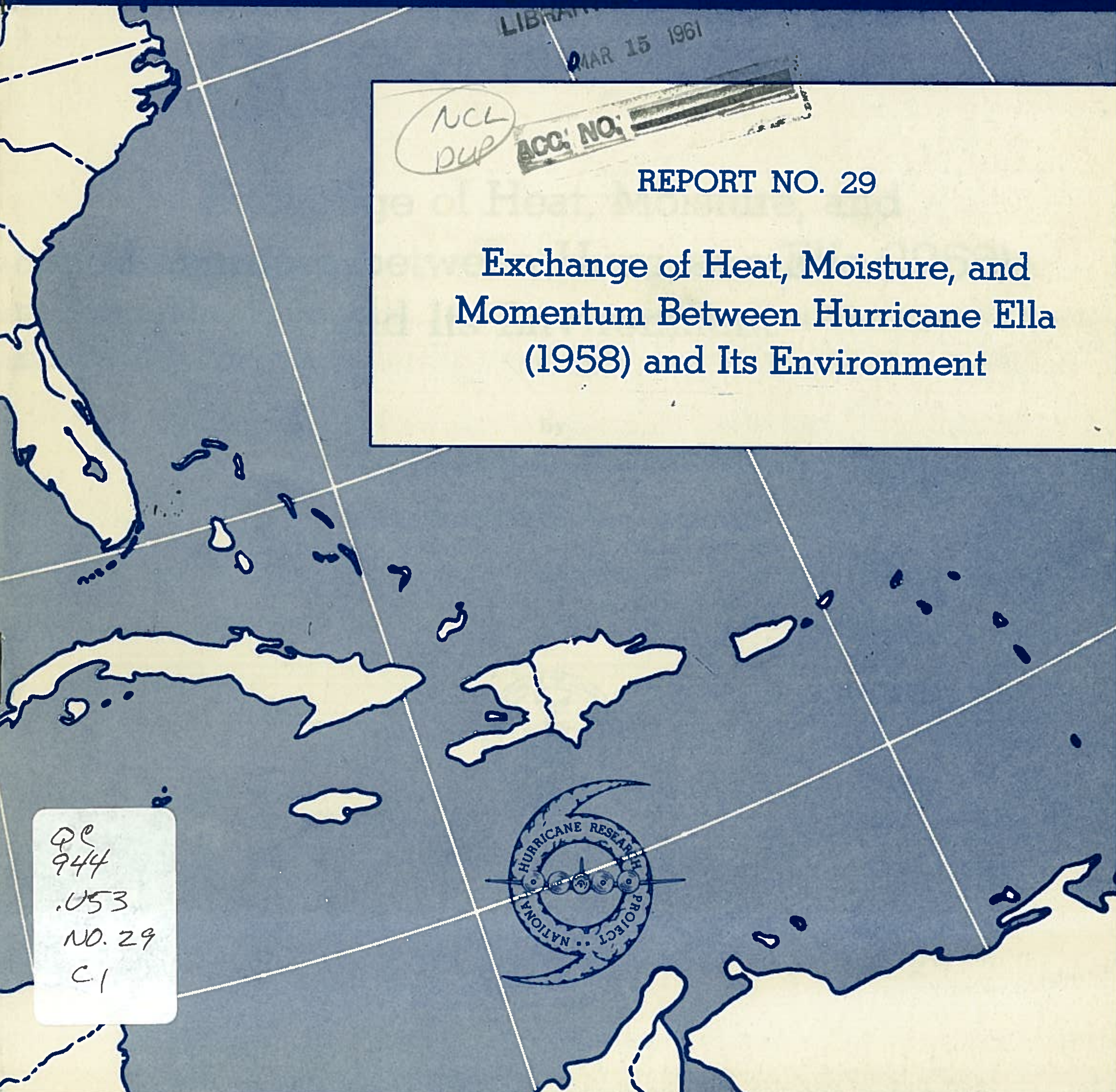

REPORT NO. 29

Exchange of Heat, Moisture, and Momentum Between Hurricane Ella (1958) and Its Environment

CONTENTS

EXCHANGE OF HEAT, MOISTURE, AND MOMENTUM BETWEEN HURRICANE ELLA (1958) AND ITS ENVIRONMENT

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ABSTRACT

On September 2, 1958, a tropical storm moved near the center of the network of upper-air observations in the Atlantic and Caribbean southeast of Florida. This made possible determination of the structure of the storm at a radius of 4.5° latitude from the center through the entire troposphere and computation of surface integrals concerning the exchange of heat, moisture, and momentum between the storm and its surroundings. It was found that a net heat export occurred indicating a net tropospheric drying and cooling after inclusion of radiation. The inward transport of relative angular momentum was insufficient to balance the flux of angular momentum to the ground as estimated from calculations for the mean hurricane. These computations then suggest weakening of the disturbance with time, and this actually occurred.

1. INTRODUCTION

From the mean distribution of temperature, wind, and moisture in hurricanes, Palmén and Riehl [1] have made estimates of the heat and energy budget in these cyclones; they have also considered the balance of absolute angular momentum. Due to lack of a sufficiently dense network of radiosonde and rawin observation stations around tropical storms it has never been possible, so far, to compute heat and momentum exchange between hurricanes and their surroundings on a synoptic basis. In this respect, a new opportunity for computations arose when, on September 2, 1958, hurricane Ella moved near the center of the network of upper-air stations southeast of Florida.

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[1] Performed this work while assigned to the University of Chicago and the U. S. Weather Bureau.

[2] Participated under contracts between the Office of Naval Research and the U. S. Weather Bureau with the University of Chicago.

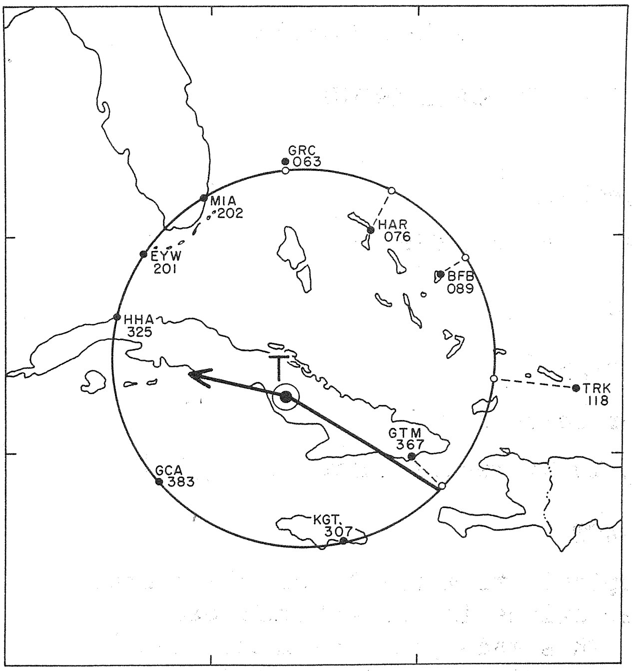

Figure 1. - Network of upper-air stations in the Bahamas circle.

Figure 1 shows the position of these stations and the path of Ella. The latter had attained hurricane strength while in the eastern Caribbean, but the eye was lost after crossing the mountains of eastern Cuba. This weakening occurred in the 12 hours preceding the time when the storm moved close to the middle of the station network of figure 1, which will be referred to as the Bahamas circle. The following calculations will be representative for the hurricane at this stage of diminished intensity, although it is probable that on the Bahamas circle - 4.5° latitude distant from the cyclone center - most of the features of the circulation were still indicative of hurricane structure.

2. STRUCTURE OF THE CIRCULATION

The basic quantities analyzed were the tangential wind component

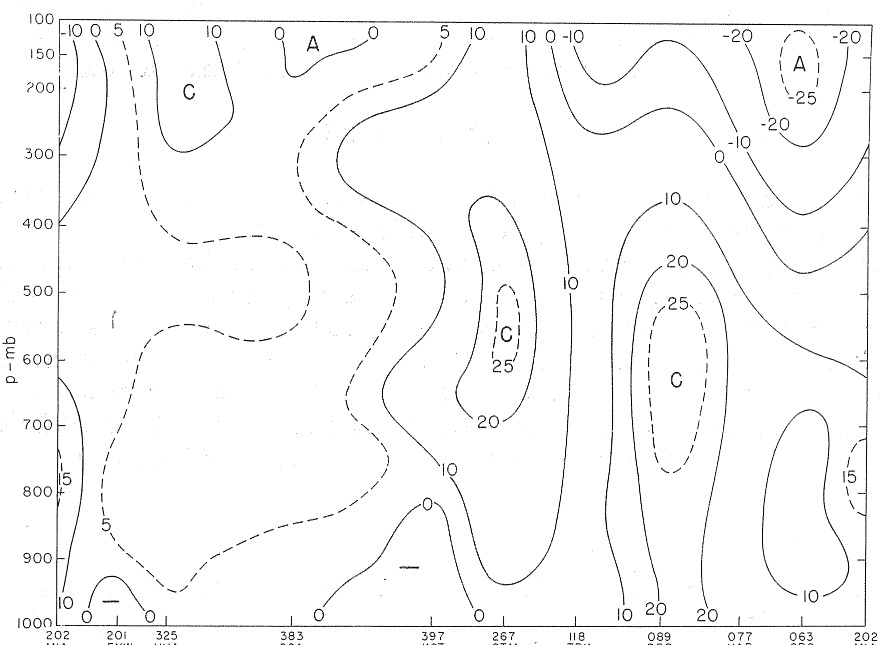

Figure 2. - Vertical cross section of the tangential component of motion (knots), September 2, 1958, 1200 GMT.

(v_θ, positive cyclonic), the radial wind component (v_r, positive outward), the specific humidity (q) and the enthalpy plus potential energy, denoted by $gz + c_pT$ where g is acceleration of gravity, z height, c_p specific heat at constant pressure, and T temperature. For each of the ten soundings on the Bahamas circle these quantities were plotted on diagrams with pressure as ordinate on a linear scale. After drawing curves mean values were obtained for layers of 100-mb. thickness from 1000 to 200 mb. and for layers of 50-mb. thickness from 200 to 100 mb. Vertical cross sections were then prepared for each of the four variables of which those for the wind components and moisture are shown in figures 2-4. As generally found in the Tropics, variations of gz and c_pT were small, and this cross section has therefore not been included.

The tangential component (fig. 2) exhibited considerable fluctuations around the circle. An axially symmetric circulation did not exist, even though the mean tangential component averaged around the circle ($\overline{v}_\theta$) may be presumed to be in quasi-gradient equilibrium. It is apparent, however, that the circulation was predominantly cyclonic in low and middle troposphere, with strongest v_θ to the right of the direction of center propagation, in agreement with the asymmetry normally observed in moving cyclones. In the high troposphere, the flow became increasingly clockwise, with strongest anticyclonic motion in the sector from northwest to northeast of Ella.

Prior to preparation of the cross section for the radial component (v_r), the storm displacement (c) was subtracted vectorially from the v_r field to yield the radial component relative to the moving center (v_{rr}). Through this step one obtains the speed with which air actually approached or moved away from the center. Variations of v_{rr} around the circle (fig. 3) were even larger

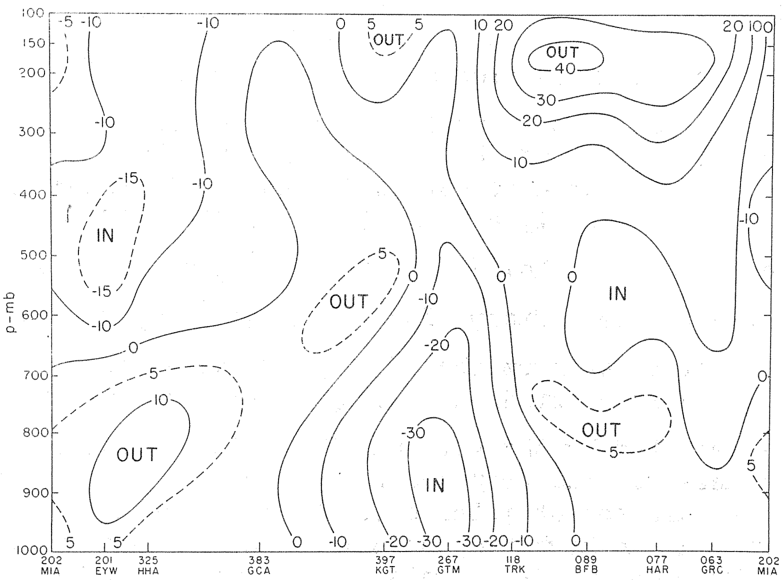

Figure 3. - Vertical cross section of the radial component of motion (knots), after vectorial subtraction of the storm displacement, September 2, 1958, 1200 GMT.

than those of v_θ, and this must be expected. While the individual wind vector is largely in gradient balance, the net radial component around a closed curve, in contrast to the net tangential component, must represent purely ageostrophic motion across the isobars. From figure 3 it is evident that all low-tropospheric inflow took place to the rear of the storm, while ahead of it there was a broad region of outbound motion even in the coordinate system moving with the center. Such an arrangement of v_{rr} is normally observed in hurricanes moving at a moderate speed in the Tropics; a general overtaking of the cyclone by the low-level flow is indicated. Most outflow occurred northeast of Ella, fairly well correlated with the area of anticyclonic flow in figure 2.

Considering next moisture, the mean vertical distribution of specific humidity was at first ascertained by averaging the ten soundings in each layer of 100-mb. thickness. The mean moisture sounding was then subtracted from the individual soundings to yield the deviations from the mean at constant pressure; these deviations were plotted on the cross section and analyzed (fig. 4). This procedure is necessary if lateral moisture gradients are to be shown clearly. Normally the vertical specific humidity gradient is much larger than the horizontal gradient, so that sections prepared without removal of the mean indicate little except general upward decrease of humidity. In figure 4, as is reasonable, lateral moisture variations were largest at some distance above the ground where vertical motions are strong. They decreased both toward the ocean surface where the vertical motion vanishes and also toward the high troposphere where the saturation specific humidity becomes small. Outstanding is the fact that negative anomalies of specific humidity prevailed where air was flowing toward and positive anomalies where air was flowing away from the center. This reveals the action of the hurricane in building up a deep moist layer in the interior through convergence of the inflow which is what produces the observed anomaly pattern around the boundary.

The mean vertical distribution of $gz + c_pT$ around the circle (fig. 5) did not deviate notably from that normally found in the area under discussion during summer. After inclusion of the latent heat content Lq (L = latent heat of condensation) the dashed curve is obtained with a mid-tropospheric minimum which is typical of practically all tropical soundings (Riehl and Malkus [2]). Near the surface $Q_o = (c_pT + Lq)_o \approx 83$ cal. gm.$^{-1}$, a value which was nearly constant around the circle. Except for additions to Q_o in the interior of the circle due to heat transfer from the ocean - probably small in this instance because the hurricane-force winds had disappeared - moist adiabatic ascent at 83 cal. gm.$^{-1}$ represented by the vertical line with arrows in figure 5, is the warmest possible ascent. The tropospheric circulation cell must end at about the pressure where this ascent intersects the mean sounding because the height of the tropopause and the high tropospheric temperature structure remain virtually unchanged from day to day. This intersection occurred near 150 mb., and for this reason all of the following calculations will be confined to the troposphere below 150 mb. As evident from figures 2-3, tangential and radial circulations did not terminate there, which points to existence of a stratospheric cell higher up.

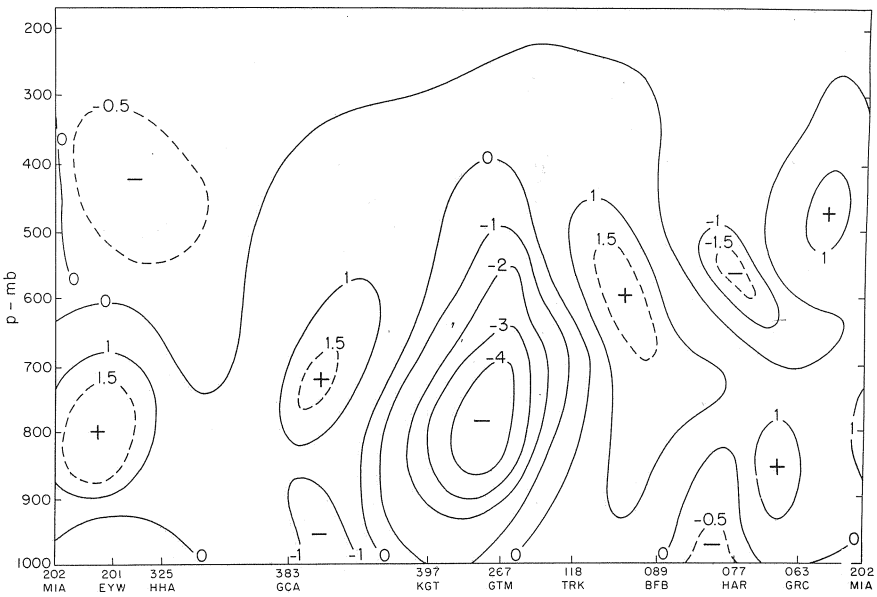

Figure 4. - Vertical cross section of the deviations of specific humidity ($gm.kg.^{-1}$) from the mean vertical distribution for the Bahamas circle stations, September 2, 1958, 1200 GMT.

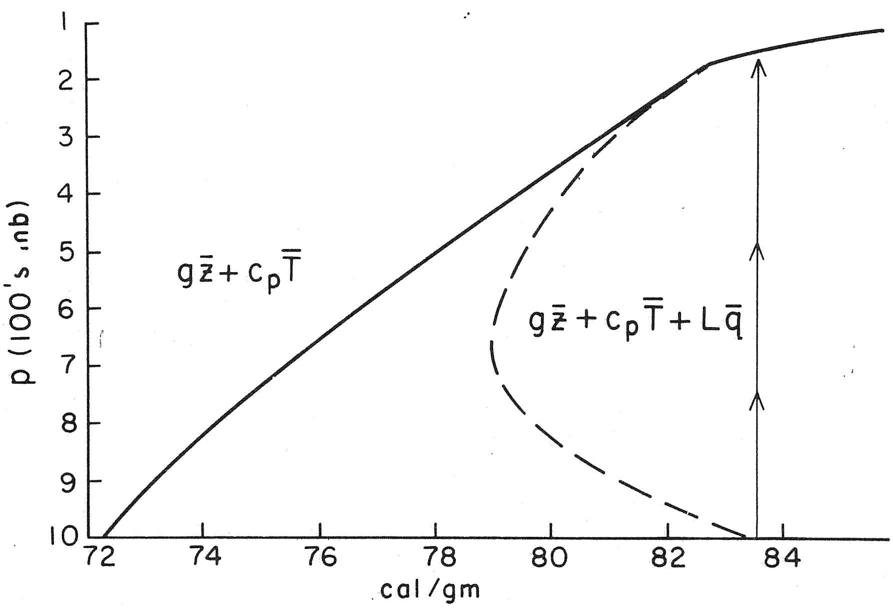

Figure 5. - Vertical profiles of $g\bar{z} + c_p\bar{T}$, and of $g\bar{z} + c_p\bar{T} + L\bar{q}$ ($cal.\ gm.^{-1}$) averaged around the Bahamas circle, September 2, 1958, 1200 GMT.

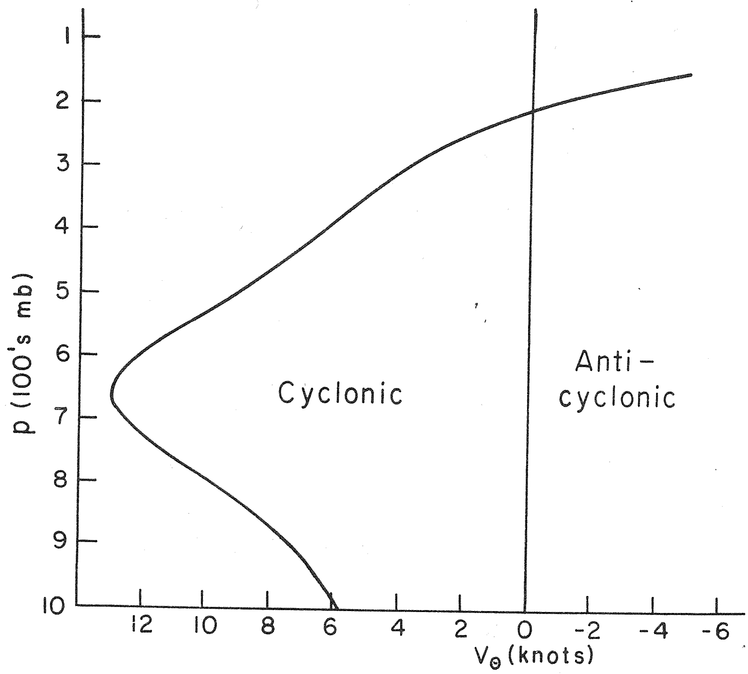

Figure 6. - Vertical profile of the tangential wind component (knots) averaged around the Bahamas circle, September 2, 1958, 1200 GMT.

The mean tangential circulation (fig. 6) was cyclonic through most of the troposphere, with net clockwise flow only above 200 mb. In this respect, the profile was more typical of large hurricanes than of smaller tropical storms where upper anticyclonic flow has been observed to extend through much deeper layers with speeds approximating those of the lower cyclonic flow. The fact that the circulation was strongest in the layer 700-600 mb. is not necessarily related to weakening of the hurricane core. Studies of individual rawin ascents around hurricanes have produced the impression that such a structure occurs frequently in the outskirts even of large hurricanes and typhoons.

The profile of the mean radial component $\overline{v}_r$ (or $\overline{v}_{rr}$), adjusted slightly so that mass inflow and outflow balance, is surprising in two respects (fig. 7). One is the depth of the inflow layer, the other the strength of the non-geostrophic wind especially in the high troposphere, where this component attained the magnitude of the total wind. Usually, the net ageostrophic component is a small residual of alternating indrafts and outdrafts which are individually quasi-gradient. Such alternation was indeed noted in figure 3. Nevertheless, the residual was large, hence may be considered as more reliable than usual in such calculations. Similar calculations were performed at 12-hour intervals while the hurricane was inside the circle. Three successive 12-hour periods yielded profiles almost identical to that of figure 6, with regular transition as Ella entered and left the Bahamas circle.

The high level of non-divergence was not expected from earlier studies (cf. Riehl [3]), but recent computations for Gulf of Mexico cyclones (Riehl [4]) have revealed similar mass circulation profiles. Because of the

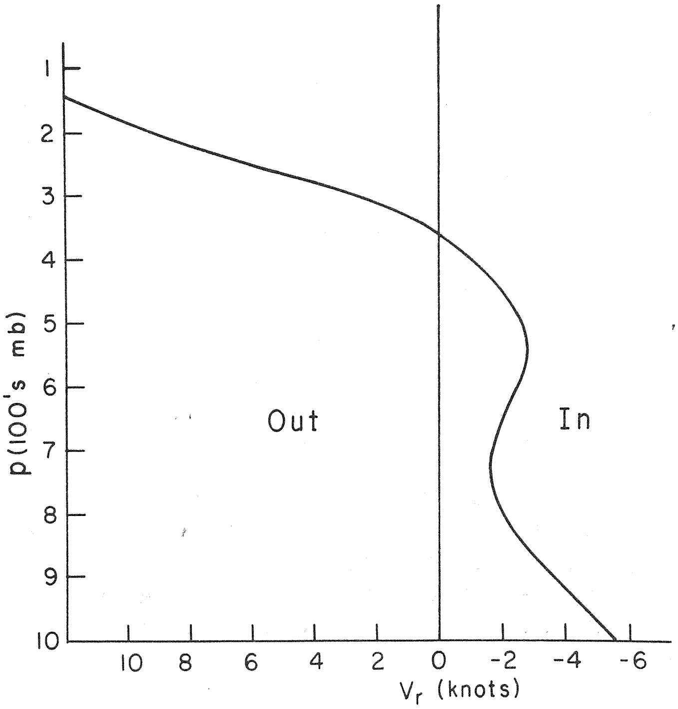

Figure 7. - Vertical profile of the radial wind component (knots) averaged around the Bahamas circle, September 2, 1958, 1200 GMT.

mid-tropospheric minimum of $gz + c_pT + Lq$ in figure 5, it was held previously that only low-tropospheric air with high heat content can penetrate into the interior of hurricanes if such storms are to be warmer than their surroundings. Recently, Simpson and Riehl [5] found from research aircraft flights that mid-tropospheric "ventilation" takes place even in the core of well-developed hurricanes; i.e., a wind component v_{rr} exists in the layer of minimum Q which brings some air laterally into the core which is not derived by ascent from the surface layer. Nevertheless, the mean component $\overline{v}_r$, averaged around circles from flight data near 600 mb., was small on these occasions compared to $\overline{v}_r$ in the surface layer, indicating that the earlier hypothesis on mass circulation is still largely valid.

It may be concluded that the bulk of mass entering the circle above 800 mb. in figure 7 did not penetrate into the central parts of the cyclone but that it ascended in the outskirts. This hypothesis is supported by research aircraft observations from three airplanes made in Ella by the National Hurricane Research Project shortly following the sounding period here analyzed. The observers (including one of the authors) found that the principal heavy rainbands were located far to the east of the center near longitude 77° - 76° W. Quantitative evaluation of the mass circulation from the aircraft traverses in low and middle troposphere cannot be made because the planes stayed on the eastern side of the center. At 235 mb., however, the B-47 jet airplane executed an irregularly shaped pentagon around the storm with mean radius of about 2° latitude. The net mass transport through this pentagon was almost zero, very different from figure 7.

From these and other data on the wind structure of hurricanes and near-hurricanes one obtains the impression that an intense and deep mass circulation commonly exists at a considerable distance from the center, and that only a small fraction of this mass penetrates to the core.

3. HEAT AND MOISTURE BUDGET

On September 2, 1200 GMT, Ella was located quite close to the geometric center of the Bahamas circle which is slightly off the northern coast of Cuba.

This position, plus the circular arrangement of the stations, suggests a cylindrical coordinate system (r, θ, z) for computing the interaction between the storm and its environment. Utilizing the relative framework discussed earlier, the export and import of heat, moisture, and momentum with respect to the travelling center can be obtained. In the relative coordinate system the heat transport through the cylinder is given by:

$$H = \iint (gz + c_p T)\, v_{rr}\, r\, d\theta \frac{dp}{g}, \tag{1}$$

and the moisture transport by:

$$M = \iint Lqv_{rr}\, r\, d\theta \frac{dp}{g}, \tag{2}$$

where p is pressure and a top has been assumed for the tropospheric circulation cell as already discussed. Much interest centers on the question whether fluxes are produced primarily by the ageostrophic mass circulation or by lateral asymmetries around the cyclone as apparent from figures 1-3. Denoting means at constant pressure around the cylinder with bars and deviations with primes, the moisture transport becomes:

$$M = \overline{M} + M' = \iint L\overline{q}\, \overline{v}_r\, r\, d\theta \frac{dp}{g} + \iint L\overline{q'v'_{rr}}\, r\, d\theta \frac{dp}{g}. \tag{3}$$

Similarly, the heat transport:

$$H = \overline{H} + H' = \iint (c_p\overline{T} + g\overline{z})\, \overline{v}_r\, r\, d\theta \frac{dp}{g} + \iint \overline{(c_p T' + gz')\, v'_{rr}}\, r\, d\theta \frac{dp}{g}. \tag{4}$$

Values of the deviations q' and v'_{rr} were determined for each layer of 100-mb. thickness (50 mb. for the top layer) from a grid of 15 points spaced evenly around the cylinder. Through numerical integration of equations 3 and 4 the following results were obtained (units 10^{14} cal. sec. $^{-1}$).

$\overline{M}$ = -1.4	$\overline{H}$ = 1.4	
M' = 0.2	H' = 0.1	M + H = 0.3
M = -1.2	H = 1.5	

Evidently the 'eddy' transports are very small compared to the transports by the mass circulation. However, since $\overline{M} + \overline{H} = 0$, the net export of heat arises only from the eddy transport term. One may question whether the net export can be accepted as reliable since neither heat nor moisture flow computations can be expected to be accurate to better than 10 percent, if that much. It may be noted, however, that the net radiation cooling of the vortex was

0.2×10^{14} cal. sec.$^{-1}$ assuming a mean cooling rate of 1.0°C. day^{-1}. Thus the calculated export has the magnitude of the cold source.

Another route may be employed to obtain a check on the validity of the moisture budget. If the time rate of change of total moisture in the vortex and the evaporation are small, the inward transport of moisture must equal the precipitation from continuity. Ordinarily no rainfall data exist over the oceans, so that the precipitation must be left as residual in moisture balance calculations. The area enclosed by the Bahamas circle, however, contains many islands reporting 6-hourly precipitation. Maps of 24-hour precipitation can be prepared and integrated by means of an equal-area grid after drawing isohyets. Such charts were made daily for August and September 1958. Of course, the margin of error in the analyses can be large, but in general the station density proved sufficient to give a good indication of the rainfall pattern.

The integrated precipitation depth for the interval September 2, 0000 GMT, to September 3, 0000 GMT, which is centered on the period of the computations, was 0.95 inch, compared to a computed rainfall of 1.08 inches from $M = -1.2 \times 10^{14}$ cal. sec.$^{-1}$. This is good agreement, especially when it is considered that an isohyetal map nearly always underestimates the actual precipitation. For comparison, integrated precipitation amounts were:

0.20 inch for September 1, 1200 GMT to September 2, 1200 GMT
0.97 inch for September 2, 1200 GMT to September 3, 1200 GMT
1.08 inches for September 3, 1200 GMT to September 4, 1200 GMT
0.14 inch for September 4, 1200 GMT to September 5, 1200 GMT

after the cyclone had left the Bahamas circle. It is seen that increase and decrease of average precipitation over the area took a very regular course as the storm first entered and later departed from the area. The good result may be ascribed to the fact that a large cyclonic wind system with an extensive precipitation shield was involved, rather than just local convection cells.

4. ANGULAR MOMENTUM TRANSPORT

Complete equations for the time rate of change of relative angular momentum with respect to the cyclone center $(v_\theta r)$ in a cylindrical coordinate system fixed in a moving storm have been worked out by Mihaljan[3]. These show that for the case of no net mass flow in or out of a cylinder the transport of absolute angular momentum through the cylinder is given by the transport of relative angular momentum measured with respect to the moving coordinate system, i.e.:

$$\frac{\delta\Omega}{\delta t} = -\iint (v_{\theta r} r)\, v_{rr}\, r\, d\,\theta\, \frac{dp}{g} \text{ plus other terms} \qquad (5)$$

depending on integration within the volume bounded by the cylinder. Here $\delta/\delta t$ denotes time differentiation following the moving storm and Ω the relative angular momentum integrated over the mass inside the cylinder. A complete

[3] M. Mihaljan, Department of Meteorology, University of Chicago, to be published.

momentum budget cannot be computed since this would require knowledge of the three-dimensional wind and mass field. Therefore, in all of the following, only the effects of interaction between the vortex and its environment through the boundary cylinder can be discussed.

We may note that $v_{\theta r} = v_\theta - c \sin \theta$, if θ is counted counterclockwise beginning with the direction from which the storm moves. Entering this definition in equation (5) and introducing mean values and deviations around the cylinder,

$$\frac{\delta \Omega}{\delta t} = - \bar{T}_\Omega - T'_\Omega + T_c + \ldots$$
$$= -\iint r \bar{v}_\theta \bar{v}_r \; r \, d\,\theta \, \frac{dp}{g} - \iint \overline{r \, v'_\theta v'_{rr}} \; r \, d\,\theta \, \frac{dp}{g} + \iint r \, c \, \overline{\sin \theta \, v'_{rr}} \; r \, d\,\theta \, \frac{dp}{g} \, . \quad (6)$$

Evaluation of equation (6) yields the following results (units 10^{23} g.cm.^{2}sec.$^{-2}$).

$-\bar{T}_\Omega = 5.4$	
$-T'_\Omega = 0.01$	$-\bar{T}_\Omega + T_c = 6.4$
$T_c = 1.0$	

There was net inward transport of relative angular momentum due to $\bar{T}_\Omega$ and T_c, while the contribution from T'_Ω was negligible. The term T_c is interesting in that it states explicitly the connection between the rate of center propagation and $\delta\Omega/\delta t$. T_c of course vanishes for stationary cyclones. For a moving disturbance, T_c will have a retarding effect on growth and maintenance of the system when net inflow occurs to the right and net outflow to the left of the direction of motion looking downstream, assuming sin θ is measured as stated above. In our case, low-level inflow and outflow had maxima just on the direction of motion (fig. 2), hence the low troposphere contributed little to T_c. The outflow occurred to the right of the direction of motion, and this position acted toward maintaining the storm's angular momentum. Since the outflow of hurricanes normally occurs in the same relative position as in this instance, the location of the clockwise outflow eddy as observed aids storm growth and maintenance from the momentum viewpoint.

In many situations the main inflow also takes place to the right of the direction of motion, and then the result will be different from that just obtained. It will be of interest in future studies to consider not only the total transport of T_c, but also its detailed distribution with height. If both lower inflow and upper outflow occur to the right of the direction of motion, the center displacement will act to equalize the angular momentum at all heights. Inflow and outflow to the left would act to produce an increasing vertical shear of v_θ, with little change of net vortex strength. Inflow to the left and outflow to the right would contribute to an increase of cyclonic momentum at all heights, and the opposite holds for inflow at the right and outflow at the left.

A complete balance of angular momentum referred to the storm center cannot be worked out for September 2, because this requires a volume distribution of data rather than just observations on one surface. It will be of interest, however, to see by means of a rough calculation, how the computed inflow compares with the transport of momentum to the ground by means of surface stresses. Direct calculation would require knowledge of the spatial distribution of v_θ at the surface; this information does not exist. We must resort to the stress calculation by Palmén and Riehl [1] for the mean hurricane; their figure 6 yielded a transport of 13.6×10^{23} g.cm.^{2}sec.$^{-2}$ to the ocean surface for the area bounded by the 4.5° radius. The difference between the mean hurricane and Ella consisted mainly in the absence of the central core of high speed in Ella. Beyond the 2° radius, the two circulations were quite comparable as far as one can judge. Further, Ella was partly over land, some of it mountainous. From Palmén and Riehl, most momentum loss to the ocean occurs outside the 2° radius in the mean hurricane because of small area and radius of the interior high-speed core. Hence it would seem conservative to assume a transport rate of three-quarters of that for the mean hurricane for Ella; the actual transport may well have been larger.

With this assumption, the transfer to the ocean becomes 10.2 units, leaving a net decrease of momentum of 3.8 units. If one assumes that time changes in v_θ are independent of the space coordinates, the mean vortex strength would decrease by about 2 knots per day from the fluxes through the cylinder and to the ground. This result should be compared with the actual mean vortex strength on September 2. On the boundary (fig. 5) the mean value of v_θ was 6 knots. From qualitative inspection of the research flights it is known that v_θ did not attain 50 knots except in a limited band in low and middle troposphere, and that at 235 mb. clockwise circulation prevailed over the whole interior. From this one may estimate that the mean vortex strength was certainly well below 20 knots, probably nearer 10 knots. It follows that the computed decrease in vortex strength was a significant fraction of the mean. Actually, Ella continued to lose intensity for at least one day after the period of calculation.

5. CONCLUSION

A fortunate distribution of upper-air observations has made it possible to show the structure of a tropical storm through the whole troposphere at a fixed radius from the center using synoptic data, and to determine the exchange of heat, moisture, and momentum between the storm and its large-scale environment. Considering structure, of particular interest are the maximum of tangential circulation near 700 mb., the strong ageostrophic components of motion with a high level of non-divergence above 400 mb., and the fact that the moisture content of the air was lower where air was moving toward the vortex in the low troposphere than where it was moving away.

Momentum and heat and moisture transport calculations pointed to an instantaneous decrease in vortex strength with time, in accord with the actual sequence of events; it must be noted, however, that the computed heat export was within the margin of error of the computations. While the equations discussed are not prognostic except by means of extrapolation, they nevertheless bring out important boundary effects influencing hurricanes. Considerable additional insight into the mechanisms governing the life cycle of hurricanes will be gained if calculations as here performed can be made successively at various stages of hurricane growth and decay, and if they can also be performed at several cylinders around a storm instead of just a single cylinder.

ACKNOWLEDGMENT

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